The fast multipole Boundary Element Method
Breaking the high frequency barrier for acoustic simulation of large structures

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Abstract

In the last 15 years, the Boundary Element Method has proven itself as an extremely powerful and versatile tool for the numerical simulation of interior and exterior acoustic radiation problems in the automotive, aerospace and consumer electronics industries.

Although Acoustic BEM codes are attractive because they allow the re-use of existing structural surface meshes, the mathematical formulation leads to a dense matrix system of equations. The matrix inversion requires a number of operations that increases rapidly with node number N : with $O(N^3)$ for direct decomposition algorithms , with $O(N^2)$ for iterative decomposition algorithms. This imposes a practical upper frequency limit on the analysis of vibrating structures with large dimensions, e.g. full-size cars or aircraft fuselages, as the ”six-elements-per-wavelength” rule leads to a computational bottleneck.

This presentation describes an innovative approach to the numerical formulation of the Acoustic Boundary Element Method : the Fast Multipole BEM method. Through the use of a multi-level grid scheme and multipolar expansions, this method only requires $O(N*\log^2 N)$ operations to resolve the matrix equations, a quantum jump in computational performance. The presentation will highlight the potential of the FMBEM algorithm by comparing computational statistics from a set of industrial cases.

Introduction

Classical Boundary Element Method (BEM) has proven to be a very efficient method for solving acoustic interior and exterior problems. Its main feature consists in defining the unknowns only on the physical surfaces as opposed to the Finite Element Method (FEM) where the complete fluid domain needs to be taken into account. The meshing operation is thus greatly simplified and exterior radiation problems are handled much easily.

However, the mathematical formulation of this method [1, 2] leads to fully-populated matrices for which solving operation can be very demanding as the complexity of the problem increases. Typical matrix inversion algorithms, i.e. LU factorization, indeed require N^3 operations, N being the number of degrees of freedom. Iterative solvers have been developed to comp for that and allow to reduce the number of operations (N^2). A typical mid-size BEM problem (about 2000 degrees of freedom) will typically solve in O(minutes) per frequency and allow full frequency analysis on a standard computer. Large size BEM problems typically counting about 10 000 degrees of freedom can be solved in O(hours) per frequency. Full frequency analysis is preferably done using high end PCs or workstations and using parallel computers. Full frequency analysis of huge models with for instance 50 000 degrees of freedom is nowadays not practically doable since it involves computation time of the O(days) per frequency with supercomputers run in parallel.

Also, since at least six boundary elements per wavelength are required for a correct representation of the problem, the size of the system increases very fast with the frequency. Thus, analysis of large structures with BEM method is limited to low to mid frequency. So there is clearly a need for a new method allowing to handle complex problems and to break the upper frequency limit for large structures.

Based on these needs, the Fast Multipole Method (FMM) has been evaluated for solving large acoustic problems. It has been validated and integrated to the commercial software LMS Virtual.Lab.

Principle of the fast multipole method will first be presented as well as the key features of the new solver. The application of the method to a practical example will then be shown with comparison of the simulation results with experimental data.

The fast multipole method

A new paradigm

Although it requires less memory and allows to solve problems much faster than classical Boundary Element Method with a number of operations of $O(N*\log N)$, the FMM was only recently applied to acoustic problems [3,4]. This method really opens up a window for acoustic simulation of huge size problems with millions of elements. New applications can be envisaged such as simulation of high frequency pressure loading on a complete vehicle or evaluation of the pass-by noise for a detailed vehicle model. Engine acoustic radiation currently possible up to about 2500 Hz could also be practically done up to 5000 Hz using the Fast Multipole Method.

Technology components

The FMM is based on a classical evaluation of the BEM operator in the near field. In the far field, a clustering of boundary elements is formed and the solution is evaluated by multipole expansion. The power of the method is that interaction between well separated sets of nodes can be treated in one time. This is illustrated in Figure 1 where we see that in classical BEM each point is interacting with all other nodes. With FMM, nodes within one group are treated as in BEM but interaction between groups is defined by only one relation. This leads to less memory and less CPU time requirements.

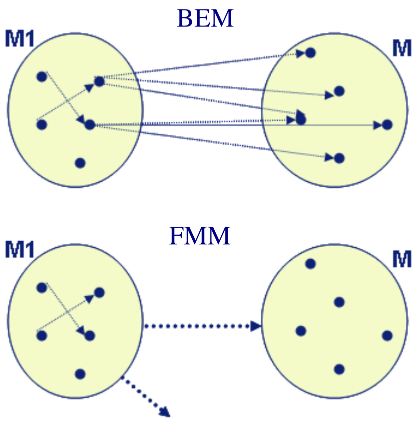

Figure 1: Interactions between nodes in conventional BEM and in Fast Multipole Method.

This describes the behavior of a single level Fast Multipole solver. The FMM solver is actually a multi-level solver which means that the size of the block will be adapted based on the distance between the blocks. Figure 2 shows the different levels of decomposition that will be used by the solver. This multi-level scheme allows to limit the number of operations to O(N*log^{2}N).

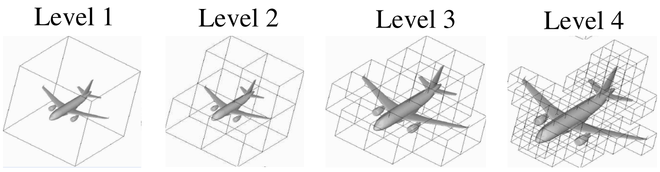

Figure 2: Multi-level Fast Multipole Decomposition

Application example

The FMM was already applied to compute exterior acoustic radiation problem and to get the pressure loading on a complete vehicle [5]. It has been applied here on a similar case but for a very large frequency range. The goal was to compute the pressure loading at some points on a car (window, door panel) due to tire noise including a complete vehicle model and on a full frequency range. The mesh of the car contains about 366 000 nodes. Discrete sources were placed at the tire location and the acoustic radiation was computed on the rigid car body up to 5000 Hz.

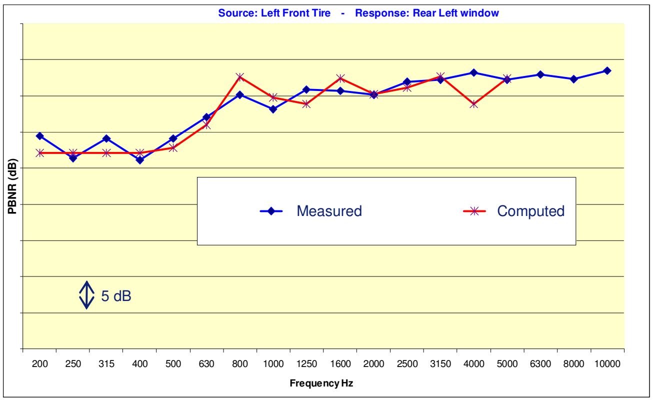

Figure 3: Measured and computed PBNR at the rear left window with a source at the left front tire.

Figure 3 shows the measured and computed Power-Based Noise Reduction Level (PBNR) at the rear left window with a source at the left front tire. We can see that there is an excellent correlation between the simulation and the measurements which demonstrates the accuracy of the method.

The performance of the solver was also investigated. Figure 4 shows the CPU time needed for each frequency using a cluster of 16, 32 and 64 processors. It can be noticed that the full frequency analysis from 400 to 5000 Hz is done in a bit more than a day (26.5 hours) with 32 processors in parallel which demonstrates the good performance of the solver.

	Hours		
	16 proc	32 proc	64 proc
400	2.9	1.1	0.7
500	1.8	1.0	0.6
630	1.7	1.0	0.6
800	1.8	1.0	0.6
1000	1.9	1.0	0.6
1250	2.1	1.2	0.7
1600	3.0	1.6	1.0
2000	3.4	1.9	1.1
2500	5.0	2.5	1.4
3150	6.6	3.0	-
4000	8.3	4.5	-
5000	12.6	6.7	-
	51.2	26.5	7.3

Figure 4: CPU time for each frequency.

Conclusions

In this paper we investigated the use of the Fast Multipole Method to solve very large acoustic problems. Method was used to compute the pressure loading on a vehicle for a full frequency range. This analysis shows very good results both in terms of accuracy and solving time and demonstrates the great potential of the method to handle full system realistic acoustic simulation. This solution was integrated to commercial software LMS Virtual.Lab.

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