Ein innovatives CFD/BEM Verfahren zur Berechnung von Strömungsgeräuschen von nicht-kompakten Körpern

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Introduction

The control and reduction of noise in ducts has become an important design issue in various industries. To cite a few examples: flow noise generated in ducts of car air-conditioning systems and flow noise generated in exhaust systems. For this reason, the numerical simulation of flow-induced noise has received a great deal of attention in the last few years.

Although flow noise can be simulated in different ways, the so-called Lighthill aeroacoustical analogy [1] has proven to be a very successful tool, because of its computational efficiency. This method effectively decouples the generation of sound by flow effects from the propagation of sound, and hence allows to split the simulation in a flow computation part, using Computational Fluid Dynamics [CFD] followed by an acoustic computation part using Computational Acoustics [CA], see [2] for an extensive review paper.

The present paper details the development and validation of a hybrid CFD-BEM methodology, which allows the use of incompressible flow data as input for the acoustic computation, even when the solid body is not acoustically compact. This innovative approach is based on a reformulation of the classical Curle equation for flow noise in the presence of solid boundaries in terms of a Boundary Element framework based on the Green kernel function. The validity of the hybrid formulation will be illustrated through the example of the sound field created by a leapfrogging vortex pair in a 2D infinite duct.

Curle Aeroacoustical Analogy

Extending the work of Lighthill, Curle [3] has derived the following expression for the acoustic density fluctuations caused by a turbulent flow in the presence of solid boundaries :

$$\rho'(\mathbf{x},t) = \frac{\partial^2}{\partial x_i \partial x_j} \iiint_V \left[\frac{T_{ij}}{4\pi c_0^2 |\mathbf{x}-\mathbf{y}|}\right] \mathrm{d}^3\mathbf{y} - \frac{\partial}{\partial x_i} \iint_{\partial V} \left[\frac{p' n_i}{4\pi c_0^2 |\mathbf{x}-\mathbf{y}|}\right] \mathrm{d}^2\mathbf{y}$$

The volume integral represents the contribution of freestream turbulence and appears as a quadrupole source. The expression Tij is the Lighthill tensor, which collects the different noise generation mechanisms associated to the flow, i.e. velocity fluctuations, non-isentropic flow and viscous stresses :

$$T_{ij} \equiv \rho v_i v_j + (p' - c_0^2 \rho')\,\delta_{ij} - \sigma_{ij}$$

The surface integral represents the contribution of the pressure fluctuations p' in the turbulent boundary layer on the solid boundaries. An implicit assumption in this derivation is the fact that the pressure fluctuations are associated to a compressible flow, and hence include both the non-propagating hydrodynamic pressure component and the propagating acoustic pressure component. In other words, it is assumed that the acoustic scattering of the solid boundary onto itself has been taken into account by the flow field.

Curle Analogy for Non-Compact Bodies

In most industrial applications for duct flow, flow velocities are quite low, typically 15-20 m/s. For these low Mach number flows (Mach<0.3), it can be shown that compressibility effects can be neglected and therefore the CFD calculation is often run as an incompressible flow analysis in order to reducte the computational effort. Although this is acceptable from the fluid dynamics point of view, one must be careful with this simplification from the acoustical point of view.

The incompressible flow neglects compressibility and therefore ignores acoustic scattering of the solid body onto itself. This is acceptable as long as the body is acoustically compact, i.e. the dimensions L of the body are much smaller than the acoustic wavelength λ. This expressed by a small value for the Helmholtz number : L/λ <<1

However, in the case of long ducts, the assumption of acoustically compact bodies does not longer hold. The dimensions of the ducts are typically a few meters, and are hence much larger than the typical acoustic wavelengths. Application of the classical formulation for these cases with an incompressible flow input leads to incorrect prediction of the sound field, as is shown in the next section.

Counter Example : Leapfrogging Vortex Pair with Incompressible Flow

Schram, Anthoine and Hirschberg [4] have shown that when the results of an incompressible flow analysis are injected into the classical Curle formulation, the resulting sound field can be significantly different from physical reality. This is illustrated by the example of the sound field created by a pair of leapfrogging vortex filaments in a twodimensional infinite duct, with a velocity of Mach=0.5, where the vortex velocity field is treated as an incompressible flow. In Figure 1 below, the acoustic density perturbation propagating to the left and right of the source region in the center of the duct is shown at ω = 1600 rad/s, above the duct cut-off frequency.

Clearly, the conventional Curle formulation based on the incompressible flow data (image b) does not yield correct results, because the acoustic scattering on the non-compact solid walls of the duct has been neglected by the flow analysis

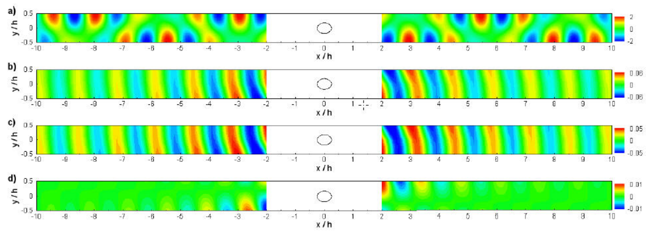

Figure 1: Acoustic density perturbation caused by a leap-frogging vortex pair at M=0.5 in a 2D duct, see [4] : a) Analytical reference solution; b)Curle formulation solution; c) Quadrupolar contribution; d) Dipolar contribution**.**

Hybrid DBEM/Curle Formulation

In order to take into account the acoustic scattering effect, even when the flow data has been obtained from an incompressible CFD calculation, we have reformulated the Curle analogy in terms of a frequency domain Direct Boundary Element formulation in the form of the following expression :

$$C(\mathbf{x})\, p_{\mathrm{L}}(\mathbf{x}) = \iiint_V T_{ij} \frac{\partial^2 G}{\partial y_i \partial y_j} \mathrm{d}^3\mathbf{y} - \iint_{\partial V} p_{\mathrm{L}} \frac{\partial G}{\partial n} \mathrm{d}^2\mathbf{y}$$

This expression is very similar to the classical Curle equation, but is actually more general in nature, as the pressure P_L in the surface integral can refer to both hydrodynamic or acoustic pressure fluctuations. In other words, the expression is valid both in the source region and outside the source region, whereas the Curle equation is limited to the acoustic pressure outside the source region.

The pressure P_L is decomposed into two components : P_L = Ph + Pa , where Ph is the incompressible hydrodynamic component and Pa is the acoustic component associated to compressibility effects. For a given dipole source location, we then also split the integration volume V into 2 parts. First, a compact region V1 around the location, e.g. at a quarter wavelength distance, where we assume that acoustic scattering can be neglected, i.e. P_L= Ph. Second, the remaining non-compact region V2 where we need to take into account the acoustic scattering, i.e. both Ph and Pa. This finally yields the following expression :

$$\begin{aligned} C(\mathbf{x})\, p_{\mathrm{a}}(\mathbf{x}) &= -\iint_{\partial V_1} p_{\mathrm{a}} \frac{\partial G}{\partial n} \mathrm{d}^2\mathbf{y} + \iiint_{V_2} T_{ij} \frac{\partial^2 G}{\partial y_i \partial y_j} \mathrm{d}^3\mathbf{y} - \iint_{\partial V_2} (p_{\mathrm{h}} + p_{\mathrm{a}}) \frac{\partial G}{\partial n} \mathrm{d}^2\mathbf{y} \\ &= -\iint_{\partial V} p_{\mathrm{a}} \frac{\partial G}{\partial n} \mathrm{d}^2\mathbf{y} + \iiint_{V_2} T_{ij} \frac{\partial^2 G}{\partial y_i \partial y_j} \mathrm{d}^3\mathbf{y} - \iint_{\partial V_2} p_{\mathrm{h}} \frac{\partial G}{\partial n} \mathrm{d}^2\mathbf{y} \end{aligned}$$

This modified Curle equation shows that in addition to the contribution of the Lighthill tensor Tij and the incompressible flow field Ph, we also have a contribution due to the scattering of the acoustic pressure field Pa, represented by the first surface integral. This expression has been incorporated into the Boundary Element solver of the commercial acoustic simulation program LMS SYSNOISE. The detailed derivation of this hybrid reformulation can be found in Schram, Martinez-Lera and Tournour [5].

Validation : Counter Example Revisited with Hybrid DBEM/Curle Formulation

The sound field produced by the leapfrogging vortex pair in a 2D infinite duct has been recomputed with the new hybrid formulation. The results are shown in Figure 2, full details on the procedure can be found in [5].

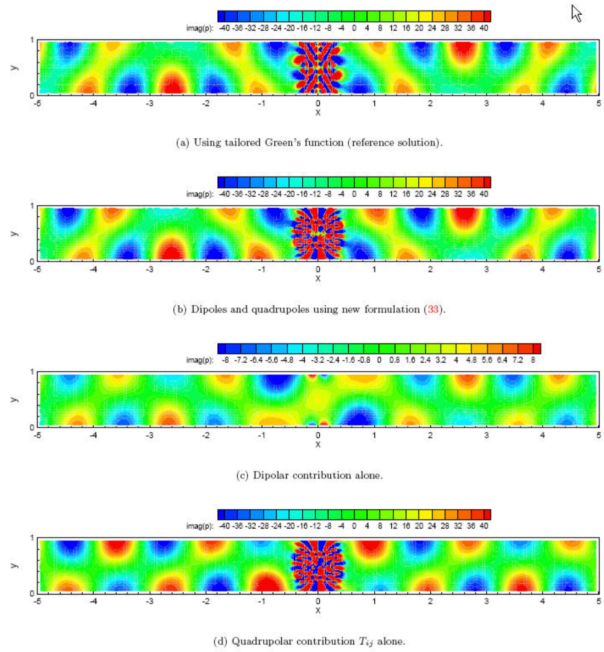

Figure 2: Sound pressure caused by a leapfrogging vortex pair at M=0.5 in a 2D duct, taken from [5]: a) Analytical reference solution; b) Hybrid DBEM/Curle solution; c) Quadrupolar contribution; d) Dipolar contribution.

Clearly, the hybrid DBEM/Curle formulation leads to a near-perfect match between the numerical and analytical solutions for the sound field in the duct. This result highlights the importance of the acoustic scattering term for non-compact bodies when using incompressible flow data as input.

References

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