FE-BE Coupling for Partly Immersed Bodies

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Introduction

Simulation of the vibro–acoustic behavior of ships necessitates dealing with fluid–structure coupled problems, since the surrounding water has a significant influence on the vibrations of the system. For the structural part, namely the ship, the finite element method (FEM) is used. The commercial finite element package ANSYS is applied for setting up the mass and stiffness matrices. The surrounding water is modeled with the fast multipole boundary element method (FBEM). For the partially immersed case, a special halfspace formulation is applied to incorporate the pressure boundary condition of the infinite water surface. The resulting algebraic system of equations is efficiently solved with a preconditioned iterative solver.

Coupled System

The FE-BE coupled system is formally written as

$$\begin{pmatrix} \mathbf{K}_{\mathrm{FE}} & \mathbf{C}_{\mathrm{FE}} \\ \mathbf{C}_{\mathrm{BE}}\mathbf{T}_{wq} & \mathbf{K}_{\mathrm{BE}} \end{pmatrix} \begin{pmatrix} \mathbf{w} \\ \mathbf{p} \end{pmatrix} = \begin{pmatrix} \mathbf{f}_{\mathrm{s}} \\ \mathbf{0} \end{pmatrix}. \qquad (1)$$

Here, $\mathbf{K}_{\mathrm{FE}} = (1 + \mathrm{j}\eta)\,\mathbf{K}_{\mathrm{s}} - \omega^2\mathbf{M}_{\mathrm{s}}$ denotes the dynamical stiffness matrix with the imaginary unit j, the angular frequency ω and the hysteretic damping parameter η. The stiffness matrix $\mathbf{K}_{\mathrm{s}}$ and the mass matrix $\mathbf{M}_{\mathrm{s}}$ are directly set up by the commercial FE package ANSYS and imported into the research code. Typically, shell elements are used for the investigated thin structures, thus $\mathbf{w}$ are the displacement and rotational degrees of freedom at all structural nodes. The nodal driving forces are denoted by $\mathbf{f}_{\mathrm{s}}$. Matrix $\mathbf{C}_{\mathrm{FE}}$ computes nodal forces on the structure, which arise from the nodal fluid pressures $\mathbf{p}$.
Application of the BEM to the Helmholtz equation leads to $\mathbf{C}_{\mathrm{BE}}$ and $\mathbf{K}_{\mathrm{BE}}$. For classical BEM implementations, these matrices are fully populated. To avoid spurious modes, a Burton-Miller approach with a Galerkin discretization is used. Thus $\mathbf{K}_{\mathrm{BE}}$ consists of the double layer potential, the hypersingular operator and a mass term. In the same way, $\mathbf{C}_{\mathrm{BE}}$ results from the single layer potential, adjoint double layer potential and a mass term. Finally, the flux q, being the normal derivative of the pressure, is computed from the structural displacements $\mathbf{w}$ by $\mathbf{T}_{wq}$.

Halfspace Formulation

For the simulation of ship-like structures, the water surface has to be incorporated into the model. Since this Dirichlet boundary has infinite size, a complete discretization of this water surface is not realizable. An exact representation is achieved by application of a special mirror technique [1]. Here, the standard fundamental solution $P(x,y) = \frac{\mathrm{e}^{\mathrm{j}\kappa\,|x-y|}}{4\,\pi\,|x-y|}$ is replaced by the modified halfspace version

$$P^*(x,y) = \frac{\mathrm{e}^{\mathrm{j}\kappa\,|x-y|}}{4\,\pi\,|x-y|} - \frac{\mathrm{e}^{\mathrm{j}\kappa\,|x'-y|}}{4\,\pi\,|x'-y|}. \qquad (2)$$

Here, a second term is added with the point x' which is mirrored at the halfspace plane (see Fig. 1).

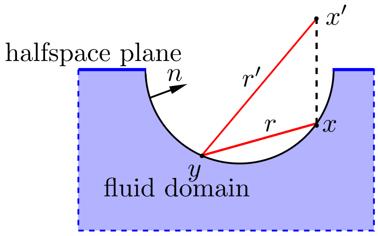

Figure 1: Mirror technique for a partly submerged body: Point x is mirrored at the halfspace plane.

With this approach, the water surface does not need to be discretized at all. As a result, the number of unknowns remains minimal. As a side effect, the condition number of $\mathbf{K}_{\mathrm{BE}}$ is improved, which is important for iterative solvers. However, the integration of the modified kernels is more expensive, since a second term has to be considered.

Fast Multipole Implementation

To overcome the fully populated BE matrices, the fast multipole method is applied, which is based on a clustering of the points [2]. One has to distinguish between a farfield, where the points x and y are well separated and the multipole expansion is used, and a nearfield, which is treated in the classical sense. An efficient implementation for the modified halfspace version is obtained, if the model is geometrically mirrored. Compared to the non halfspace version, some modifications have to be taken into account:
In the **nearfield**, one also has to consider the contributions of the mirrored clusters, which are close to the halfspace plane. As the number of these clusters is small, the expense for setting up the nearfield is hardly increased. This is important, since the integration of the nearfield is a significant expense for the Helmholtz case. Thus, most of the interaction is taken into account by the **farfield**. When using a multilevel cluster tree, an efficient representation of the mirrored part, being mainly in the farfield, is achieved. For both the nearfield and the farfield only the contributions of the mirrored part into one direction has to be considered. Apart from an increase of translation operators in the farfield, the memory consumption remains constant.

Solver

In the fully coupled system (1), the vector of unknowns consists of pressure and structural degrees of freedom. Using the equivalent Schur complement formulation

$$\left(\mathbf{K}_{\mathrm{BE}} - \mathbf{C}_{\mathrm{BE}}\mathbf{T}_{wq}\,\mathbf{K}_{\mathrm{FE}}^{-1}\mathbf{C}_{\mathrm{FE}}\right)\mathbf{p} = -\mathbf{C}_{\mathrm{BE}}\mathbf{T}_{wq}\,\mathbf{K}_{\mathrm{FE}}^{-1}\,\mathbf{f}_{\mathrm{s}}, \quad (3)$$

a reduced system is introduced, where only the unknown pressures occur. For a detailed comparison of both systems see [3]. In the following, (3) is solved with a GMRES. A simple diagonal scaling of $\mathbf{K}_{\mathrm{BE}}$ is applied as preconditioner. In every iteration step the inverse of $\mathbf{K}_{\mathrm{FE}}$ is needed, which is efficiently computed by a LU factorization.

Simulation Results

As a test case, the container vessel depicted in Fig. 2 is investigated in the frequency range between 3 Hz and 25 Hz. A harmonic simulation is performed every 0.2 Hz.

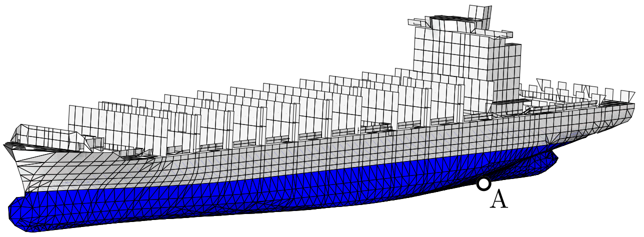

Figure 2: FE model of the investigated container vessel.

The structure is excited by 330 forces at the rear part of the vessel. The blue elements are in contact with the water. Since a conforming coupling scheme is applied, the boundary elements are directly generated from these elements. The structural part consists of approximately 35k degrees of freedom and 1517 nodes are in contact with the water.

Figure 3 visualizes the fluid pressure at node A (cf. Fig. 2) for the fully coupled FE-BE approach. Additionally, the results for the so called one way coupling is plotted. In this case, matrix $\mathbf{C}_{\mathrm{FE}}$ in (3) is simply set to zero. This is equivalent to a dry vibration analysis of the structure and a subsequent acoustic analysis. Obviously, there is a significant difference between the fully coupled case and the one way coupling. This example clearly demonstrates the necessity of a fully coupled simulation procedure. It is not sufficient to solve the FE and BE problems independently of each other.

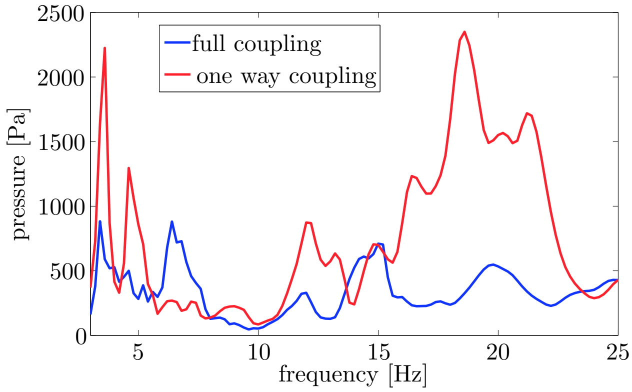

Figure 3: Pressure at node A of the vessel.

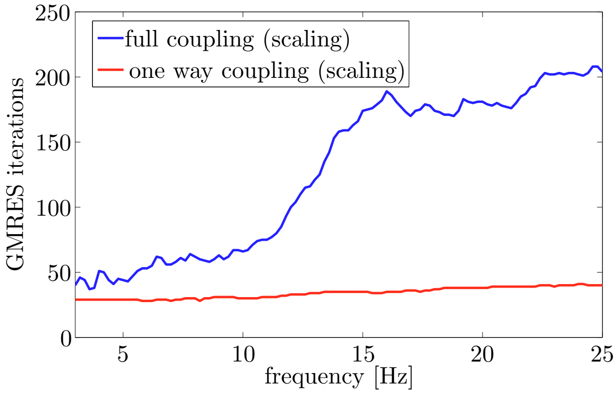

Figure 4: GMRES iteration steps, full coupling versus one way coupling. A diagonal scaling is used as preconditioner.

For an efficient simulation, a fast convergence of the iterative solver is desirable. Figure 4 compares the GMRES iteration steps for the full coupling and the one way coupling. In case of the latter one, convergence is rapid and almost independent of the frequency, though only a simple diagonal scaling is applied as preconditioner. This is due to the good condition number of $\mathbf{K}_{\mathrm{BE}}$, resulting from the halfspace formulation. In the fully coupled case, the number of iterations increases for higher frequencies. This is as expected, since the second part of the Schur complement was neglected for preconditioning. Thus, the one way coupled system is used for preconditioning of the fully coupled system. For higher frequencies, the influence of the coupling increases resulting in a higher number of iteration steps.

Conclusion

In this paper, a fully coupled FE-BE simulation scheme for partly submerged bodies is presented. The water surface is incorporated by a special halfspace formulation. Due to the fast multipole method, even large scale problems can be handled. The approach turned out to be efficient, even for industrial applications.

Acknowledgement

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