Room division into convex sub-spaces and its benefits to calculation time and diffraction simulation with ray tracing

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Introduction

Ray- or beam tracing are frequently used in geometrical room and city acoustics, but suffer from the deficiency of lacking diffraction simulation. This investigation presented here is part of a project to implement Quantized Pyramidal Beam Tracing (QPBT)[Ste03] where Stephenson's sound particle diffraction module is embedded. This module is based on the uncertainty principle and requires the detection of sound particles passing by a diffraction edge[SS07]. A convex sub-division (CSD) by introducing virtual, i.e. acoustic transparent, walls is advantageous for the automatic diffraction detection as well as for a reduction of computation time (CT), especially in city acoustics which is aimed at here mainly. Sub-divisions like Binary Space Partitioning, Octrees and more already exist[FDFH95], but do not explicit aim at a CSD optimized for diffraction detection.This work is restricted to 2D, where edges are actually vertices (see Fig. 1)

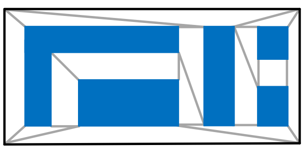

Figure 1: A simple 2D city map perforated by buildings and its division in convex sub-spaces ($t \approx 0.87$)

Definitions

The simulation is based on a 2D geometrical structure, whichever consists of one or more polygons. Each polygon has to be closed and can be the outer boundaries as well as inner obstacles. The only condition made is the mathematically positive surrounding of the simulation environment, so obstacles are sorted clockwise and outer boundaries counterclockwise. In the first step, vertices disturbing the convexity (indentions) are detected by evaluating the angles to the two neighbour vertices and are called 'inner edges' (with regard to a 3D scene).

CSD of connected geometries

In that approach an inner edge is chosen (one after another) and a matching vertex to split off the greatest possible convex polygon is tried to find (see Fig. 2). Between these two vertices a virtual wall is inserted.

Disadvantages:

- Only connected polygons
- Complicated cases of intersected virtual walls
- virtual walls not preferring the bisecting line - unfavorable for diffraction.

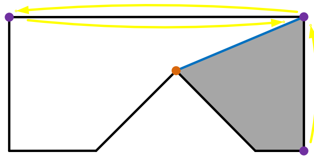

Figure 2: Simple example of CSD separating the largest convex polygon. Counterclockwise partner vertices to build the end of the virtual wall are tested, until at the upper left corner the splitted polygon would not be convex any more. So the blue wall is chosen to generate the grey convex sub-space

CSD of geometries with obstacles

So, in the new approach it is tried to split the geometry by introducing virtual walls close to the bisecting line on inner edges. In contrast to the former method, no convex sub-spaces are necessarily generated at each sub-division step. So the created sub-polygons have to be recursively sub-divided again until all polygons are convex. The Idea: To find a suitable end point for a virtual wall, a simple form of ray tracing has to be performed starting with a ray on the bisecting line of the inner edge. Many difficult cases are avoided by accepting only the closest intersection point of the ray with walls (or already found virtual walls). But a problem is: The introduction of a virtual wall on this ray would add a new vertex to the geometry (see. Fig. 3, end of green line). That should be avoided. Instead this end point is shifted and one of the two end points of the intersected wall is chosen as the ray's end point, generating the lower change of angle. However, due to that changing of angle a new obstacle could now intersect that ray. To prevent that, this ray tracing and shifting procedure with the modified angle is repeated until there is found a non-interrupted ray. Now, a virtual wall is inserted and the sub-polygons are updated. In further steps virtual walls are treated like double-sided real walls for the CSD. The whole procedure is updated until no inner edge exists any more and hence the whole space is perfectly sub-divided into convex spaces with virtual walls opimized for the diffraction detection.

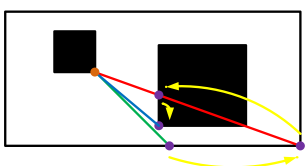

Figure 3: Simple example of a CSD step. The green bisecting line intersects with the lowest wall and the end point is moved to the right (red line). A new obstacle appears in the line of sight and the intersection point is moved down to create the virtual wall (blue line)

The speed-up-effect of CSD

To determine the speed-up of classical ray tracing, first of all the calculation times of a ray tracing iteration step to the next wall from classical ray tracing t_{nc} and from convex ray tracing t_c have to be defined. Both times can be approximated by a constant time due to update and mirroring and an intersection test time proportional to the number of walls to check. With different numbers of vertices points ($n_c = n_{nc}$) it came out that (on a modern personal computer) these times can be estimated by linear regression of measured calculation times in ns to

$$t_{nc} = 0.14 \cdot n_{nc} + 0.6, \qquad t_c = 0.1 \cdot \frac{n_c}{2} + 0.5 \quad (1)$$

Already in convex geometries the CT changes by the factor 2 due to the abort after finding the only intersection point and from 0.14 to 0.1 because the distance to the intersection point does not have to be calculated. The relative error between speed-up-factor $S = \frac{t_{nc}}{t_c}$ of the regression and the measurement has been investigated. This error is very small for all vertex numbers ($\frac{\Delta S}{S_{Meas}} < 4\%$).

But in real non convex geometries which are convex sub-divided, a ray has to perform on average more than one iteration to find the next real wall. Therefore, a shape-factor $t = \frac{V}{P}$ is introduced as the sum of lengths from all virtual walls V relative to the overall real wall length P. t is almost independent of the number of vertices and mostly in the range $0 < t < 2$, seldom greater than 5 (see Fig. 4) (Every virtual wall has to be counted twice, because it belongs to the two sub-spaces it separates). Assuming diffuse sound fields, mean free path lengths

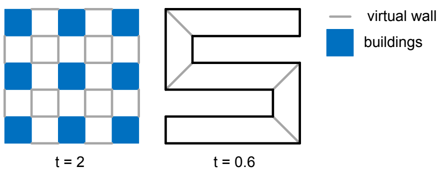

Figure 4: Two example of different shape factors t

(MFPL) may be defined both for the original geometry l_{nc} and for the sub-divided geometry l_c

$$l_{nc} = \pi \cdot \frac{A}{P}, \qquad l_c = \pi \cdot \frac{A}{P+V} = \frac{l_{nc}}{1+t}, \qquad (2)$$

where A is the total area of ray propagation. The number of iterations to find the next real wall increases with the MFPL $\frac{l_{nc}}{l_c}$ by the same factor $(1+t)$. In the additional iterations the ray is transmitted to the next convex sub-space (see Fig. 5) and modifies the speed-up equation, inserting equation 2, to

$$S = \frac{t_{nc}}{t_c \cdot \frac{l_{nc}}{l_c}} = \frac{0.14 \cdot n_{nc} + 0.6}{\left(0.1 \cdot \frac{n_c}{2} + 0.5\right) \cdot (1+t)}. \qquad (3)$$

The average number of vertices of each convex sub-space n_c is statistically about $4 - 5$ in real spaces. Thus, the speed-up equation can be approximated as

$$S \approx \frac{0.2 \cdot n_{nc} + 0.9}{1+t}, \qquad (4)$$

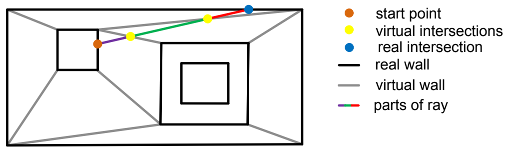

Figure 5: One non convex iteration step equals three faster convex iteration steps ($t \approx 1.15$).

such that for $n_{nc} \to \infty$ the CT hardly depends on the number of vertices n_{nc} anymore, but only on the shape factor t (see Fig. 6). This equation could be verified with some non convex geometries. Here the relative error variating t is even smaller ($\frac{\Delta S}{S_{Mess}} < 2\%$). The CT of the CSD procedure performed only once at the beginning, can be neglected compared with the huge gain in CT of the later RT.

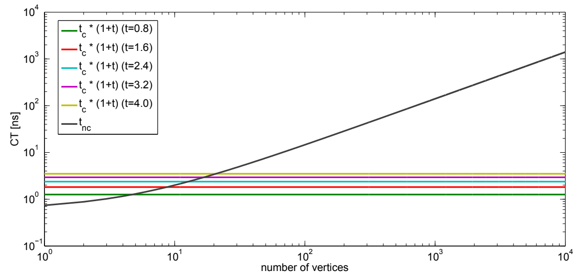

Figure 6: Comparison of CT to find next real wall with CSD $t_c \cdot (1+t)$ and without CSD t_{nc}

Conclusion

At least in 2D the CSD allows a acceptable speed-up of ray tracing by typically factors of $7 < S < 1330$(with $n_{nc} = 100...10000$ and $t = 0.5...2$). Also the detection of diffraction events is improved. In principal, diffraction can now be added by combining the transmission of rays to the next sub-space with a change of direction depending on the by-pass-distance (Stephenson's module based on the uncertainty relation). The same benefits presented for ray tracing can surely be projected to beam tracing and three dimensional simulations, although CSD is much more complicated in 3D[SS07].

References

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