Full Coupling of the Finite Element and Fast Boundary Element Method for Structural-Acoustic Problems

Dominik Brunner[1], Matthias Fischer[1], Christian Cabos[2], Lothar Gaul[1]

[1] *Institut für Angewandte und Experimentelle Mechanik, Universität Stuttgart, Email: brunner@iam.uni-stuttgart.de*

[2] *Germanischer Lloyd, Hamburg, Email: christian.cabos@gl-group.com*

Introduction

Often, structural-acoustic simulations can be performed neglecting the influence of the acoustic fluid onto the vibration behavior of the structure. In the case of fluids with high density, a strong coupling between structure and fluid has to be applied. The structural vibrations are simulated by the finite element (FE) method whereas the exterior acoustic problem is efficiently modelled by the boundary element (BE) method.

FE Formulation of the Elastic Structure

The structural part is assumed to be linear elastic. The FE formulation in the frequency domain reads

$$\mathbf{K}_{\mathrm{FE}}\,\mathbf{u} + \mathbf{C}_{\mathrm{FE}}\,\mathbf{p} = \mathbf{f}_{\mathrm{s}},$$

where $\mathbf{K}_{\mathrm{FE}}$ denotes the frequency dependent dynamical stiffness matrix, $\mathbf{u}$ and $\mathbf{f}_{\mathrm{s}}$ are the nodal displacements and nodal forces of the structure respectively and $\mathbf{p}$ are the nodal pressures of the acoustic fluid. The coupling matrix $\mathbf{C}_{\mathrm{FE}}$ takes into account the effect of the acoustic pressure onto the structure.
To obtain a versatile simulation tool, the mass- and stiffness matrices which set up $\mathbf{K}_{\mathrm{FE}}$ are imported from the commercial FE-program ANSYS. Thus, all available element types of ANSYS can be used for the simulation of the structure.

BE Formulation of the Acoustic Fluid

Starting point of the boundary element (BE) formulation of the fluid is the time harmonic Helmholtz equation. To overcome the non-uniqueness problem of exterior acoustic problems a Galerkin type Burton-Miller approach is used. On the coupling interface, the flux $q(x) = \frac{\partial p(x)}{\partial n}$ can be expressed by the structural displacements

$$q(x) = \rho_{\mathrm{f}}\,\omega^2\,u_n(x), \tag{1}$$

where ρ_{f} denotes the fluid density and u_n is the displacement in normal direction. Triangulation of the boundary leads to the system of equations

$$\underbrace{\left(\frac{1}{2}\mathbf{I} + \mathbf{K} + \frac{\mathrm{i}}{\kappa}\mathbf{D}\right)}_{\mathbf{K}_{\mathrm{BE}}}\mathbf{p}\,\underbrace{-\rho_{\mathrm{f}}\,\omega^2\left(\mathbf{V} + \frac{\mathrm{i}}{2\kappa}\mathbf{I}' - \frac{\mathrm{i}}{\kappa}\mathbf{K}'\right)}_{\mathbf{C}_{\mathrm{BE}}}\mathbf{u} = 0, \tag{2}$$

where $\mathbf{K}$ and $\mathbf{D}$ are matrices arising from the double layer potential and hypersingular operator and $\mathbf{V}$, $\mathbf{K}'$ consist of the single- and adjount double potential, respectively [2].

Fast Multipole Algorithm

Setting up and storing the BE matrix is of order $\mathcal{O}(N^2)$. To overcome this bottleneck, the fast multipole method is applied. One has to evaluate potentials of the type

$$\Phi(x_b) = \sum_{a=1}^{A} \frac{\mathrm{e}^{\mathrm{i}\kappa|x_b - y_a|}}{|x_b - y_a|} q_a, \tag{3}$$

where $|x_b - y_a|$ is the distance between source- and field-point and q_a is the source strength. This expression can be reformulated by means of the multipole expansion [1]

$$\Phi(x_b) = \frac{\mathrm{i}\kappa}{4\pi}\int_{S^2} \mathrm{e}^{\mathrm{i}\kappa(x_b - z_b)\cdot s} M_L(s, D) \sum_{a=1}^{A} \mathrm{e}^{\mathrm{i}\kappa(z_a - y_a)\cdot s} q_a \,\mathrm{d}s,$$

where $M_L(s, D)$ denotes the translation operator [2]. The efficiency can be further improved by applying a multilevel clustertree, leading to a fast multilevel multipole (FMM) algorithm. It has a quasi linear numerical complexity of order $\mathcal{O}(N \log^2 N)$.

Solution of the Coupled System

The coupled fluid structure problem can be written in matrix form as

$$\begin{pmatrix} \mathbf{K}_{\mathrm{FE}} & \mathbf{C}_{\mathrm{FE}} \\ \mathbf{C}_{\mathrm{BE}} & \mathbf{K}_{\mathrm{BE}} \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \mathbf{p} \end{pmatrix} = \begin{pmatrix} \mathbf{f}_{\mathrm{s}} \\ \mathbf{0} \end{pmatrix}. \tag{4}$$

By introducing the Schur-complement $\mathbf{S}$, the exact solution can be obtained by

$$\mathbf{p} = -\mathbf{S}^{-1}\mathbf{C}_{\mathrm{BE}}\mathbf{K}_{\mathrm{FE}}^{-1}\mathbf{f}_{\mathrm{s}} \quad \text{and} \tag{5}$$

$$\mathbf{u} = \mathbf{K}_{\mathrm{FE}}^{-1}(\mathbf{f}_{\mathrm{s}} - \mathbf{C}_{\mathrm{FE}}\mathbf{p}). \tag{6}$$

One should not apply a direct solver on the system (4), since this would be of order $\mathcal{O}(N^3)$ in case of fully polulated matrices. Instead of this, one can apply a GMRES, where the FMM approach computes a fast matrix-vector product. Two different solvers are investigated:

1. System (4) is solved by a single GMRES. Since preconditiong is essential, one can use the diagonal matrices $\mathbf{K}_{\mathrm{FE}}$ and $\mathbf{K}_{\mathrm{BE}}$ to construct a preconditioner. In case of $\mathbf{K}_{\mathrm{FE}}$ an incomplete LU (ILU) and a LU were applied, whereas a ILU was used to approximate the inverse of $\mathbf{K}_{\mathrm{BE}}$.
2. One can also apply a GMRES solving the reduced equation (5). The inverse of the Schur-complement can be approximated by $\mathbf{K}_{\mathrm{BE}}^{-1}$. The same preconditioner as mentioned above can be used. This approach is advantageous, if one can apply a direct solver for the FE-part, which is based on a factorization that can be reused at every GMRES iteration.

Example 1: Spherical Shell Structure

A spherical shell structure, totally submerged in water, which is driven by a unit point force at the north pole is investigated. For this problem an analytical solution is available in form of a series solution [3]. Figure 1 shows the analytical solution of the displacement at the north pole in comparison to the numerical solution of the FE/BE–approach.

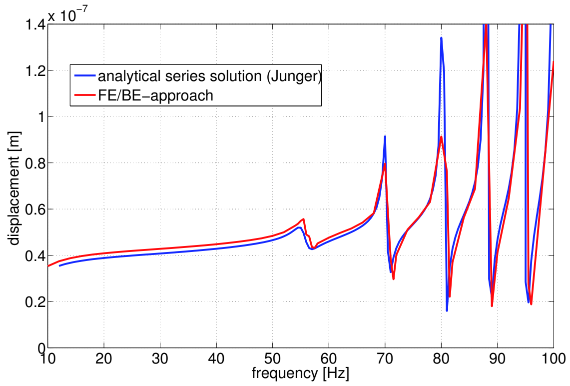

Figure 1: Analytical- versus FE/BE-solution. The sphere is modelled by 2,600 triangular SHELL63 elements.

Figure 2 compares the number of GMRES-iterations for the two solvers and different preconditioners. The implementation of the numerical package PETSc is used for the ILU/ LU solvers. The second solver employing a GMRES on the reduced system shows the best efficiency with respect to the number of iterations.

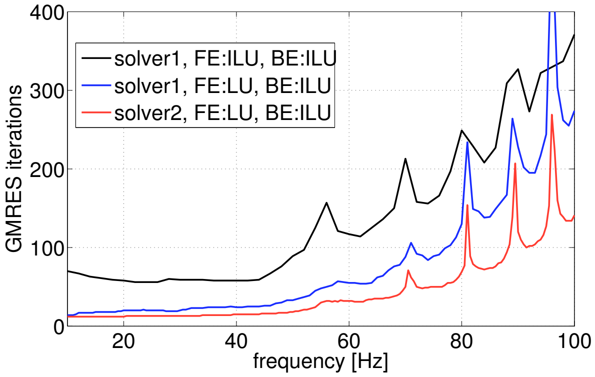

Figure 2: Number of iterations for the different solvers. In this case, the sphere is modelled by 1,300 triangular SHELL63 elements.

Example 2: Submarine–Like Structure

As a second more realistic example a submarine–like structure as depicted in Fig. 3 is investigated. It consists of a hull with semi-spherical caps and internal stiffeners. The structure is modelled by 10,500 SHELL63 quadrilateral elements. It is totally submerged in water. The six driving forces are chosen such that the system fulfills the static equilibrium condition.

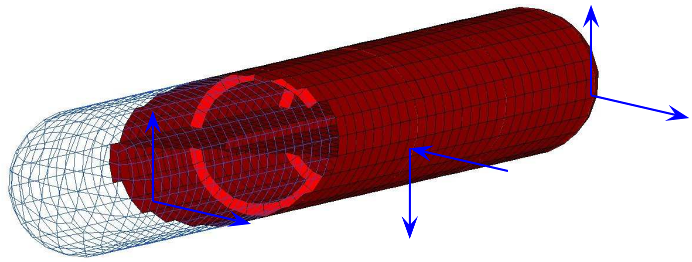

Figure 3: Submarine like structure driven by six point forces.

Two different scenarios are compared: First, the weakly coupled solution. In this case the effect of the acoustic pressure onto the vibrations of the structure is neglected. Secondly, the fully coupled solution. Here, the feedback of the pressure is taken into account. Figure 4 shows the two frequency sweeps for a symmetric point excitation as shown in Fig. 3. The absolute value of the displacement vector of the middle driving node is plotted. Due to the influence of the strong coupling, a significant frequency shift is observable.

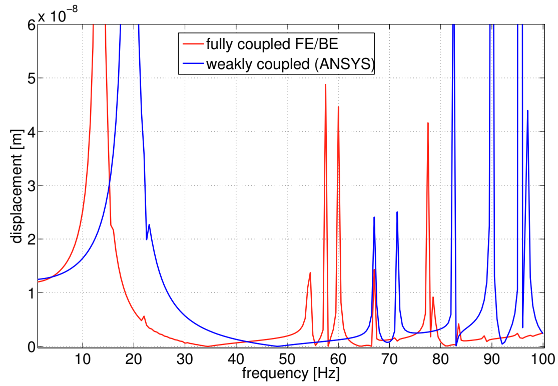

Figure 4: Weakly- versus fully coupled solution of the submarine like structure.

Conclusion

The effectiveness of the proposed FE-Fast BE approach could be shown with two examples. The first one shows the correctness of the numerical results, whereas the second one demonstrates the applicibility of the method for realistic problems. Due to the good complexity of the FMM algorithm, even frequency sweeps can be computed with this coupling scheme.

References

[1] Rokhlin, V.: Diagonal forms of translation operators for the Helmholtz equation in three dimensions. Applied and Computational Harmonic Analysis, 1(1993), 82-93

[2] Fischer, M: The Fast Multipole Boundary Element Method and its Application to Structure-Acoustic Field Interaction. Dissertation 2004, Institute of Applied and Experimental Mechanics, University Stuttgart

[3] Junger, C; Feit D.: Sound, structures, and their interaction. MIT Press, Cambridge, 1972