# Prediction of Sound Transmission Loss of Multi-layered Small Sized Elements

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## Introduction

Often it is a problem to get a good estimate by prediction of the sound transmission loss $R$ of multi-layered structures, like glazing, panels, etc..

A practical solution could be based on the transfer matrix approach for infinite plates for the prediction of the transmission coefficient $\tau$ that is a function of the angle of incidence of sound. Since in reality a more or less diffuse sound field exists in rooms, we have random incidence. If an ideal diffuse sound field is assumed the prediction will underestimated grossly the sound transmission loss due to low values for grazing incidence. Beranek [1] suggests to limit the angles of incidence up to 78° for limp single layer panels. For other types of structures the cut-off angle usually varies between 70° to 85° to get a good estimate of the single number rated value, but for the frequency dependence of stiffness controlled panels this ad-hoc correction does not give a satisfactory agreement at all. To improve the prediction, an approach that considers the finite size of the structure is combined with a non-uniform directional distribution of incident power.

## Prediction of transmission coefficient $\tau$

### Sound transmission coefficient $\tau_{inf}(k_0,\phi)$

A transfer matrix approach is used to predict the transmission coefficient $\tau_{inf}(k_0,\phi)$ of an multi-layered infinite structure that is a function of the angle of incidence $\phi$ of sound and of the wavenumber $k_0$ in air [2]. For each layer the equation of motion is set-up, like the Helmholtz equation for fluids, the Kirchhoff equation for thin plates and Biot theory for porous absorbers. If a fibre material is used as porous absorber then usually it is sufficient to assume a fluid layer with complex equivalent sound speed and density. A set of linear equations is found by applying continuity and equilibrium conditions at the boundary layers. They have to be solved to get $\tau_{inf}(k_0,\phi)$.

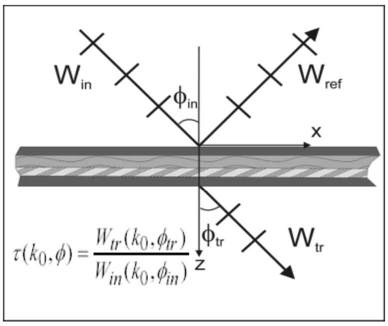


**Figure 1**: Transmission coefficient $\tau_{inf}(k_0,\phi)$

### Averaging over angle of incidence $\phi$

$\tau_{inf}(k_0,\phi)$ is averaged over $\phi$ to get the transmission loss $\tau$ for random incidence like in reality by using equation (1).

$$\tau(k_0) = \frac{\int_0^{\pi/2} \tau_{\inf}(k_0,\phi)\sin(\phi)\cos(\phi)\,D(\phi)\,d\phi}{\int_0^{\pi/2} \sin(\phi)\cos(\phi)\,D(\phi)\,d\phi} \qquad [-] \qquad (1)$$

$D(\phi)$ is a weighting function for the distribution of the incident power. In case of a diffuse field $D(\phi)$ is unity for all $\phi$ and for Beranek's field incidence it is unity up to the cut-off angle $\phi_{cut\text{-}off}$ and for bigger values it is zero. Kang [3] investigated the distribution of incident power on a surface for rooms of different shape with ray-tracing models. He found that for reverberant chambers the Gaussian distribution of equation (2) approximates the incident power best if $\beta$ is in the range between 1 and 2.

$$D(\phi) = e^{-\beta\,\phi^2} \qquad [-] \qquad (2)$$

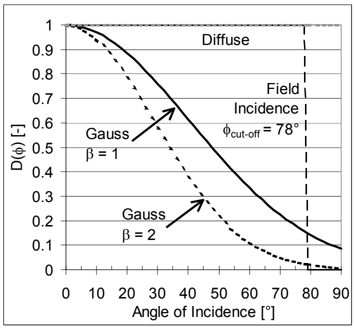


**Figure 2:** Weighting function $D(\phi)$ for directional distribution of energy of incident sound field

### Spatial Windowing

Villot [4] presents a method to calculate the transmission coefficient $\tau_{fin}$ of a finite structure for a oblique angle of incidence as given by equation (3).

$$\tau_{fin}(k_0,\phi) = \tau_{\inf}(k_0,\phi)\cdot\left[\sigma(k_0,\phi)\cdot\cos(\phi)\right]^2 \qquad [-] \qquad (3)$$

The first factor on the right hand side is found with the transfer matrix method for an infinite structure and depends only on the structure itself. The second factor is a function of geometry only and is governed by the forced radiation efficiency $\sigma(k_0,\phi)$ for an oblique angle of incidence. $\sigma(k_0,\phi)$ can be found by assuming an infinite structure that is on one side partially excited with airborne sound over a finite area of length $L_x$ and width $L_y$ and also radiates sound by this area only on the other side of the structure. Novak [5] gives $\sigma(k_0,\phi)$ for this situation in terms of a triple integral.

$$\sigma(k_0,\phi)=\frac{2k_0}{\pi^3 S}\int_0^{2\pi}\int_{k_x}\int_{k_y}\frac{1}{\sqrt{k_0^2-k_x^2-k_y^2}}\frac{\sin^2\left(\left(k_x-k_0\sin\phi\cos\theta\right)L_x/2\right)}{\left(k_x-k_0\sin\phi\cos\theta\right)^2}\cdot\frac{\sin^2\left(\left(k_y-k_0\sin\phi\sin\theta\right)L_y/2\right)}{\left(k_y-k_0\sin\phi\sin\theta\right)^2}dk_x dk_y d\theta \quad (4)$$

$S$ is the area of the excited surface of the structure and integration is performed over all azimuth angles of incidence $0 \le \theta \le 360°$ and over all wavenumbers $k_x$ and $k_y$ that radiate sound and thus have to meet the condition $k_x^2 + k_y^2 < k_0^2$. The great advantage of the method is that the triple integral can be evaluated independently from the structure for a given element size $(L_x, L_y)$ and set of discrete angles $\phi$ using numerical methods. After $\tau_{fin}(k_0,\phi)$ is computed equation (1) has to be applied to get $\tau_{fin}(k_0)$ for incident sound fields.

## Comparison of prediction and measurement

Sound transmission measurements have been carried out at different test specimen in a test facility with a small opening (1,25 m x 1,50 m) for glazings according to ISO 140. $\tau_{inf}(k_0,\phi)$ and $\sigma(k_0,\phi)$ has been computed for the dimensions of the opening at discrete angels with a spacing of 0.25°. The results of prediction and measurement for a double glazing that consists of two sheets of 6 mm float glass separated by 16 mm air are shown in figure 3 for the infinite and the finite case as well as for different directional distributions of incidence energy. For the infinite structure $R$ is shown for the field incidence case of Beranek, an ideal diffuse field and the Gaussian distribution $(\beta = 1)$. The latter two underestimate $R$ in the whole frequency range below the coincidence frequency $f_c$. The field incidence case first approaches the results of the Gaussian distribution and the diffuse field respectively, then increases rapidly resulting in overestimation just below coincidence and approaches the others again at higher frequencies.

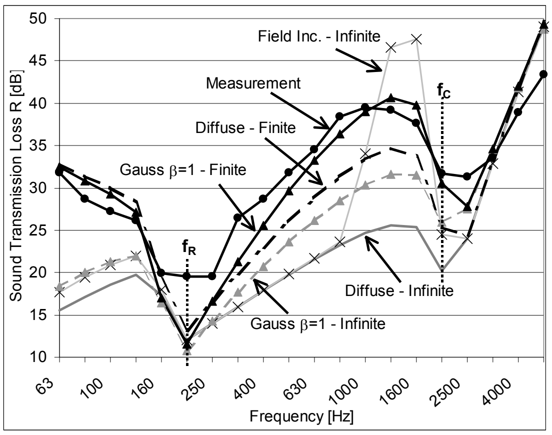


**Figure 3:** *R* of a double glazing (glass-6mm air-16mm glass-6mm)

For the finite plate $R$ is only presented for the diffuse field and the Gaussian distribution. The agreement with experiment of both is good in the very low frequency range below the mass-spring-mass resonance $f_R$. For the diffuse field and finite size $R$ is bigger than for infinite extension but still underestimates experiment between $f_R$ and $f_c$. The prediction with the Gaussian distribution agrees well with measurement well above $f_R$, but close to the resonance it also underestimates $R$ like all the other cases.

The second test specimen is a panel (1,20m x 1,45m) made from a single layer of plasterboard (12mm) that is mounted to a metal frame (75mm CW/UW-channels) on both sides. The 75mm deep cavity is empty. The results are presented in figure 4. Again the method of an infinite plate with a diffuse sound field underestimates $R$ grossly. At low frequencies the prediction is improved by considering the finiteness of the structure and between $f_R$ and $f_c$ by the Gauss distribution of incident power. Again agreement is poor close to $f_R$.

## Conclusions

A good agreement of predicted and measured sound transmission loss $R$ was obtained with a transfer matrix approach for an infinite plate taking into account the finiteness of the structure was by spatial windowing afterwards and approximating the distribution of incident power by a Gaussian distribution. The finite size of the structure has large effect on $R$ at low frequencies and at angles close to grazing incidence. Only close to the mass-spring-mass resonance the agreement of prediction and measurement was bad. Probably this is caused by the damping of the spring that is neglected in the model so far.

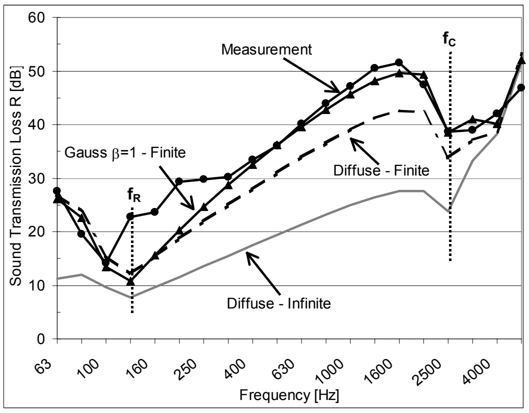


**Figure 4:** *R* of a gypsum board panel with a 75mm air cavity

## Literature


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