Decompositions for Aeroacoustic Simulations in Complex Domains

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Introduction

For CAA, an accurate and feasible direct simulation that considers both the generation of sound and its propagation into the far-field is hard to realize with one numerical method in a single computational domain. However, a direct approach contains automatically the interaction of the acoustic perturbations with the flow-field, a property which lacks the popular acoustic analogy models. The proposed method is basically a direct simulation, but it simplifies the problem that has to be solved for individual regions in the computational domain. The idea - to use a non-overlapping domain decomposition method where the equations, methods, grids and time steps are adapted to meet the local requirements (Utzmann et al. [1]) - is taken some steps further. The coupling method is very general and has also been applied to a coupling between a direct numerical simulation (DNS) code with an acoustic solver (Babucke et al. [2]).

Numerical Methods

In the domains, the Navier-Stokes equations, the nonlinear Euler equations and the linearized Euler equations (LEE) are solved by a variety of methods. New methods such as arbitrary high-order finite volumes (FV) on unstructured grids (Dumbser et al. [3]) and Lax-Wendroff like arbitrary high-order finite differences (FD) on structrured grids (Lörcher et al. [4]) as well as known methods such as the ADER discontinuous Galerkin (DG) schemes (Dumbser et al. [5]) and the ADER finite volume schemes (Schwartzkopff et al. [6, 7]) are available in the decomposition framework. All methods ensure excellent wave propagation capabilities through their high-order implementation.

Domain Decomposition

The coupling mechanism is able to maintain high order of accuracy globally. Two or more different domains Ω_i are coupled at their common boundary $\partial\Omega = \Gamma$ over the data in the ghostelements. Depending on the discretization method, this element can be a ghostcell (finite volume and discontinuous Galerkin methods) or a ghostpoint (finite difference methods). The data between the domains are exchanged by interpolating the values from the neighbor-grid onto either the Gauss integration points of the ghost cells (FV and DG methods) or onto the ghostpoints (FD methods) themselves. For symmetry reasons, uneven interpolation orders are preferred. A subsequent integration in order to obtain mean-values (FV) or a mapping onto the degrees of freedom (DG) can follow. If a domain couples with a DG domain, no interpolation is needed, as the solution is represented by a polynomial inside each domain. Hence, this polynomial can be evaluated at each position that is needed. Domains with completely different time steps are allowed in order to use the largest time step possible in each domain. For this subcycling, the so-called Cauchy-Kovalevskaja procedure takes a key position (Utzmann et al. [1]). It answers the question how to treat the ghostelements of the domain with the smaller Δt, which have to be provided with an updated value before the domain can proceed with its time stepping. By replacing time derivatives with spatial derivatives, a Taylor series in time can be used to calculate the state in a ghostelement at an arbitrary time. If there are more subcycles of a domain until the next common timestep is reached, the coefficients of the Taylor series can be stored.

Example: Von Karman Vortex Street

The versatility and feasibility of the approach is shown for a 2D Von Karman vortex street (Fig. 1). The diameter of the cylinder is $D = 1.$, the freestream density, velocity and pressure are $\rho_\infty = 1.$, $u_\infty = 0.2$, $v_\infty = 0.$ and $p_\infty = 0.71428571428$. The dynamic viscosity is chosen as $\mu = 0.00133333$, hence the Reynolds number based on the diameter is $Re = 150$. At this Reynolds number, vortex pairs are shed periodically from the downstream side of the cylinder and the laminar flow is basically two dimensional. This case has been studied extensively in the past, so the results can be compared with both numerical (NASA [9]) and experimental data (Roshko [8]). The contour plot of the pressure in Fig. 1 shows a close up of the calculation domain and its composition of different domains, methods, grid types and orders of accuracy. A zoom into the actual grids is plotted in Fig. 2. The unstructured domain contains triangular elements, the structured ones contain cartesian elements. The overall calculation domain extents are $[1200 \times 1200]$, while the unstructured inner region around the cylinder is only

Table 1: CPU times, area fractions, elements and subcycles.

Domain	CPU(%)	Area(%)	Elements	$\Delta t/\Delta t_{01}$
01	38.2	0.03	8054	1
02	21.8	0.67	38920	8
03	1.7	0.07	3920	8
04	27.9	2.83	160000	8
05	5.6	96.4	348358	32
Coupling	4.8	-	-	-
Total	100.0	100.0	559252	-

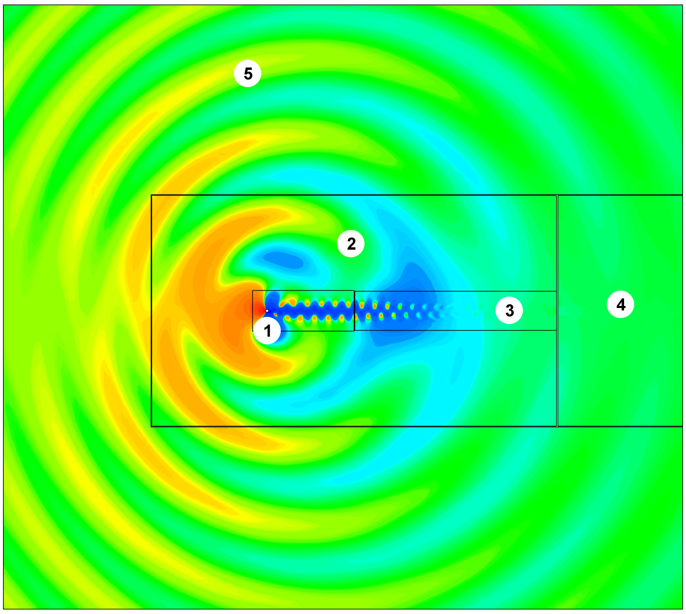

Figure 1: Von Karman vortex street at $t = 2500$; *Domain 1, cylinder:* Navier-Stokes equations, unstructured meshes, nonlinear FV scheme, $\mathcal{O}4$. *Domain 2, near field:* nonlinear Euler equations, structured mesh, FV scheme, $\mathcal{O}4$. *Domain 3, wake:* Navier-Stokes, structured mesh, FV scheme, $\mathcal{O}4$. *Domain 4, damping zone:* Navier-Stokes, structured mesh, FV scheme, $\mathcal{O}2$. *Domain 5, acoustic far field:* Linearized Euler equations, structured mesh, FD scheme, $\mathcal{O}8$.

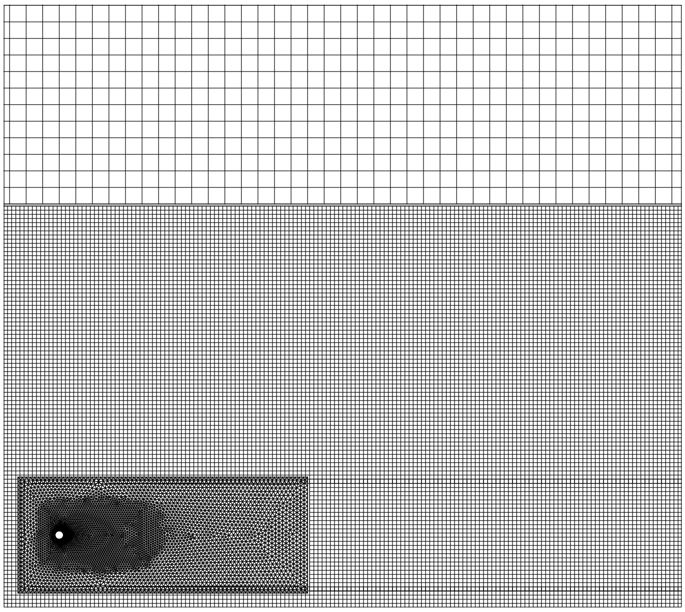

Figure 2: Grid topology: Depicted are the fine unstructured triangular grid around the cylinder, the coarser structured mesh in the near field / wake region and the coarse acoustic grid for the far field.

$[35 \times 14]$. See Table 1 for area fractions, CPU times and the number of elements. Also the time step ratios are given: The Δt's are 8 and 32 times larger in the structured regions than the Δt of the unstructured innermost domain. At the outer boundary of the acoustic far field domain, a simple sponge layer is employed to avoid reflections. The computation was performed on a single Intel Xeon 5150 2.66GHz core, the overall wall-clock time was 70507s. The simulation time is $t = 2500$, which is far beyond reaching periodicity of the emitted sound in the whole domain (after $t = 600$, the first acoustic waves reach the upper and lower domain boundary). The wavelength of the acoustic waves is determined by extracting the sound pressure along a line $y = [50, 550]$ in y-direction at $x = 0.$ in the far field. The wavelength $\lambda = 27.16$, which should correspond to the frequency of the laminar separation, translates to a Strouhal number of $Str = \frac{D \cdot f}{u_\infty} = 0.184$, which is in good agreement with the experimental and numerical references (Roshko [8]: Values range from $0.179 - 0.182$, NASA [9]: Values range from $0.150 - 0.183$). Note, that the *8th order* calculation in the far field is very inexpensive (5.6% of the CPU time for 96.4% of the total area, Table 1).

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