A Coupled FEM/BEM Approach for the Modeling of Active Noise and Vibration Control

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Introduction

In recent years an increasing attention has been paid to active control techniques, since they provide an effective way for vibration suppression and noise reduction of thin-walled lightweight structures. The application of smart materials is an often used concept for actively reducing the structural vibration and sound radiation. Piezoelectric ceramics are the most widely used smart materials in active noise and vibration control, because they are efficient actuators and sensors and can easily be bonded on or imbedded into conventional structures. The design of smart structures requires fast and reliable simulation tools. Therefore, the present paper proposes a coupled finite element/boundary element approach, based in the frequency domain, which enables the modeling of piezoelectric smart lightweight structures for predicting the structural vibration and the sound emission. The approach presented here models the passive vibrating structure as well as the piezoelectric actuators and sensors by using the finite element method (FEM). The boundary element method (BEM) is applied for the modeling of the corresponding sound field. In order to verify the developed numerical approach, simulations of a simple active system are carried out and the results are compared with analytical reference solutions. As a test case, a simply supported rectangular plate coupled with an acoustic cavity and with surface attached piezoelectric patch actuators is considered. A direct velocity feedback control (DVFB) is used to determine the control signals for the actuators.

Finite Element Formulation

The linear finite element equations of motion for a coupled structural-piezoelectric system are given by [1]

$$\begin{bmatrix} \mathbf{M}_u & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\varphi}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_u & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\boldsymbol{\varphi}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_u & \mathbf{K}_{u\varphi} \\ \mathbf{K}_{u\varphi}^T & -\mathbf{K}_\varphi \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\varphi} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_u \\ \mathbf{f}_\varphi \end{bmatrix}, \tag{1}$$

where $\mathbf{u}$ is the vector with the nodal structural displacements and rotations and $\boldsymbol{\varphi}$ is the vector with the nodal values of the electric potentials. The Matrices $\mathbf{M}_u$, $\mathbf{C}_u$ and $\mathbf{K}_u$ are the structural mass, damping and stiffness matrix, respectively. Matrix $\mathbf{K}_\varphi$ is the dielectric matrix. The piezoelectric coupling arises in the piezoelectric coupling matrix $\mathbf{K}_{u\varphi}$. The external loads are stored in the mechanical load vector $\mathbf{f}_u$ and in the electric load vector $\mathbf{f}_\varphi$. It should be noted that the vector $\mathbf{u}$ contains the nodal displacements of the passive structure as well as the nodal displacements of the piezoelectric material. Applying the FOURIER transform to the given system of equations (1) leads to an equivalent system of equations in the frequency domain

$$\begin{bmatrix} -\Omega^2\mathbf{M}_u + i\Omega\mathbf{C}_u + \mathbf{K}_u & \mathbf{K}_{u\varphi} \\ \mathbf{K}_{u\varphi}^T & -\mathbf{K}_\varphi \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{u}} \\ \tilde{\boldsymbol{\varphi}} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{f}}_u \\ \tilde{\mathbf{f}}_\varphi \end{bmatrix}, \tag{2}$$

where the vectors $\tilde{\mathbf{u}}$ and $\tilde{\boldsymbol{\varphi}}$ represent the complex amplitudes of the structural displacements and electric potentials, which are related to the time dependent values $\mathbf{u}$ and $\boldsymbol{\varphi}$ by

$$\mathbf{u}(t) = \mathrm{Re}\left(\tilde{\mathbf{u}}(\Omega)e^{i\Omega t}\right) \qquad \boldsymbol{\varphi}(t) = \mathrm{Re}\left(\tilde{\boldsymbol{\varphi}}(\Omega)e^{i\Omega t}\right). \tag{3}$$

Boundary Element Formulation

In many industrial applications, acoustic problems are modeled by the HELMHOLTZ equation

$$\nabla^2\tilde{p} + k^2\tilde{p} = 0\,. \tag{4}$$

Here $\tilde{p}$ is the acoustic pressure and k is the wave number. The boundary conditions of an acoustic problem are either the acoustic pressure $\tilde{p}$ or the normal velocity $\tilde{v}_n$. For harmonic acoustic fields the acoustic pressure and the normal velocity are linked by the relationship

$$\frac{\partial\tilde{p}}{\partial\mathbf{n}} = -i\rho_0\Omega\tilde{v}_n\,. \tag{5}$$

The boundary integral equation form of the HELMHOLTZ equation (4) can be written as

$$\int_{O_p} \tilde{p}\frac{\partial g}{\partial\mathbf{n}}dO + c_p\tilde{p} = -i\rho_0\Omega\int_{O_p} g\tilde{v}_n dO\,, \tag{6}$$

where g is the fundamental solution of (4). The application of a boundary element discretization together with the collocation method on the integral equation (6) yields the BE matrix equation

$$\mathbf{H}\tilde{\mathbf{p}} = -i\rho_0\Omega\mathbf{G}\tilde{\mathbf{v}}_n\,, \tag{7}$$

with the two influence matrices $\mathbf{H}$ and $\mathbf{G}$.

Structural-Acoustic Coupling

It is well known that the presence of a surrounding fluid strongly influences the vibrational behavior of a thin walled mechanical structure. For that reason, the fluid-structure interaction needs to be considered in the numerical simulation. Due to the vibro-acoustic coupling the normal velocities of the fluid and the displacements of the structure coincide at the fluid-structure interface, which results in

$$\tilde{\mathbf{v}}_n = -i\Omega \mathbf{T}^T \tilde{\mathbf{u}}\,, \tag{8}$$

where $\mathbf{T}$ is a transformation matrix between the FE and BE nodes. The additional mechanical load vector

$$\tilde{\mathbf{f}}_{up} = -\mathbf{T}\mathbf{L}\tilde{\mathbf{p}} \tag{9}$$

represents a second coupling term resulting from the sound pressure acting on the vibrating structure. $\mathbf{L}$ is a coupling matrix that includes the shape functions of the discretized FE and BE model. Substituting equation (8) into (7) and adding up the additional load vector (9) on the left-hand side of (2) leads to the coupled system of equations

$$\begin{bmatrix} -\Omega^2\mathbf{M}_u + i\Omega\mathbf{C}_u + \mathbf{K}_u & \mathbf{K}_{u\varphi} & \mathbf{TL} \\ \mathbf{K}_{u\varphi}^T & -\mathbf{K}_\varphi & \mathbf{0} \\ \rho_0\Omega^2\mathbf{G}\mathbf{T}^T & \mathbf{0} & \mathbf{H} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{u}} \\ \tilde{\boldsymbol{\varphi}} \\ \tilde{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{f}}_u \\ \tilde{\mathbf{f}}_\varphi \\ \mathbf{0} \end{bmatrix} . \tag{10}$$

Numerical Example

To illustrate the accuracy of the proposed approach, numerical simulations of a test case were carried out.

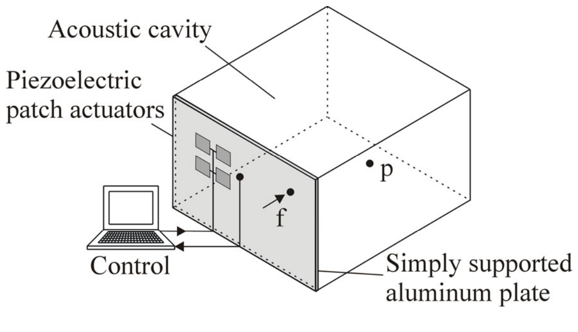

Figure 1: Smart plate coupled with an acoustic cavity

The considered system, shown in Fig. 1, consists of a simply supported rectangular aluminum plate that is coupled with an acoustic cavity and has four surface attached piezoelectric patch actuators. The side opposite from the aluminum plate is open and the other four boundaries of the cavity are acoustically hard surfaces. A direct velocity feedback control (DVFB) was chosen in order to actively increase the structural damping of the plate. The controller is assumed to be a constant gain g_c between the normal velocity $\dot{u}_c$ of a given point on the surface of the aluminum plate and the voltage φ_c, which is fed back to the four piezoelectric patch actuators. The influence of the controller can be taken into account by substituting the control law

$$\tilde{\varphi}_c = g_c i\Omega\tilde{u}_c \tag{11}$$

into the coupled FE/BE system of equations (10). Based on the resulting system of equations, frequency response functions were computed using the FEM software package COSAR and MATLAB. Therefore, the rectangular aluminum plate was meshed with layered SemiLoof-finite shell elements and the discretization of the boundaries of the acoustic cavity was performed with quadratic quadrilateral boundary elements. The following figures show the uncontrolled and controlled frequency response of the sound pressure $\tilde{p}$ regarded at the center of the acoustic cavity due to a harmonic excitation force. In order to verify the developed approach, analytical reference solutions [2] are compared with the numerical results.

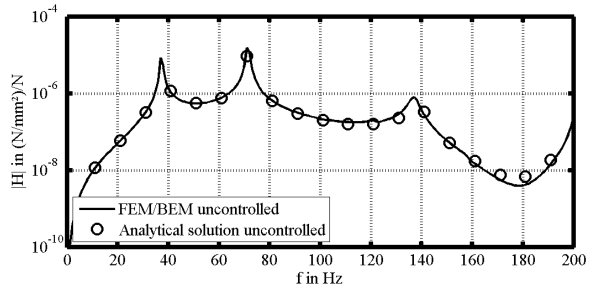

Figure 2: Comparison of the numerical and analytical frequency responses without control

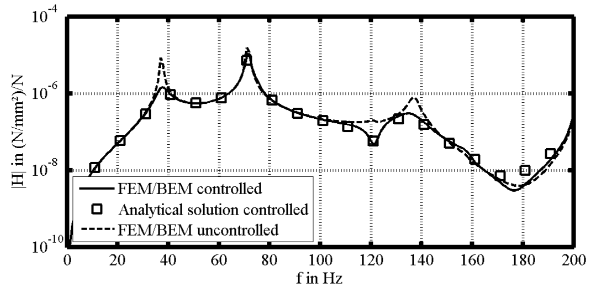

Figure 3: Comparison of the frequency responses with and without control

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[2] Fahy, F.: Sound and Structural Vibration. Radiation, Transmission and Response. Academic Press, London, 1994