

# Nonclassical nonlinear acoustics of solids: fundamentals and applications

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## Introduction

Contemporary acoustics studies elastic waves and vibrations in gases, liquids and solids in a frequency band from infrasound ( $<20$  Hz) to terasound ( $10^{12}$ – $10^{13}$ ) Hz and in a dynamic range ( $10^{-12}$ – $10^6$ ) W/m<sup>2</sup>. The high-intensity end is the field of nonlinear acoustics whose theoretical fundamentals for air-acoustics were laid by Euler, Lagrange, Poisson and Stokes over the 18<sup>th</sup>–19<sup>th</sup> centuries. In solids, nonlinear phenomena have been studied for about 50 years and found to be caused by unharmonic lattice vibrations. Though such a “classical” nonlinearity is locally small it accumulates along the propagation distance and results in a noticeable nonlinear waveform distortion and dissipation.

A gradual pace of the history of nonlinear acoustics of solids has been disturbed by a dramatic turn over the last decade. A gigantic increase in local nonlinearity was revealed for simulated and realistic damaged areas in a wide class of solid materials. A physical reason for that is associated with inherently nonlinear vibrations of non-bonded contacts (contact acoustic nonlinearity, CAN [1]) due to mechanical constraint between the fragments of defects. The CAN was shown to increase a local nonlinearity of the non-bonded contact by 3–4 orders of magnitude and to provide nonclassical nonlinear vibration spectra of planar defects [2]. In this paper, the generation mechanisms of the nonclassical nonlinear spectral components are discussed and applications for defect selective imaging using laser scanning vibrometry and air-coupled ultrasound are shown.

## Nonclassical Higher Harmonics & Wave Modulation

An intense acoustic wave interaction with a damaged area results in its nonlinear vibrations caused by “clapping” and “rubbing” of the defect fragments. “Clapping”, apparently, leads to asymmetrical modulation of the contact stiffness: it is higher for compression and lower for contact extension. Such a “bimodular” behaviour of an initially closed defect driven by a harmonic acoustic strain  $\varepsilon(t) = \varepsilon_0 \cos \nu t$  is similar to a “mechanical diode” and results in the pulse-type stiffness modulation  $\Delta C(t)$ . Since  $\Delta C(t)$  is a pulse-type periodic function of the driving frequency  $\nu$ , the spectrum of nonlinear acoustic vibrations in the damaged area ( $\Delta C(t) \cdot \varepsilon(t)$ ) contains a number of its higher harmonics  $n\nu$ . The stiffness modulation index can be as high as  $\sim 1$  thus providing a very efficient harmonic generation by “clapping” defects [2].

Shear traction dynamics of the damaged area results in “rubbing” controlled by the friction between its contacts. In this case, the stiffness modulation is caused by transition between “stick” and “slide” phases: higher contact stiffness

provided by the static friction in the “stick” phase drops substantially as the contact surfaces start sliding. Such a transition takes place when the harmonic driving force recovers from zero, i.e. twice for the acoustic wave period. Therefore, the contact stiffness modulation  $\Delta C(t)$  is a  $2\nu$ -pulse-type function which comprises its higher harmonics  $2n\nu$ . As a result, the spectrum of nonlinear shear vibrations of the defect ( $\Delta C(t) \cdot \varepsilon(t)$ ) contains a number of odd harmonics  $(2n+1)\nu$  only.

In a similar way, if an additional wave is injected into the damaged area, a strong stiffness inter-modulation will take place and produce locally multiple combination frequency components  $m\nu_1 \pm n\nu_2$  (wave modulation pattern).

## Ultra-Subharmonics

Assume now that besides the nonlinearity, the damaged area exhibits some resonance properties (eigen frequency  $\omega$ ). In this case, the acoustic wave impact produces the combination frequencies of the driven and free oscillations ( $\nu \pm \omega$ ). If  $\nu = 2\omega$ , the difference frequency component  $\nu - \omega = \omega$  is in resonance and the output is a subharmonic vibration at half the input frequency:  $\omega = 2\nu$ . Such phenomena fall into a class of parametric resonance phenomena well known in the case of a swing. The parametric resonances feature a “jump” of the subharmonic amplitudes (instability) beyond a certain threshold: their avalanche-like growth turns the output spectrum into a series of the subharmonics integer multiples of  $\nu/2$  (ultra-subharmonics). For higher driving amplitudes, the resonance range of  $\nu$  expands and parametric resonances is observed in wide frequency bands.

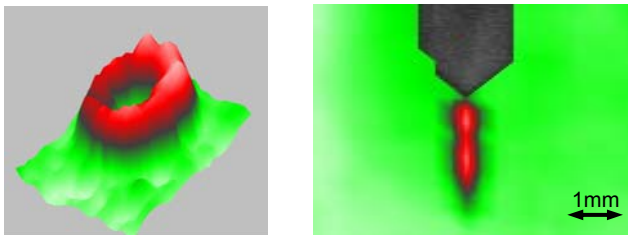
## Ultra-Frequency Pairs & Chaotic Spectrum

The subharmonic generation is modified if one assumes a more complicated structure of the defect represented e.g. as a pair of coupled nonlinear oscillators (eigen frequencies  $\omega_1$  and  $\omega_2$ ). In this case [3], the nonlinear impact of the driving wave results in a generation of a pair of combination frequencies ( $\nu - \omega_1$ ) and ( $\nu - \omega_2$ ). If  $\nu \approx \omega_1 + \omega_2$ , then both coupled oscillators are simultaneously brought to resonance (combination decay instability). The nonlinear spectrum acquires the frequency pair:  $\Omega_{1,2} = (\nu/2) \pm \Delta$ , where  $\Delta = |\omega_1 - \omega_2|/2$ . Successive higher-order nonlinear interaction between  $\nu$  and  $\Omega_{1,2}$ , results in a spectrum of ultra-frequency pairs around the subharmonics  $(2n+1)(\nu/2) \pm \Delta$  and the higher harmonics  $n\nu \pm 2\Delta$ . Further increase in driving amplitude turns the spectrum into noise-like pattern typical for chaotic vibrations.

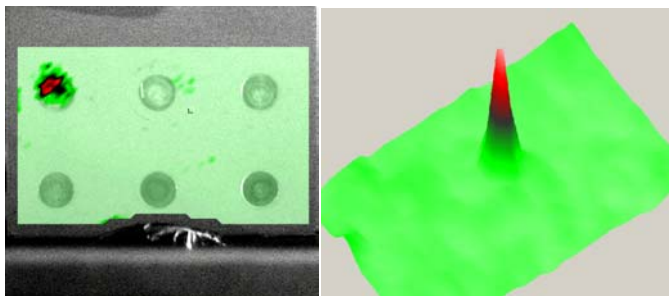
## Nonlinear Defect Selective Imaging

The nonlinear spectral components are generated locally within the defect area and, therefore, only the source of the nonlinearity is expected to be primarily seen in the nonlinear vibration patterns. To produce intense acoustic (flexural) waves required for the nonlinear regime, an ultrasound welding piezoelectric stack transducer was driven with a CW electric signal in a frequency range 20-40 kHz. A traditional methodology for imaging of nonlinear acoustic vibrations [2] is based on scanning laser vibrometry (SLV) which provides an extreme sensitivity (pico-meter range) in measuring and imaging vibration fields by evaluating the laser light scattered back from the vibrating object. After point-by-point scanning of the specimen surface, A/D conversion and FFT, the C-scan images are obtained for any spectral line within the frequency bandwidth of 1 MHz.

A number of experiments and tests have been performed in our laboratory (see [2]) to study the opportunities for nonlinear imaging using both simulated and realistic defects. Some examples are given below (Figures 1, 2) to illustrate both the diversity and generality of this approach for various samples, nonlinear vibration modes, and materials.



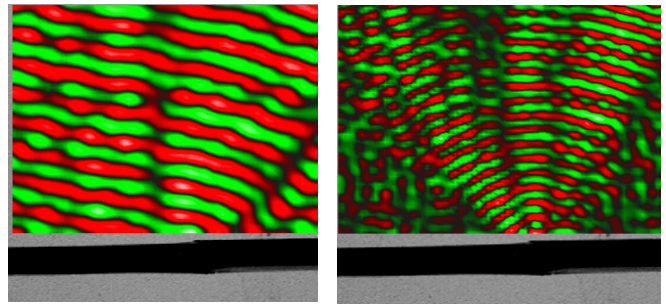
**Figure 1:** Third harmonic SLV-image of an oval delamination in a composite plate (left); ultra-subharmonic (70 kHz) SLV-imaging of fatigue crack in metal driven by 20 kHz-input (right).



**Figure 2:** Combination frequency SLV-imaging of a defective rivet (red) in an aluminium aircraft component (left); ultra-frequency pair SLV-image of impact in fiber-reinforced plastic (right).

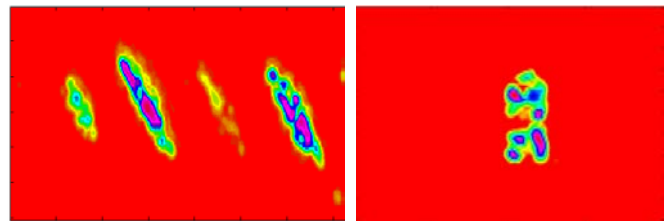
Our experiments also prove that planar defects as localized sources of nonlinear vibrations efficiently radiate nonlinear ultrasound in surrounding air. This effect is illustrated in Figure 3 by using air-coupled laser vibrometry proposed in [4] for imaging acoustic fields in air. The image of the second harmonic field in Figure 3 reveals the evidence of nonlinear air-coupled emission (NACE) by cracked defects.

Since the NACE comes from the defects it can be used to locate and visualize them [5]. A practical version of NACE combines a low-frequency (40 kHz) acoustic excitation with



**Figure 3:** Fundamental frequency (40 kHz, left) and second harmonic (right) air-coupled emission in air by a cracked rod of composite material.

a high-frequency (440 kHz) focused AC-ultrasonic transducer as a receiver. The FFT of the output signal is then applied for computer imaging of the 11<sup>th</sup> harmonic NACE over a specimen surface (Figure 4).



**Figure 4:** NACE image (120x180mm) of impact damage in carbon fiber-reinforced plate (left) and delamination area (5x10mm) in glass fiber-reinforced plastic (right).

In conclusion, nonclassical nonlinear acoustics proposes a much broader approach to nonlinear acoustic phenomena in solids and new opportunities of practical applications. Multiple frequency components of nonclassical nonlinear spectra provide abundant information on properties and location of defects. Both the nonlinear SLV and NACE demonstrate new possibilities of nonlinear acoustic applications in material characterization and non-destructive testing.

## References

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