Model Reduction Techniques with Additional Interface Reduction for an Efficient FE/BE Coupling

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Introduction

When predicting the vibro-acoustic behavior of thin-walled structures submerged in a dense fluid, the feedback of the acoustic pressure onto the structure may not be neglected and strong coupling schemes need to be applied. A fully-coupled simulation scheme, based on a finite element (FE) formulation for the structural part and a boundary element (BE) formulation for the acoustic part, turned out to be well suited for the prediction of large-scale fluid-structure coupled problems [1]. For the FE-method model reduction techniques exist, such as the Craig Bampton method or the Rubin method, to represent the dynamic behavior of a component substructure. However, they are not numerically efficient when applied unmodified to the fully coupled FE-BE formulation, since the size of the reduced, fully-populated system matrix is directly related to the high number of nodes on the FE-BE interface. Therefore, in this work interface reduction methods are investigated. Here the size of the reduced order model is decreased by reducing the number of retained interface modes, while only marginally increasing the reduction error.

FE-BE Coupled Formulation

The structural part is modeled by the FE method. By deriving the equations of motion from d'Alembert's principle and assuming linear elastic material properties the FE equation is obtained

$$\mathbf{M}_\mathrm{s}\ddot{\mathbf{u}} + \mathbf{K}_\mathrm{s}\mathbf{u} = -\mathbf{C}_\mathrm{FE}\mathbf{p} + \mathbf{f}_\mathrm{s} \ , \qquad (1)$$

with the mass matrix $\mathbf{M}_\mathrm{s}$, the stiffness matrix $\mathbf{K}_\mathrm{s}$, the nodal displacement vector $\mathbf{u}$, the pressure vector $\mathbf{p}$ and the nodal force vector $\mathbf{f}_\mathrm{s}$. The coupling matrix $\mathbf{C}_\mathrm{FE}$ takes into account the influence of the acoustic pressure onto the fluid. For time–harmonic behavior the mass and stiffness matrix is combined to form the dynamic stiffness matrix $\mathbf{K}_\mathrm{FE} = -\omega^2\mathbf{M}_\mathrm{s} + \mathbf{K}_\mathrm{s}$, where ω denotes the angular frequency. The BE method is used for the acoustic domain. The BEM is well-suited for the solution of the exterior acoustic problem, since it intrinsically fulfills the Sommerfeld radiation condition. Starting point for the BE formulation is the Helmholtz equation. The Neumann boundary datum may be expressed by the structural displacement in normal direction on the interface $q(x) = \varrho_\mathrm{f}\omega^2 u_n(x)$, where ϱ_f is the density of the fluid. Triangulation of the boundary leads to the system of equations

$$\mathbf{K}_\mathrm{BE}\mathbf{p} + \mathbf{C}_\mathrm{BE}\mathbf{u} = 0 \ , \qquad (2)$$

with the BE-matrix $\mathbf{K}_\mathrm{BE}$ on which the fast multilevel multipole method may be applied in order to circumvent the well-known drawback of fully populated boundary element matrices. Combining (1) and (2) yields the FE-BE coupled formulation.

FE-Interface Reduction

Both the Craig-Bampton method and Rubin's method are based on a consistent Rayleigh-Ritz coordinate transformation $\mathbf{u} = \mathbf{\Theta}\,\mathbf{z}$, where $\mathbf{z}$ denotes the generalized coordinates and $\mathbf{\Theta}$ is a rectangular coordinate transformation matrix of component modes. Combining (1) and (2) and using the transformation matrix $\mathbf{\Theta}$ yields

$$\left(\mathbf{K}_\mathrm{BE} - \mathbf{C}_\mathrm{BE}\mathbf{\Theta}\tilde{\mathbf{K}}_\mathrm{FE}^{-1}\mathbf{\Theta}^\mathrm{T}\mathbf{C}_\mathrm{FE}\right)\mathbf{p} = -\mathbf{C}_\mathrm{BE}\tilde{\mathbf{K}}_\mathrm{FE}^{-1}\mathbf{f}_\mathrm{s} \ , \qquad (3)$$

where $\tilde{\mathbf{K}}_\mathrm{FE} = \mathbf{\Theta}^\mathrm{T}\mathbf{K}_\mathrm{FE}\mathbf{\Theta}$ stands for the reduced dynamic stiffness matrix. The term within parentheses in (3) denotes the Schur-complement using the reduced stiffness matrix. In [1] a numerical efficient solver based on the Schur-complement using the full-size dynamic stiffness matrix is described. An LU-factorization is used instead of the computation of the inverse of $\mathbf{K}_\mathrm{FE}$. The LU-factorization of $\tilde{\mathbf{K}}_\mathrm{FE}$ will be much faster than using the full-order system matrix $\mathbf{K}_\mathrm{FE}$, if the reduced-order system is small.

For the Craig-Bampton method, $\mathbf{\Theta}$ is populated by fixed-interface modes $\mathbf{\Phi}_\mathrm{CB}$ and constraint modes $\mathbf{\Psi}_\mathrm{CB}$. Free-interface modes $\mathbf{\Phi}_\mathrm{R}$ and attachment modes $\mathbf{\Psi}_\mathrm{R}$ are used to form $\mathbf{\Theta}$ in case of Rubin's method. For both methods the number of constraint/attachment modes is equal to the number of degrees of freedom on the FE-BE coupling interface. For FE-BE coupling the constraint/attachment modes are clearly dominating the size of $\mathbf{\Theta}$. A significant reduction of the size of the reduced order system might thus be obtained by applying interface reduction methods. Two methods are investigated. For the Craig-Bampton method, the dominant subspace of $\mathbf{\Psi}_\mathrm{CB}$ is selected by the solution of the eigenvalue problem of the Guyan reduced system, i.e. solving $\left(-\lambda_j^2\mathbf{\Psi}_\mathrm{CB}^\mathrm{T}\mathbf{M}_\mathrm{s}\mathbf{\Psi}_\mathrm{CB} + \mathbf{\Psi}_\mathrm{CB}^\mathrm{T}\mathbf{K}_\mathrm{s}\mathbf{\Psi}_\mathrm{CB}\right)\mathbf{\Upsilon}_j = 0$, as proposed in [2]. Hereby, $\mathbf{\Upsilon}_j$ is normalized to unit kinetic energy. Only N_1 vectors of $\mathbf{\Upsilon}$ associated with the lowest strain energy are retained. This method will be referred to as `CB-it`. The coupling forces on the FE-BE interface act only in normal direction. Therefore, for the modified method `CB-n-ir`, a coordinate transformation is defined, which rotates one degree of freedom (DOF) on each interface node such that it is parallel to the surface normal at this point. Instead of six constraint modes per node, only one normal constraint modes per node is computed. The in-plane DOFs are then considered to be free DOFs. In case of Rubin's method iterative, reduction schemes are developed, named `Ru-ir` and `Ru-n-ir`. One (normal) attachment mode is first assigned to form the reduction basis $\boldsymbol{\Theta}$. All other (normal) attachment modes are then tested, whether they can be well represented by the current reduction basis. If not, the reduction basis is augmented by the m-orthogonal part of the considered attachment mode.

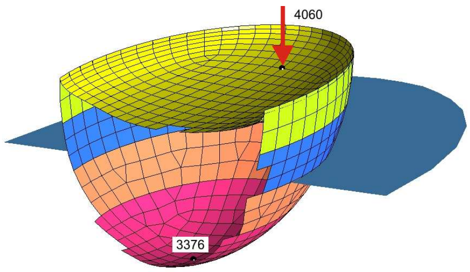

Figure 1: Section cut of the numerical model.

Numerical example

The FE-BE coupled model of a partly submerged hull as depicted in Fig. 1 is investigated to show the applicability of the proposed interface reduction methods. The model has a length of 10.00 m, a width of 5.00 m and a total height of 3.75 m. The hull has a shell thickness of 0.05 m, the cover of 0.01 m. Steel (E=207 GPa, ν=0.3, ρ_s=7669 $^{kg}/_{m^3}$) is used as material for the hull. The FE-model consists of 1484 elements and 1159 nodes yielding 6954 DOFs, of which 25% are in contact with the surrounding fluid. Rayleigh damping is assumed with the parameters $\alpha = 10^1/_s$ and β=1.0 10^{-7} s. Since a conforming coupling scheme is applied, the BE-mesh is directly created out of the FE-mesh. The system is excited by a nodal force in y-direction at node 4060. The system behavior is investigated in a frequency range between 10 and 100 Hz. The reflecting water is taken into account by a modified fundamental solution [3]. The compressibility of the fluid is neglected. Fig. 2 and 3 show the dynamic flexibility frequency response function (FRF) and the pressure FRF on node 3376 for the interface reduction methods `CB-n-ir` compared to the full-order solution. Obviously, it is sufficient to use normal constraint modes (circles) instead of the original constraint modes (dash-dotted line). The displacement and pressure is well approximated by the method `CB-n-ir` if 80 or 40 dominant vectors are retained. It is worth noting, that for the classic Craig-Bampton method 1938 constraint modes

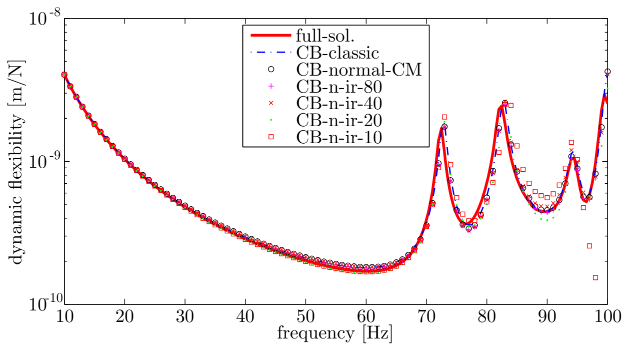

Figure 2: Dynamic flexibility FRF for the interface reduction method CB-n-ir compared to full-order solution.

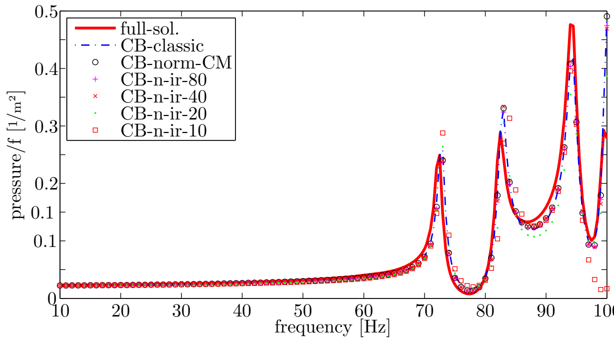

Figure 3: Acoustic pressure FRF for the interface reduction method CB-n-ir compared to full-order solution.

are retained. If the number of retained modes is further reduced, a loss of accuracy is observed. The method `Ru-n-ir` is also capable to predict the fully-coupled behavior as shown for the dynamic flexibility FRF in Fig. 4. For 130 retained attachment modes small deviations for the FRF are observed. If 56 normal attachment modes are used, the FRFs differ between 30 and 70 Hz, though the resonance behavior is still well predicted.

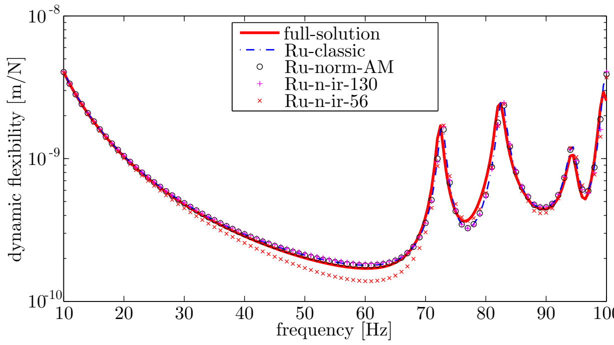

Figure 4: Dynamic flexibility FRF for the interface reduction method Ru-n-ir compared to full-order solution.

Conclusion

In this work, interface reduction methods have been presented enabling an even faster solution of the fully FE-BE coupled system while introducing an only marginal additional error.

Acknowledgement

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References

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