Nonlinear modeling of ultrasound contrast agents with Wiener series

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Introduction

Ultrasound contrast agents consist of microbubbles with diameters in the range of 1-5 μm, small enough to pass through the capillaries of the pulmonary system. To enable an examination time of several minutes and prevent them from dissolving, these bubbles need to be stabilised by a shell. The gas core of a microbubble is a strong scatterer of ultrasound because of the fundamentally different acoustic impedances of gas and water. Furthermore, these bubbles are resonant at frequencies used in diagnostic ultrasound imaging. The oscillation of a contrast agent microbubble is highly nonlinear, which can be exploited for detection purposes. In this paper, nonlinear modelling of ultrasound contrast agent with Wiener series is evaluated for the numeric evaluation of pulsing sequences for contrast agent detection.

Nonlinear modelling

To optimise detection schemes, it is necessary to model the oscillation behaviour of the contrast agent. Currently, contrast agents are modelled by nonlinear differential equations set up from physical insight into bubble oscillation and parameters determined from optical observations of single microbubbles. Simulating bubble behaviour with such a model, however, involves a high computational cost.

$$\begin{aligned} y[n] = h_0 &+ \sum_{k_1=0}^{\infty} h_1[k_1]x[n-k_1] \\ &+ \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} h_2[k_1,k_2]x[n-k_1]x[n-k_2] \\ &+ \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty} h_3[k_1,k_2,k_3]x[n-k_1]\times \\ &\qquad\qquad x[n-k_2]x[n-k_3] \\ &+ \dots . \end{aligned} \qquad (1)$$

Equation (1) presents a general black-box model for a nonlinear system with memory. The kernels h_n, which describe the system behaviour, can be determined from simulations or measured data with suitable identification algorithms, for instance as presented in [1]. Such a model then enables a computationally cheap evaluation of bubble behaviour and thus detection schemes.

Identification results

To evaluate pulsing schemes, a free gas bubble with a resting diameter of $r_0 = 0.75$ μm was simulated with a modified Rayleigh-Plesset equation as given in [4]. The parameters used for the model were $c = 1480\ \mathrm{m\,s^{-1}}$, $\mu = 10^{-3}\ \mathrm{Pa\,s}$, $\rho = 998\ \mathrm{kg\,m^{-3}}$, $\sigma = 0.072\ \mathrm{N\,m^{-1}}$, $\delta_t = 0$, $S_f = 0\ \mathrm{kg\,s^{-1}}$, $\chi = 0\ \mathrm{kg\,s^{-2}}$.

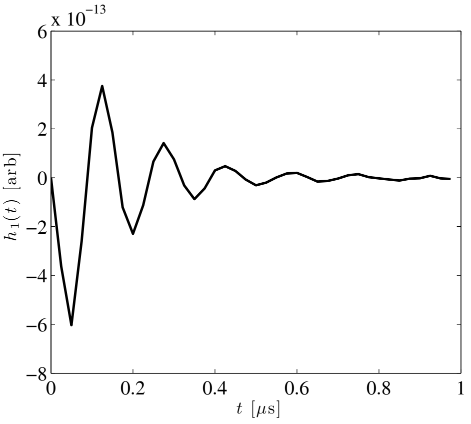

Figure 1: First order kernel of identified contrast agent. It can be seen that the linear component of a contrast agent microbubble features the behaviour of a damped oscillator.

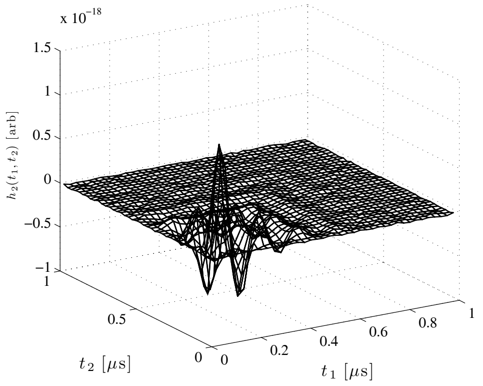

Figure 2: Second order kernel of identified contrast agent. Since the kernel contains components at locations with time lags greater zero, the system under inspection is a nonlinear system with memory.

A constant kernel of $h_0 = 0.75 \cdot 10^{-6}$, which corresponds to the bubble resting radius, was determined. Linear, quadratic and cubic kernels are shown in Figures 1, 2 and 3 respectively. The linear kernel which describes the system's linear response displays the behaviour of a damped oscillator. The quadratic and cubic components show meaningful amounts at off-diagonal positions which indicate that the system in question is a nonlinear system with memory.

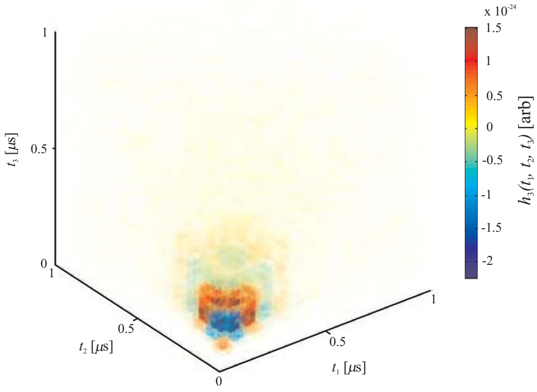

Figure 3: Third order kernel of identified contrast agent. Similar to the second order kernel, the memory of the system can be seen.

Evaluation of pulse sequences

The most common imaging techniques for contrast agents are pulse inversion (PI) and contrast pulse sequencing (CPS). They consist of pulses with amplitudes according to Figure 4 which are transmitted sequentially with the responses to each pulse being added. With both methods, signal originating from linear scatterers is eliminated.

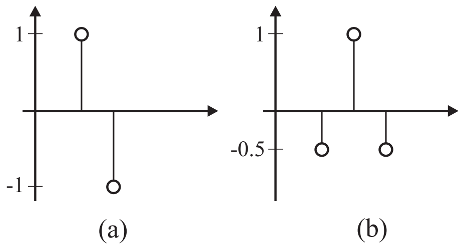

Figure 4: (a) Pulse inversion imaging. (b) Contrast pulse sequencing

While the signal obtained from pulse inversion contains only the even-order harmonic response, the signal obtained from CPS also contains odd-order harmonic components. The quality of the imaging technique, however, depends on the properties of the imaging device (e.g. transducer bandwidth), the propagation properties (linear and non-linear propagation) and, most importantly on the dynamic properties of the contrast agent itself. To evaluate a pulsing scheme, a system as shown in Figure 5 is presented.

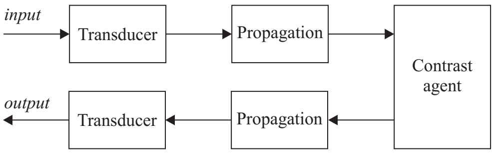

Figure 5: Block diagram for pulsing scheme evaluation system.

The evaluation system consists of a transducer model and a propagation model which can account for nonlinearities in the propagation path. The microbubble radius is computed from the model identified earlier and a far field approximation of the sound emitted by the bubble is made according to [2].

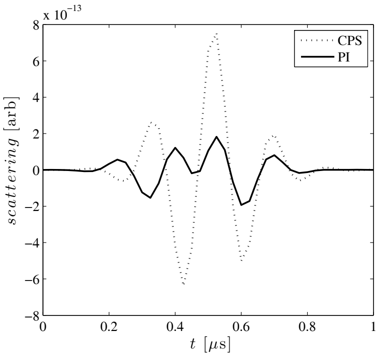

Figure 6: Comparison of pulse inversion and contrast pulse sequencing imaging modes. With CPS, a peak signal increase of 12.3 dB over PI can be achieved.

The comparison of signal strength obtained by using the PI and CPS pulsing schemes can be seen in Figure 6. The signal of a microbubble obtained by CPS is 12.3 dB higher than the signal obtained from PI. This is consistent with earlier results such as published in [3].

Conclusion

A method for nonlinear system modelling was applied to ultrasound contrast agents. A Wiener model of a microbubble allows calculation of the bubble oscillation with a low computational cost. The pulse sequence evaluation system gave realistic estimates of signal strength gained by various pulsing schemes. This enables evaluation and optimisation of pulsing schemes for the detection of contrast agents.

References

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