# Calibration of the two microphone transfer function method to measure acoustical impedance in a wide frequency range

Rene Boonen, Paul Sas,
*K.U.Leuven, Department of Mechanical Engineering, Celestijnenlaan 300 B, B-3001, Heverlee, Belgium*
*email: rene.boonen@mech.kuleuven.be*

## Introduction

In many acoustical simulations, particularly when using models with lumped elements or electrical analog circuits, the acoustical impedance of a component needs to be determined accurately. A widely used acoustical impedance measurement method is the "two microphone transfer function method", which is standardized in ISO-10534-2.

When the acoustical impedance is needed over a wide frequency band, for example from 10Hz to 10kHz, this method shows limitations. In this paper, a new calibration method has been proposed such that acoustical impedances can be measured with high accuracy over a wide frequency range. First, the estimation of the speed of sound has been eliminated. Next, by calibration at two different reference sections, the calibration of the sensor mismatch has become superfluous. Exchanging positions of the sensors is not necessary. Finally, a recursive procedure has been proposed to refine the microphone position calibration [1]. As result, the calibration procedure has been reduced to the accurate determination of the sensor positions.

## Principle of the two microphone transfer function method

In this section, the two microphone transfer function method is discussed. Figure 1 presents the set-up for acoustical impedance measurement. It consists of an acoustic wave guide with at the left end an excitation source and at the right end the impedance $Z_l$ to be measured. This impedance includes everything present at the right side of the reference section. Two microphones at two distinct positions $x_1$ and $x_2$ measure the sound pressure inside the duct. From the transfer function between the two microphones, the reflection coefficient and consequently, the unknown connected impedance will be determined.

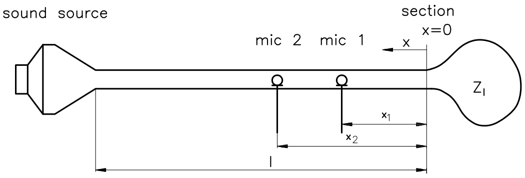


**Figure 1:** Wave guide closed with unknown acoustical impedance $Z_l$.

The transfer function $T_{12}$ between the pressures at the positions $x_1$ and $x_2$ in terms of the wavenumber $k$ is:

$$T_{12} = \frac{p(x_1,k)}{p(x_2,k)} = \frac{e^{j\,k\,x_1} + \Gamma_l\, e^{-j\,k\,x_1}}{e^{j\,k\,x_2} + \Gamma_l\, e^{-j\,k\,x_2}} \tag{1}$$

From the load reflection coefficient $\Gamma_l$ the load impedance $Z_l$ is then determined:

$$Z_l = Z_0\,\frac{1+\Gamma_l}{1-\Gamma_l} = j\,Z_0\,\frac{\sin\,k\,x_1 - T_{12}\,\sin\,k\,x_2}{\cos\,k\,x_1 - T_{12}\,\cos\,k\,x_2} \tag{2}$$

## Calibration of the set-up according to ISO 10534-2

Equation (2) becomes inaccurate when the reflection coefficient $\Gamma_l$ approaches unity. Therefore, prior calibration of the set-up is necessary to obtain accurate results.

The ISO 10534-2 standard prescribes the following calibration actions:

- The velocity of sound needs to be determined accurately using measurements of ambient temperature and atmospheric pressure.
- The distance between the pressure sensors need to be measured accurately.
- The mismatch between the amplitude and phase of the pressure sensors needs to be calibrated. In short, the procedure is to measure the transfer function $T_{12}$ of the two pressures at position $x_1$ and $x_2$, then interchange the two pressure sensors from location $x_1$ to $x_2$ and from $x_2$ to $x_1$ respectively, measure the transfer function $T_{21}$ and calculate the calibration factor $\delta$ such that:

  $$\delta^2\,T_{12}\,T_{21} = 1 + 0\,j \qquad (j = \sqrt{-1}) \tag{3}$$

  This calibration factor $\delta$ is complex and frequency dependent.

The ISO-calibration method does not take the distance error into account when interchanging the sensors. The consequence is that the measurement range is limited, as well in frequency as in impedance amplitude and phase [1]. To demonstrate this, a simulation was carried out wherein an error of 0.5 mm was introduced when exchanging the sensors. The result is presented in figure 2. The calibration of the sensor mismatch shows errors at frequencies above 150 Hz and consequently the impedance measurement range is limited to 35 dB around the wave guide characteristic impedance until 400 Hz.

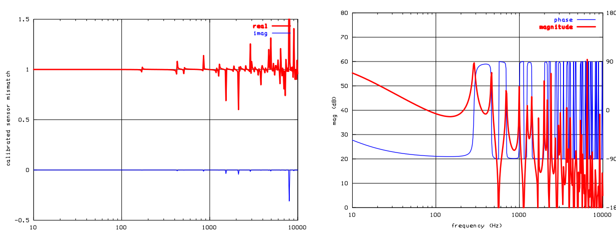

**Figure 2:** *left:* calibration result of sensor mismatch; *right:* resulting closed end impedance; when applying the ISO 10534-2 standard.

## Improved calibration

In the new method, it is sufficient to calibrate the sensor positions, expressed as the travelling times of the acoustic waves from the respective sensor positions to the reference section. Therefore, two transfer functions are measured between the two sensor outputs with the duct closed at the reference section and then with the shifted reference section, as shown in figure 3.

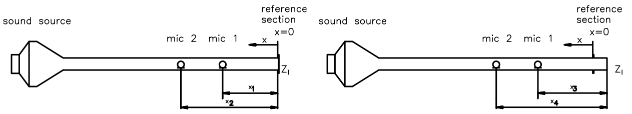


**Figure 3:** *left:* calibration setup with duct end closed at reference section and *right:* closed at shifted reference section.

The speed of sound is eliminated by replacing $k\,x_i$ by $\omega\,t_i$, $(i = 1, 2, 3, 4)$, in expression (1), wherein $t_1$, $t_2$, $t_3$ and $t_4$ are the times the wave needs to travel the distances $x_1$, $x_2$, $x_3$ and $x_4$ respectively.

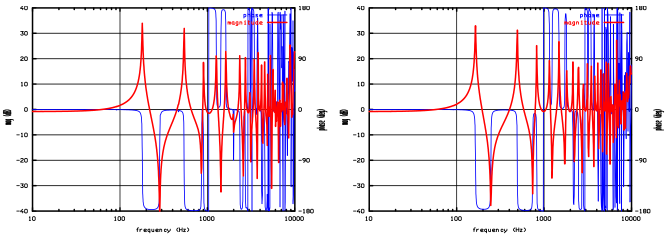

**Figure 4:** *left:* measured tranfer function between the sensors with duct closed at reference section and *right:* duct closed at shifted reference section.

These traveling times $t_1$, $t_2$, $t_3$ and $t_4$ are determined from the measured transfer functions, which are presented in figure 4. The traveling times $t_1$, $t_2$, $t_3$ and $t_4$ are:

$$t_1 = \frac{1}{4\,f_1}\,, \quad t_2 = \frac{1}{4\,f_2}\,, \quad t_3 = \frac{1}{4\,f_3} \text{ and } \quad t_4 = \frac{1}{4\,f_4} \qquad (4)$$

wherein the frequencies $f_1$, $f_2$, $f_3$ and $f_4$ correspond to the frequencies determined by the quarter wavelength between the reference section and the corresponding sensor positions.

The next step is the division of the two transfer functions. The sensor mismatch is then removed from the resulting transfer function, presented in figure 5left. This results in:

$$T = \frac{Z^2\,\cos\,\omega\,t_1\,\cos\,\omega\,t_4 - Z_0^2\,\sin\,\omega\,t_1\,\sin\,\omega\,t_4 - j\,Z\,Z_0\,\mathit{sin}\,\omega\,t_1\,\cos\,\omega\,t_4 - j\,Z\,Z_0\,\mathit{cos}\,\omega\,t_1\,\sin\,\omega\,t_4}{Z^2\,\cos\,\omega\,t_2\,\cos\,\omega\,t_3 - Z_0^2\,\sin\,\omega\,t_2\,\sin\,\omega\,t_3 - j\,Z\,Z_0\,\mathit{sin}\,\omega\,t_2\,\cos\,\omega\,t_3 - j\,Z\,Z_0\,\mathit{cos}\,\omega\,t_2\,\sin\,\omega\,t_3} \qquad (5)$$

wherefrom the closed end impedance $Z$ will be calculated. This result (see figure 5right) determines the measurement range, and must be as large as possible.

Often, the travelling times $t_1$, $t_2$, $t_3$ and $t_4$ are still not sufficiently accurate when determined from the equations (4). The impedance is extremely sensitive to errors in the travelling times. Therefore, the acoustical impedance $Z$ of the closed duct itself will be used to refine these travelling times.

Figure 6 presents the situations wherein respectively the travelling times $t_1$ and $t_2$ will be corrected. When a pressure maximum occurs at one of the microphone positions, the gradient of the pressure is zero and a small deviation of the microphone position does not affect the accuracy of the result. The other microphone position, where the gradient is generally not zero, determines fully the observed impedance. So, the deviation of the microphone position can be accurately determined.

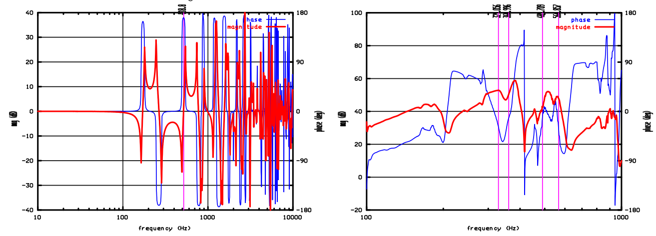

**Figure 5:** *left:* division of the two transfer functions; *right:* resulting closed end impedance.

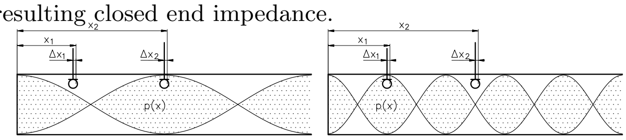


**Figure 6:** Refining situations for $x_1$ and $x_2$.

In this way, the correction of the travelling time $t_1$ will be determined when a half wave length stands at position $x_2$ (see figure 6left). As result, a load impedance appears at the reference section which is composed of the hard wall impedance and a short piece wave guide with length $\Delta x$. Determining the travelling time $\tau_1$ in this short piece wave guide results in:

$$\tau_1 = \frac{t_2}{\pi}\,\arctan\,\frac{Z_0}{j\,Z} \qquad (6)$$

which is the correction on the travelling time $t_1$. The impedance $Z$ is the one calculated from equation (5). In the same way, the travelling times $t_2$, $t_3$ and $t_4$ will be corrected. This correction of the travelling times can be repeated until maximum accuracy has been obtained.

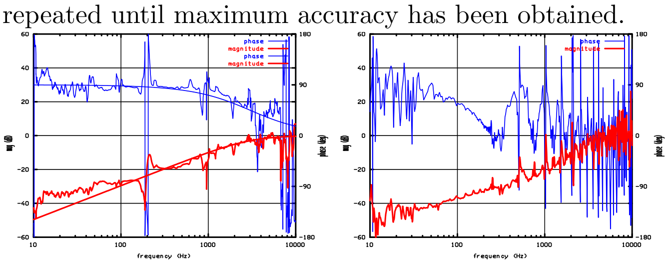

**Figure 7:** *left:* calibration result of sensor mismatch; *right:* resulting closed end impedance; after improved calibration.

Figure 7 compares the open end impedance, calibrated with the new method and the one calibrated with the ISO-method.

## Conclusion

This theoretical investigation about the calibration of the two microphone transfer function method has demonstrated that a small distance variation between the microphone position has a large effect on the impedance accuracy. A new improved calibrition method has been proposed to calibrate the sensor position deviation. The next step in the research is to correct for finite sensor dimensions.

## References

[1] Boonen R., Sas P., "Calibration of the two microphone transfer function method to measure acoustical impedance in a wide frequency range", proc. of the ISMA2004 Conference, (2004), Leuven, Belgium, pp. 325-336 (available at *www.isma-isaac.be*)