From ray to beam tracing and diffraction – an analytical prognosis formula for the trade-off between accuracy and computation time

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Motivation

In geometric acoustics, the classical deterministic but indirect mirror image source method (MISM) [1] and straight-forward but statistical ray tracing methods (RT) [2] are in use. RT looses accuracy because of spatially extended receivers while the MISM suffers from an exponentially increasing computation time with the reflection order [3]. Beam tracing (BT), i.e. tracing of spatially extended rays, [4], can be understood as an efficient combination of both. All these energetic methods suffer from neglected wave effects, mainly diffraction. An efficient energetic beam diffraction method, based on the uncertainty relation, has recently again successfully been tested [5]. But any recursive introduction of diffraction lets the number of rays and, hence, the computation time (CT) explode. The basic solution is to re-unify beams in cases of overlap. This is not possible with rays. So, as a part of a project to implement the method of QPBT [6], the gain of efficiency by using BT instead of RT is investigated where BT serves as reference method for accuracy. The rest-reverberation error (after L reflections) is neglected - as it is possible in most practical cases with absorption. As only a principal study is aimed at, the investigation has been restricted to 2D.

The CT of Ray Tracing

The algorithm of RT comprises nested loops over a constant number of M rays, up to L reflections, and (in the simplest case) over all K 'walls′ (plane polygon surfaces, and, within that, in 3D over all vertices). The CT for the last shall be summarized by an empirical CT unit CTU. Then the CT is

$$CT_{RT}(L) = M \cdot L \cdot CTU \qquad (1)$$

With the sound particle simulation method (SPSM, as a version of RT) the immitted 2D-intensity I' (power per width with a source of sound power P) is computed from N sp crossings of detectors of area S_D around the receivers (e.g. circles) by summing up the relative energies e_i (=1 from start) weighted with the inner crossing distances w_i [7] (maybe in a time interval):

$$I' = P/(M \cdot S_D) \cdot \sum_{n=1}^{N} e_i \cdot w_i \qquad (2)$$

For normal distributions (with constant energies), the relative error of the intensity is $D = 1/\sqrt{N}$ (3)

In a diffuse sound field, the expected number of sp crossing a detector is $N = M \cdot L \cdot C_D / C_R$ [3] (where the C are the circumferences of detector and room resp.), hence, with equ. 1,2, for an accuracy D, the number of sp to be emitted is (for the total energy)

$$M = (C_R / C_D)/(L \cdot D^2) \qquad (4)$$

and

$$CT_{RT}(D) = (C_R / C_D)/ D^2 \cdot CTU \qquad (5)$$

The CT of Beam Tracing

First, BT works efficiently (practically only) if the geometry is sub-divided into convex sub-spaces separated by 'transparent walls' [8]. At these walls also diffraction events close to 'inner edges' may be discovered efficiently. The algorithm of BT is recursive as with the MISM: From the source, first as many beams as walls are emitted (K), recursively split up into again and again K, later converging to K/2 (mirroring walls must be 'in front').

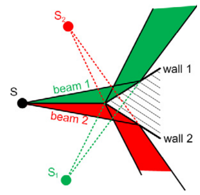


fig. 1 beam from the left, reflected at 2 walls, split up into 2 beams

So, the number of 'constructible' MIS of *l*th order is about

$$m_{CMIS}(l) \approx K \cdot (K/2)^{l-1} \qquad (6)$$

But, from this exponentially growing number, only a vanishing number are 'visible' (valid). This can be estimated by reasons of energy conservation (the total energy of all the MIS of *l*th order must compensate the 1/r –distance -law in 2D). As can be derived from considering the space of the MIS (see fig. 2) the number of MIS on average visible *from every receiver* is only proportional to the ***first order*** of *l*:

$$m_{AVMIS}(l) \approx 2 \cdot \pi \cdot (\Lambda^2 / S_R) \cdot l \qquad (7)$$

with the mean free path length $\Lambda = \pi \cdot S_R / C_R$ (S_R= room area). **This is *not* the number of beams** to be constructed. **'Beams are MIS with built-in visibility'.** BT is a very efficient straight-forward method to find only MIS, which may be visible from at least *any* receiver point. Immissions are counted according the 1/r – law ($I' = P/(2\pi \cdot r)$) simply if a receiver lies within a beam. But as beams do not cover all receivers, several beams per MIS are necessary to reach all. To estimate the number m_B of beams (= MIS visible from *any* point) the split-up-factor $q(l) = m_B(l)/m_B(l-1)$ is considered. As beams become narrower with each split-up, this decreases and will approach 1. By some geometric-statistical considerations turns out:

$$q(l) \approx (l + 1/2) \cdot K_m / m_B(l-1) + 1 \qquad (8)$$

where K_m is the average number of walls in the convex sub-divided rooms which may be assumed to be small (typically K_m=3-5). From equ.8 follows the recursion formula

$$m_B(l) \approx (l + 1/2) \cdot K_m + m_B(l-1) \qquad (9)$$

and from that the ***quadratic law*** $m_B(l) \approx K_m (1 + l + l^2/2)$ (10).

Why is this by one order of *l* more than the average number? This can be understood considering the grid of mirrored rooms in the MIS space (fig.2):

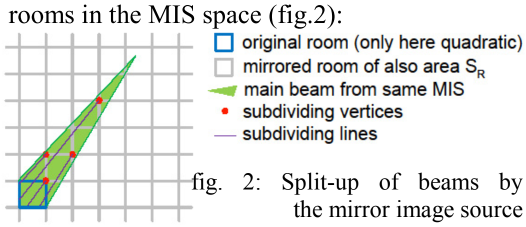


fig. 2: Split-up of beams by vertices in the mirror image source space

By each vertex of the MIS space a 'main beam' from locally the same MIS is split up into many sub-beams representing another mirroring permutation visible from another receiver point.

The number of splitting vertices is the area of the main beam which is about $l \cdot \Lambda^2 / 2$ divided through S_R times K_M.

The CT for BT is the total number of beams to be constructed *up to* Lth reflection order which is in the order of L^3

$$M_B(L) = \sum_0^L m_B(l) \approx K_m\left(1 + \ldots + L^3/6\right) \qquad (11)$$

times an empirical CTU' for projecting a beam onto all the walls which is about 3 CTU for RT. Hence

$$CT_{BT}(L) \approx K_m \cdot L^3 / 2 \cdot CTU \qquad (12)$$

Comparison of Ray and Beam Tracing

So, for same reflection order, the speed-up factor BT/RT is

$$Q(L) = CT_{RT} / CT_{BT} \approx 2M / \left(K_m \cdot L^2\right) \qquad (13)$$

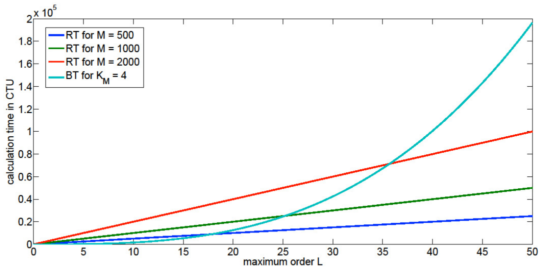


fig. 3: Computation time as a function of reflection order L: 3.order growth with BT (eq.11) versus linear growth (eq.1) with RT

However, it is more significant to compare the CT of RT and BT for a given accuracy of the computed intensities D. **Case a: For the total immitted intensities** the speed-up is

$$Q_a(L, D) \approx 2(C_R / C_D) / \left(K_m \cdot L^3 \cdot D^2\right) \qquad (14a)$$

So, with the cube of the reflection order L, RT becomes faster than BT (Q<1). For typ. 1dB accuracy resp. $D \approx 0.26$ or $N = D^{-2} \approx 15$ immitted sp, $K_m \approx 4$ and $C_R / C_D \approx 10$ (the detectors should be one order smaller than small walls), $Q_a \approx 75 / L^3$, hence this is the case for *L>4.2.*

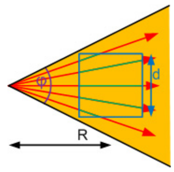
Case b: If the accuracy of the intensity within a small and late time interval of the echogram Δt is of interest, only the sp counted within the small distance $c\Delta t$ are relevant, hence the number of emitted sp must be higher by the factor $p = (L \cdot \Lambda)/(c\Delta t) = T / \Delta t$, where T is the total real run time the sp are traced over. The result for this case b is

$$Q_b(L, D, \Delta t) \approx 2(C_R / C_D) \cdot \Lambda / \left(c\Delta t \cdot K_m \cdot L^2 \cdot D^2\right) \qquad (14b)$$

where $L = cT / \Lambda$. For a reasonable accuracy (rest-reverb. error <1%) only 1/3 of the reverberation time T60 or a 20dB-decay has to be followed, with T60 for ex. 1.5s, T=0.5s. To compute room acoustical parameters reasonably accurate (as for ex. the 'Deutlichkeit') typ. Δt=10ms, such that typ. $p \approx 50$. For all these parameters $Q_b \approx p \cdot Q_a \approx 3750 / L^3$. Then RT becomes faster than BT ($Q_b < 1$) only if *L>15.5* (seldom necessary with common absorption degrees >0.1), hence, more often BT is faster.

Case c, if the accuracy of the intensity of a single MIS is of interest: As each of the MIS (of different identity i.e. mirroring sequence) is represented by a beam, in this case, one may imagine that simply *N* sound particles have to travel within a beam, see fig. 4 - even within the narrowest of highest order - such that the accuracy *D* is reached. Therefore the speed-up is a constant: $Q_c \approx D^{-2}$ (14c)

fig. 4 rays in a beam representing a certain MIS, with detector



So, for ex. to compute single MIS with 1dB accuracy, BT is always about 15 times faster than RT (in 2D). However, the maximum detector diameter *d* must be small enough and obey $d <\approx 4\pi \cdot \Lambda / \left(K_m \cdot L\right)$ (15).

Due to the spatial extension of the detectors, further inaccuracies occur in the echograms and, hence, in the room acoustical parameters computed from that. First, by the temporal 'smearing' of the echogram peaks in the range of a mean free path length within the detector Λ_d / c; second, by wrong detected MIS (invisible from the detector centre).

Numerical experiments show good congruence with the quadratic growth of the number of beams and the CT and quite exact matching for a regular polygons.

For non-convex but convex subdivided rooms, the comparison results are also valid interpreting the order L as the sum of reflections and transmissions to other convex rooms (where t is the ratio transparent/ real circumference, typ. *t*=0.2…1). For higher *t*, i.e. cleft geometries as cities, this means an advantage for ray tracing.

Conclusion

While for lower orders beam tracing is faster, beginning at a critical order, ray-tracing becomes faster, depending on the case and wanted accuracy. These orders are typically 3 – 10 for case a, and rather 10-30 for case b. So usually, RT is better for calculation of only intensities (for ex. noise maps). In case of wanted echogram accuracy, the preference depends on many parameters. Only for an exact identification of MIS, BT, with even at high order less beams than rays, is always much more efficient than RT by a constant factor D^{-2}. However, it is doubtful whether this case is relevant.

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