Error corrections for the calibration of the two microphone transfer function method for acoustical impedance measurement.

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Introduction

In many acoustical simulations, particularly when using models with lumped elements or electrical analog circuits, the acoustical impedance of a component needs to be determined accurately. A widely used acoustical impedance measurement method is the "two microphone transfer function method", which is standardized in ISO-10534-2.

When the acoustical impedance is needed over a wide frequency band, this method shows limitations. In this paper, a improved calibration method has been proposed such that acoustical impedances can be measured with high accuracy over a wide frequency range. First, the estimation of the speed of sound has been eliminated. Next, by calibration at two different reference sections, the calibration of the sensor mismatch has become superfluous. Exchanging positions of the sensors is not necessary. Finally, a recursive procedure has been proposed to refine the microphone position calibration [1]. As result, the calibration procedure has been reduced to the accurate determination of the sensor positions.

Principle of the two microphone transfer function method

In this section, the two microphone transfer function method including wave guide damping is discussed. Figure 1 presents the set-up for acoustical impedance measurement. It consists of an acoustic wave guide with at the left end an excitation source and at the right end the impedance Z_l to be measured. This impedance includes everything present at the right side of the reference section. Two microphones at two distinct positions x_1 and x_2 measure the sound pressure inside the duct. From the transfer function between the two microphones, the reflection coefficient and consequently, the unknown connected impedance will be determined.

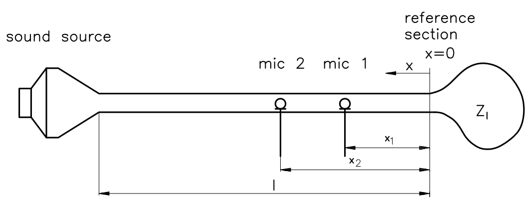

Figure 1: Wave guide closed with unknown acoustical impedance Z_l.

The transfer function T_{12} between the pressures at the positions x_1 and x_2 in terms of the propagation constant γ is:

$$T_{12} = \frac{p(x_1,\gamma)}{p(x_2,\gamma)} = \frac{\mathrm{e}^{j\,\gamma\,x_1} + \Gamma_l\,\mathrm{e}^{-j\,\gamma\,x_1}}{\mathrm{e}^{j\,\gamma\,x_2} + \Gamma_l\,\mathrm{e}^{-j\,\gamma\,x_2}} \tag{1}$$

From the load reflection coefficient Γ_l the load impedance Z_l is then determined:

$$Z_l = Z_0\,\frac{1+\Gamma_l}{1-\Gamma_l} = j\,Z_0\,\frac{\sin\,\gamma\,x_1 - T_{12}\,\sin\,\gamma\,x_2}{\cos\,\gamma\,x_1 - T_{12}\,\cos\,\gamma\,x_2} \tag{2}$$

Equation (2) becomes inaccurate when the reflection coefficient Γ_l approaches unity. Therefore, prior calibration of the set-up is necessary to obtain accurate results.

The ISO 10534-2 standard does not include wave guide damping effects and the deviation of microphone positions after interchanging them. This limits the accuracy of the method [1].

Improved calibration

In the new method, the sensor positions are calibrated by shifting the reference section. The distances are expressed as the travelling times of the acoustic waves from the respective sensor positions to the reference section. Therefore, two transfer functions are measured between the two sensor outputs with the duct closed at the reference section and then with the shifted reference section, as shown in figure 2.

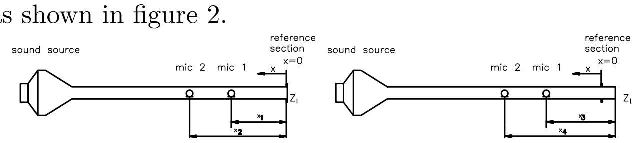

Figure 2: *left:* calibration setup with duct end closed at reference section and *right:* closed at shifted reference section.

The speed of sound is eliminated by replacing $\gamma\,x_i$ by $\beta\,t_i$, $(i = 1,2,3,4)$, wherein $\beta = \omega\,(1 - j\,\xi)$, with ξ the wave guide damping coefficient, in expression (1), wherein t_1, t_2, t_3 and t_4 are the times the wave needs to travel the distances x_1, x_2, x_3 and x_4 respectively.

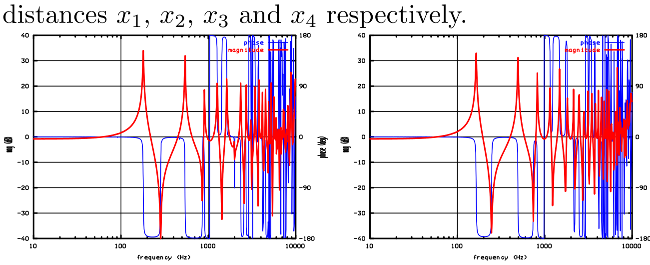

Figure 3: *left:* measured tranfer function between the sensors with duct closed at reference section and *right:* duct closed at shifted reference section.

These traveling times t_1, t_2, t_3 and t_4 are determined from the measured transfer functions, which are presented in figure 3. The traveling times t_1, t_2, t_3 and t_4 are:

$$t_1 = \frac{1}{4\,f_1},\quad t_2 = \frac{1}{4\,f_2},\quad t_3 = \frac{1}{4\,f_3}\ \text{and}\quad t_4 = \frac{1}{4\,f_4} \tag{3}$$

wherein the frequencies f_1, f_2, f_3 and f_4 correspond to the frequencies determined by the quarter wavelength between the reference section and the corresponding sensor positions.

The next step is the division of the two transfer functions. The sensor mismatch is then removed from the resulting transfer function, presented in figure 4left. This results in:

$$T = \frac{\begin{array}{c} Z^2 \cos\beta\, t_1 \cos\beta\, t_4 - Z_0^2 \sin\beta\, t_1 \sin\beta\, t_4 - \\ j\, Z\, Z_0\, sin\, \beta\, t_1 \cos\beta\, t_4 - j\, Z\, Z_0\, cos\, \beta\, t_1 \sin\beta\, t_4 \end{array}}{\begin{array}{c} Z^2 \cos\beta\, t_2 \cos\beta\, t_3 - Z_0^2 \sin\beta\, t_2 \sin\beta\, t_3 - \\ j\, Z\, Z_0\, sin\, \beta\, t_2 \cos\beta\, t_3 - j\, Z\, Z_0\, cos\, \beta\, t_2 \sin\beta\, t_3 \end{array}} \quad (4)$$

wherefrom the closed end impedance Z will be calculated. This result (see figure 6left) determines the measurement range, and must be as large as possible.

The wave guide damping will be estimated from the phase of the transfer function (4), assuming that the closed end impedance Z approaches ∞. Figure 4right presents the estimated phase compared to the measured phase of the transfer function presented in figure 3left. They match very well until 5 kHz.

Often, the travelling times t_1, t_2, t_3 and t_4 are still not sufficiently accurate when determined from the equations (3). The impedance is extremely sensitive to errors in the travelling times. Therefore, the acoustical impedance Z of the closed duct itself will be used to refine these travelling times.

Figure 5 presents the situations wherein respectively the travelling times t_1 and t_2 will be corrected. When a pressure maximum occurs at one of the microphone positions, the gradient of the pressure is zero and a small deviation of the microphone position does not affect the accuracy of the result. The other microphone position, where the gradient is generally not zero, determines fully the observed impedance. So, the deviation of the microphone position can be accurately determined.

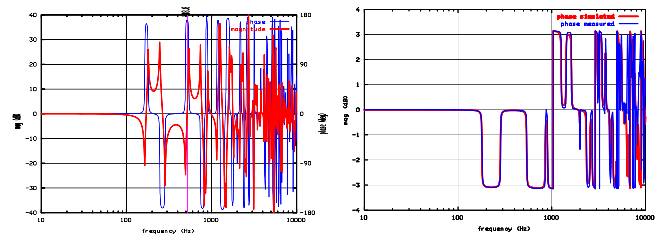

Figure 4: *left:* division of the two transfer functions; *right:* overlay of estimated and measured phase.

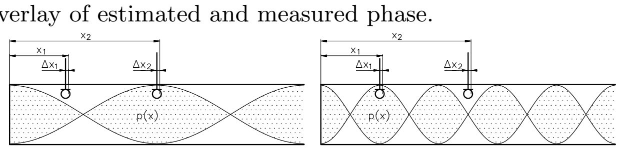

Figure 5: Refining situations for x_1 and x_2.

In this way, the correction of the travelling time t_1 will be determined when a half wave length stands at position x_2 (see figure 5left). As result, a load impedance appears at the reference section which is composed of the hard wall impedance and a short piece wave guide with length Δx. Determining the travelling time τ_1 in this short piece wave guide results in:

$$\tau_1 = \frac{t_2}{\pi} \arctan \frac{Z_0}{j\, Z} \quad (5)$$

which is the correction on the travelling time t_1. The impedance Z is the one calculated from equation (4). In the same way, the travelling times t_2, t_3 and t_4 will be corrected. This correction of the travelling times can be repeated until maximum accuracy has been obtained.

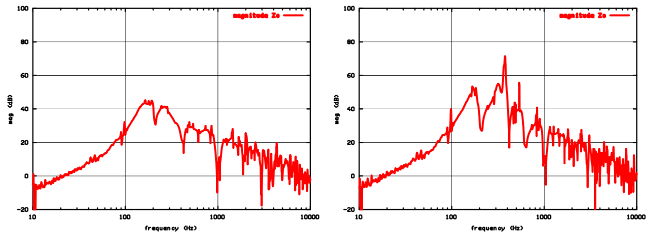

Figure 6: *left:* Closed end impedance resulting from transfer function measurements using expression (4), *right:* Closed end impedance resulting after refining.

As result, the closed end impedance increases, enhancing the measurement range (see figure 6right). The measurement range is now limited by the deviations between subsequent transfer function measurements. From simulations, presented in figure 7, a deviation of 0.15% in amplitude has already a large effect.

In order to improve accuracy, a third calibration at another reference section has been carried out. The singularities are now overlapped (until 1 kHz) and the low frequency performance improves. The resulting impedance is selected where the closed end impedance has its maximum amplitude. The resulted open end impedance, presented in figure 8, has been considerably improved.

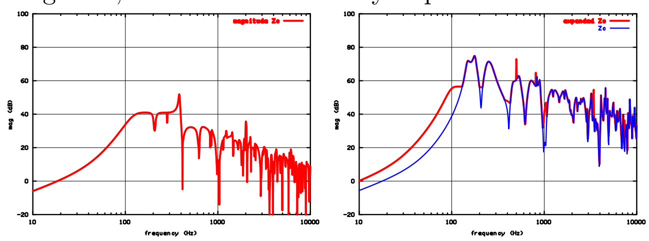

Figure 7: *left:* Simulated closed end impedance before refining. *right:* Simulated closed end impedance after refining (thin line), combined with third reference section (thick line).

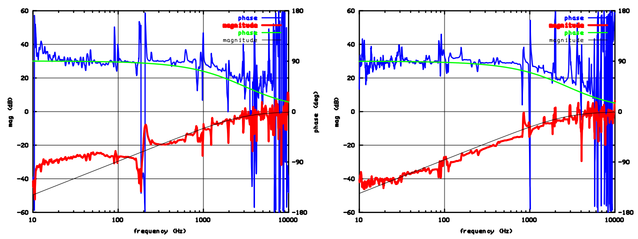

Figure 8: *left:* Resulting open end impedance using two reference sections, *right:* Resulting open end impedance using three reference sections. (Thin line is spherical radiator.)

Conclusion

An improved calibration method has been proposed. The method uses shifted reference sections to calibrate the microphone positions. Wave guide damping has been taken into account. A third calibration enhances the low frequency measurement range and removes several singularities.

References

[1] Boonen R., Sas P., Desmet W., Lauriks W., Vermeir G., "Calibration of the two microphone transfer function method to measure acoustical impedance in a wide frequency range", proc. of the ISMA2006 Conference, (2006), Leuven, Belgium, pp. 4501-4512 (available at *www.isma-isaac.be*)