Computation of the Surface Velocity of a Cylindrical Layered Dielectric Device Caused by Partial Discharges

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Introduction

Electric discharges that do not completely bridge the electrodes are called partial discharges (PD) [1]. Inside a solid dielectric PD are caused by small gas filled cavities or inclusions consisting of dirt or textile fibres. During a partial discharge event an electromagnetic pulse and light is emitted. But the most interesting thing is that also an elastic wave is generated which propagates through the solid dielectric. PD take place in the interior parts of high-voltage cable devices. These interior parts form a layered cylinder and are composed -from the inner to the outer- by a copper core, a layer of cross-linked polyethylene (XLPE) and a layer of liquid silicone rubber (LSR).

During a partial discharge a tiny amount of electrical energy is released. In [2] a value of $W_{el} = 150\,\mu J$ is given. It is further stated that a fraction of 0.002% from the electrical energy is transformed in to acoustic energy to the solid dielectric. Due to the small source strength of the buried source and due to the damping behaviour of XLPE and LSR and the fact the diameter is large compared to the wavelength one is primarily interested in predicting the particle velocity at the surface. In this study the wave propagation will be treated as two dimensional problem in the radial and circumferential plane.

Modelling the cylindrical structure

This leads to a 2-D model as illustrated in Fig.1. The inner cylinder is rigid. The elastic layers numerated by 1 and 2 have isotropic homogeneous material properties and are rigidly bonded at their interface. The source is located at the interface of layer 1 and layer 2 at radius r_s.

The angular dependence of a harmonically oscillating load at frequency ω is described by a Fourier series given in Eq.(1)

$$F(\theta,t) = e^{j\omega t}\sum_{n=-\infty}^{n=\infty} F_n e^{jn\theta}, \qquad n = 0, \pm 1, \pm 2, \ldots \quad (1)$$

The vector field of the particle velocity $\mathbf{v}$ is composed by a dilatational wave part $\mathbf{v_L}$and a transverse wave part $\mathbf{v_T}$. By introducing the scalar potential ϕ and the vector potential ψ it can be summarised:

$$\mathbf{v} = \mathbf{v_L} + \mathbf{v_T}, \quad \mathbf{v_L} = \nabla\phi, \quad \mathbf{v_T} = \nabla \times \psi \quad (2)$$

The general solution for the scalar potential can be writ-

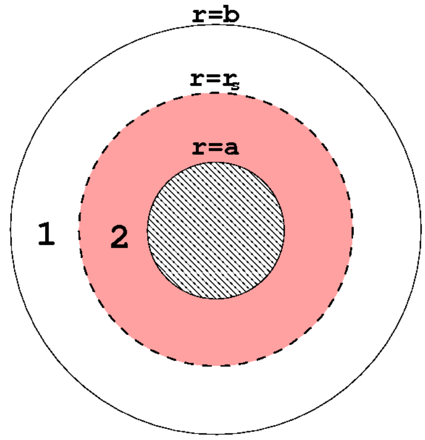

Figure 1: 2-D model of layered cylinder. From the inner to the outer, rigid layer, layer made of cross-linked polyethylene (XLPE) ($a \leq r \leq r_s$), layer made of an elastomer (LSR) ($r_s \leq r \leq b$). The dashed circle at $r = r_s$ is the place where the source is located.

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$$\phi(r,\theta,t) = e^{j\omega t}\sum_{n=-\infty}^{n=\infty}(A_n H_n^{(1)}(k_L r) + B_n H_n^{(2)}(k_L r))e^{jn\theta} \quad (3)$$

and for the vector potential only ψ_z remains

$$\psi_z(r,\theta,t) = e^{j\omega t}\sum_{n=-\infty}^{n=\infty}(C_n H_n^{(1)}(k_T r) + D_n H_n^{(2)}(k_T r))e^{jn\theta} \quad (4)$$

where k_L, k_T are the wave numbers of longitudinal and transverse waves and $H_n^{(1)}$, $H_n^{(2)}$ are the Hankel functions of first and second kind respectively. In further treatments the term $e^{j\omega t}$ will be omitted.

Zero Order Analysis

For the two dimensional model the velocities (v_r, v_θ) can be computed by setting Eq. (3) and (4) in Eq. (2). The stresses (σ_{rr}, $\tau_{r\theta}$) are related with the velocities by the constitutive equation for example given in [3]. For the case of zero order ($n = 0$) only an expression of the scalar potential remains. Equation (3) rewritten for ($n = 0$):

$$\phi(r) = AH_0^{(1)}(k_L r) + BH_0^{(2)}(k_L r) \quad (5)$$

This means only dilatational waves in radial direction can propagate which is given by the radial velocity v_r

and radial stress σ_{rr}. In Eq.(5) $H_0^{(2)}$ represents an outward travelling wave and $H_0^{(1)}$ represents inward travelling wave. From expressions given in [3] leads to

$$v_r = \frac{\partial \phi}{\partial r} \tag{6}$$

for the radial velocity and for radial stress

$$\sigma_{rr} = -\frac{2G}{j\omega}\left[\frac{\partial v_r}{\partial r} + \frac{\mu}{1-2\mu}\left(\frac{v_r}{r} + \frac{\partial v_r}{\partial r}\right)\right] \tag{7}$$

where G is the shear modulus and μ is the Poisson's ratio. From Eq.(5), (6) and (7) the general field equations are

$$v_r = -k_L\left[AH_1^{(1)}(k_L r) + BH_1^{(2)}(k_L r)\right] \tag{8}$$

$$\sigma_{rr} = -\frac{2G}{j\omega}\left[A\widehat{H^{(1)}} + B\widehat{H^{(2)}}\right] \tag{9}$$

where the abbreviation $\widehat{H^{(m)}}$ is introduced with $m =$ 1, 2.

$$\widehat{H^{(m)}} = k_L H_2^{(m)} - \frac{H_1^{(m)}}{r} + \frac{\mu}{1-2\mu}\left(k_L H_2^{(m)} - \frac{2}{r}H_1^{(m)}\right) \tag{10}$$

The surface velocity is determined by first dividing the cylinder into two source free regions, namely region 1 ($r_s < r \leq b$) and region 2 ($a \leq r < r_s$), see Fig.1. Hereafter the field quantities $v_{r,1}$, $\sigma_{r,1}$ of region 1 and $v_{r,2}$, $\sigma_{r,2}$ of region 2 are determined. This is done separately by Eq.(8) and (9) together with the corresponding boundary conditions at $r = b$ for region 1 and $r = a$ for region 2 as given in Table 1.

Table 1: Boundary conditions at $r = a$, $r = b$ and interface condition at $r = r_s$.

$r = a$	$r = r_s$	$r = b$
$v_{r,2} = 0$	$v_{r,1} + v_0 = v_{r,2} - v_0$	$v_{r,1}$
$\sigma_{rr,2}$	$\sigma_{rr,1} + p_0 = \sigma_{rr,2} + p_0$	$\sigma_{rr,1} = 0$

Concerning shortage of space a detailed derivation can not given here. But finally setting the known field quantities into the interface condition given in Table 1 the particle velocity $v_{r,1}$ in region 1 is evaluated.

$$v_{r,1} = -\frac{2v_0}{H}\frac{G_2}{G_1}\frac{H_a}{H_b}X \tag{11}$$

where the abbreviations are given in Eq. (12) to (16).

$$H = \frac{G_2}{G_1}\frac{H_a}{H_b}X + Y \tag{12}$$

$$X = -\frac{\widehat{H^{(2)}}(k_{L,1}b)}{\widehat{H^{(1)}}(k_{L,1}b)}H_1^{(1)}(k_{L,1}r) + H_1^{(2)}(k_{L,1}r) \tag{13}$$

$$Y = -H_1^{(1)}(k_{L,2}r) + \frac{H_1^{(1)}(k_{L,2}a)}{H_1^{(2)}(k_{L,2}a)}H_1^{(2)}(k_{L,2}r) \tag{14}$$

$$H_a = \widehat{H^{(1)}}(k_{L,2}r) - \frac{H_1^{(1)}(k_{L,2}a)}{H_1^{(2)}(k_{L,2}a)}\widehat{H^{(2)}}(k_{L,2}r) \tag{15}$$

$$H_b = -\frac{\widehat{H^{(2)}}(k_{L,1}b)}{\widehat{H^{(1)}}(k_{L,1}b)}\widehat{H^{(1)}}(k_{L,1}r) + \widehat{H^{(2)}}(k_{L,1}r) \tag{16}$$

By setting $r = b$ in Eq.(11) one achieves the particle velocity on the surface of the layered cylinder.

Discussion

The problem discussed focuses on two dimensional wave propagation in radial direction and circumferential direction. An exact expression is derived to evaluate the radial velocity of zero order at the surface of a layered cylinder caused by a source internal to the structure. Although the example chosen here is quite simple the derived equations are very lengthy. It is expected that equations become very cumbersome in cases of more complicated source functions $n > 0$, given in Eq.(1). In a further study it should be figured out whether to solve the layered cylinder 1) by an analytical approach for example [4] or 2) by a semi-analytical approach for example [5]. The analytical approach presented in [4] is an matrix method which is numerically stable over a wide range of axial wave numbers and circumferential orders. In [5] a semi analytical approach is described, where the radial wave propagation is computed by a finite element algorithm whereas the wave propagation in circumferential and axis direction is solved analytical.

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