The vibro-acoustic influence of a thin fluid layer, sandwiched between two structural skins.

B.A.T. Petersson and M.J. Greaves*

Institut für Technische Akustik, 10587 Berlin, Deutschland, Email: b.a.t.petersson@tu-berlin.de

Introduction

Often sandwich and multi-layered structures contain fluid film between the constituents. For different material properties of the structural layers or different layer thicknesses, a rather substantial dissipation can be obtained in a frequency range below the critical frequency of the constituents. This paper presents a model for the analysis of the dissipation mechanism [1], often termed squeeze-film damping and give an example of its application to jet engine nacelles. There are a variety of models in the literature. Trochidis [2] used an incompressible model whilst Fox and Whitton [3] considered the vibration of a single plate over a fixed base analytically and employed an elaborated model that incorporated gas compressibility, inertia and thermal conductivity but which assumed the gas pressure to be constant over the layer thickness. Möser [4] extended the work of Trochidis and included compressibility with both plates free to vibrate. No thermal effects were included. Chow and Pinnington [5] used a hybrid approach combining a compressible model, ignoring thermal effects, with a Statistical Energy Analysis approach for higher frequencies. More recently, Beltman [6] compared a number of different model types and geometries to a low Helmholtz number approximation, developed originally by Zwikker and Kosten [7] and extended by Tijdeman [8].

Theoretical Analysis

Consider two parallel thin plates, separated a distance d and excited by a line force at the top plate. No radiation or fluid loading is included in the model. There are two reasons for this. Firstly, the effect of acoustic radiation into unenclosed air will have a small effect on the behaviour of the plates. Secondly, and more importantly, any sound radiation would contribute to the system losses and hence corrupt the analysis of the squeeze-film damping. The initial approach to the coupling of the plates is similar to that employed in [9]. The plate and the air gap share the same wavenumber in the x-direction (k_x) and y-direction (k_y) but variation is allowed in the z-direction (k_z). The pressure in the air gap at the face of plate 1 at $z = 0$ and the pressure at the face of plate 2 at $z = d$ are expressed as p_0, and p_d respectively. Writing the governing equations for two thin plates coupled by an air layer gives,

$$B_1' \nabla^4 v_1 + m_1'' \frac{\partial^2 v_1}{\partial t^2} = \frac{\partial}{\partial t}(p_F - p_0), \tag{1}$$

$$B_2' \nabla^4 v_2 + m_2'' \frac{\partial^2 v_2}{\partial t^2} = \frac{\partial}{\partial t} p_d. \tag{2}$$

Harmonic excitation and response of the form, $v = \hat{v}e^{-i\omega t}$ and $p = P_0 + \hat{p}e^{-i\omega t}$, is assumed, and writing the pressures at the plate faces as functions of the plate velocities gives $p_0 = z_0^I v_1 + z_0^{II} v_2$ and $p_d = z_d^I v_1 + z_d^{II} v_2$. Upon applying a spatial Fourier transform, equations (1) and (2) become

$$\left[\nabla^4 - k_{b1}^4 - \frac{i\omega}{B_1'} z_0^I\right] \hat{v}_1 - \frac{i\omega}{B_1'} z_0^{II} \hat{v}_2 = -\frac{i\omega}{B_1'} \hat{p}_F, \tag{3}$$

$$\frac{i\omega}{B_2'} z_d^I \hat{v}_1 + \left[\nabla^4 - k_{b2}^4 + \frac{i\omega}{B_2'} z_d^{II}\right] \hat{v}_2 = 0. \tag{4}$$

The modelling of the air gap essentially follows Morse and Ingård [10]. The key assumptions of the present approach are that the dimensions and wavelength are large in comparison to the mean free path, there is no internal heat generation and that the flows involved are laminar. Taking as the governing equations the linearised Navier Stokes equation, the linearised conservation of mass and the linearised equation of continuity for heat flow,

$$\rho_0 \frac{\partial}{\partial t} \mathbf{u} = -\nabla P + \left(\frac{4}{3}\mu + \beta\right) \nabla (\nabla . \mathbf{u}) - \mu \nabla \times (\nabla \times \mathbf{u}), \tag{5}$$

$$\rho_0 \nabla . \mathbf{u} + \frac{\partial}{\partial t} \rho = 0, \tag{6}$$

$$\rho_0 T_0 \frac{\partial S}{\partial t} = K \nabla^2 T. \tag{7}$$

With the field variables as the sum of a constant and a small harmonic perturbation, and assuming that there is no mean fluid velocity gives,$P = P_0 + \hat{p}e^{-i\omega t}$, $S = S_0 + \hat{\sigma}e^{-i\omega t}$, $T = T_0 + \hat{\tau}e^{-i\omega t}$, $\rho = \rho_0 + \hat{\delta}e^{-i\omega t}$ and $\mathbf{u} = \hat{\mathbf{u}}e^{-i\omega t}$. Separating the velocity into a rotationless part $\mathbf{u}_l$ and a divergence free part $\mathbf{u}_r$ allows the governing equations to be combined into one fourth-order differential equation describing the temperature perturbation,

$$\frac{K}{\rho_0 C_P}\left[\frac{1}{i\omega} - \frac{\gamma}{\rho_0 c_0^2}\left(\frac{4}{3}\mu + \beta\right)\right] \nabla^4 \hat{\tau} + \left\{1 - \frac{i\omega}{\rho_0 c_0^2}\left[\frac{\gamma K}{C_P} + \left(\frac{4}{3}\mu + \beta\right)\right]\right\} \nabla^2 \hat{\tau} + \left(\frac{\omega^2}{c_0^2}\right) \hat{\tau} = 0. \tag{8}$$

Solving this equation will give an expression for the temperature variation, which subsequently will allow solutions for the pressure and longitudinal velocity perturbations. To obtain a complete solution, it is also be necessary to solve the equation governing the rotational velocity component,

$$i\omega \rho_0 \mathbf{u}_r = \mu \nabla \times (\nabla \times \mathbf{u}_r). \tag{9}$$

*Presently at QinetiQ, Ively Road, Farnborough, Hampshire GU14 0LX, UK

Eight boundary conditions are required to solve for the 8 unknowns: Zero temperature perturbation at both plate faces ; zero x-direction transverse velocity at both plate faces ; zero y-direction transverse velocity at both plate faces ; continuity between plate velocity and the z-direction fluid velocity at both plate faces. A substitution of (4) into (3) gives

$$\left[\nabla^4 - k_I^4\right]\left[\nabla^4 - k_{II}^4\right]\hat{v}_1 = -\left[\nabla^4 - k_{b2}^4 + \frac{i\omega}{B_2'} z_d^{II}\right]\frac{i\omega}{B_1'}\hat{p}_F \quad (10)$$

where k_I and k_{II} are

$$k_{I,II}^4 = \frac{1}{2}\left[k_{b1}^4 + k_{b2}^4 + i\omega z_0^I\left(\frac{B_1' + B_2'}{B_1' B_2'}\right)\right]$$

$$\pm\frac{1}{2}\sqrt{\left\{\begin{array}{l}\left[\left(k_{b1}^4 - k_{b2}^4\right) - i\omega z_0^I\left(\frac{B_1' - B_2'}{B_1' B_2'}\right)\right]^2 \\ +4\frac{(i\omega)^2}{B_1' B_2'}\left(z_0^{II}\right)^2\end{array}\right\}}. \quad (11)$$

If the losses in the plate are ignored, consideration of the power balance shows that the power dissipated in the system can be equated to the power input to the plate system. Disregarding losses in the plates is reasonable since the plate losses in typical materials will be small compared to the viscous losses in the gap. Similarly, the reversible energy of the system can be considered to be the sum of the contributions from the plates and the air gap, but since the air gap value will represent a small contribution in comparison to the plates it may be ignored. Finally, from a "physical" point of view, the reversible energy can be equated to the sum of the kinetic energies in the upper and lower plate, since the whole system would have to be considered. However, from an engineering point of view, it would be possible to view the fluid layer and the bottom plate as an added damping treatment and hence only the energy in the top plate would be considered. In this case, the former will be adopted. Accordingly, the loss factor is given by

$$\eta = \frac{W'_{IN}}{\omega\left(E'_{kin1} + E'_{kin2}\right)} \quad (12)$$

or, expressed in another way

$$\eta = \frac{\frac{1}{4\pi^2}\Re\left[\int\limits_0^{+\infty}\Theta\left|\hat{p}_F\right|^2 dk_x\right]}{\omega\left[\int\limits_{-\infty}^{+\infty}\frac{1}{2}m_1''\left|v_1\right|^2 dx + \int\limits_{-\infty}^{+\infty}\frac{1}{2}m_2''\left|v_2\right|^2 dx\right]}. \quad (13)$$

In the latter form Θ is the wave mobility, the ratio of the plate velocity to the exciting pressure and the y-dependence is dropped.

Results

In Figure 1 the results obtained with the model described above are compared with those reported in [5]. With the present model being a more complete representation, deviations can be observed. Nonetheless there is a clear fundamental agreement . The markedly higher experimental results at high frequencies most likely are to be ascribed radiation losses. A maximum can be seen slightly below the critical frequency of the directly excited plate. Above this maximum the influence of the viscous effects rapidly diminishes and the loss factor drops markedly. Figure 2

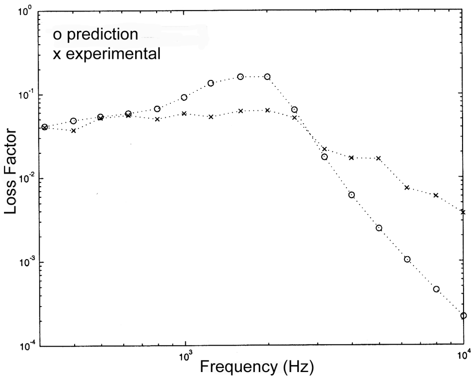

Figure 1: Comparison of predicted and experimental results.

shows the results after some optimisation with respect to the conditions for the jet engine nacelle. In this case the parameter is the altitude.

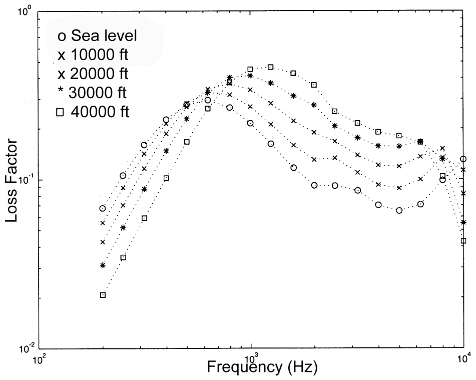

Figure 2: Optimized liner at various altitudes.

References

[1] Greaves M.J., 2001, Loughborough University Ph.D. thesis.

[2] Trochidis, A., 1977, Technischen Universität Berlin, Ph.D. thesis.

[3] Fox, M. J. H. and P. N. Whitton, Journal of Sound and Vibration, 1980. 73(2): p. 279-295.

[4] Möser, M., Acustica, 1980. 46(2): p. 210-217.

[5] Chow, L. C. and R. J. Pinnington, Journal of Sound and Vibration, 1987. 118(1): p. 123-139.

[6] Beltman, W. M., Journal of Sound and Vibration, 1999. 227(3): p. 555-586.

[7] Zwikker, C. and C. W. Kosten, 1949, New York: Elsevier Publishing Company.

[8] Tijdeman, H., Journal of Sound and Vibration, 1975. 39(1): p. 1-33.

[9] Cremer, Heckl and Petersson 2004, Berlin: Springer-Verlag.

[10] Morse, P. M. and K. U. Ingard 1968, Princeton, New Jersey: Princeton University Press.