A Combined TMM-WBM Prediction Technique for Finite-Sized Multilayered Structures

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Introduction

The transfer matrix method (TMM) is widely used to simulate the vibroacoustic behaviour of multilayered structures, because of the low computation cost and easy implementation of different types of layers [1]. Standard TMM assumes infinite layers and represents the plane wave propagation in the layers. In the low- and mid-frequency range, the modal behaviour of both structure and rooms can largely influence the sound insulation properties as measured in labo and in situ. Therefore a wave based model (WBM) has been developed, which models the direct sound transmission through a structure placed between two reverberant rooms. Up till now, single, double and multilayered walls - consisting of acoustically thin plates separated by air cavities - could be investigated [2]. Multilayered structures with fluid, elastic and poro-elastic layers are incorporated in an extended WBM by means of the TMM. The dynamic properties of the structure, calculated with the TMM, are used as boundary and continuity conditions in the WBM. The combined TMM-WBM model is numerically validated. Comparison is made with sound transmission loss measurements of sandwich panels.

Wave Based Model

The geometry of the wave based model (see [2]) is shown in Figure 1. A rectangular structure with dimensions L_{px} and L_{py}, consisting of N plates separated by air cavities, is placed between two rectangular 3D rooms. In source and receiving room, uniform damping is taken into account as function of the reverberation time T. To calculate the sound transmission loss, a harmonic volume point source is placed in the source room. Flanking is not taken into account.

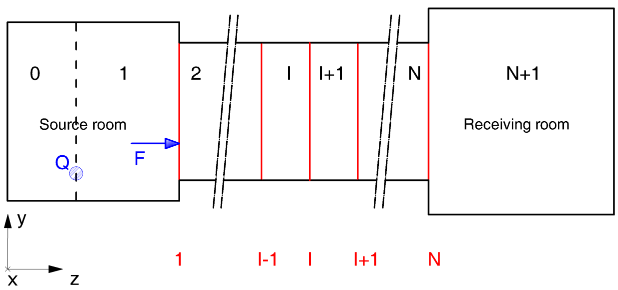

Figure 1: Geometry of the wave based model.

In the wave based method, the field variables are approximated in terms of wave function expansions. For the acoustic pressures in the rooms and cavities:

$$\underline{p}_a(x,y,z) = \sum_{m=0}^{M}\sum_{n=0}^{N}\left(e^{-j\underline{k}_{zmn}z}\underline{A}_{mn} + e^{j\underline{k}_{zmn}z}\underline{B}_{mn}\right) \times \cos\left(\frac{m\pi}{L_x}x\right)\cos\left(\frac{n\pi}{L_y}y\right), \tag{1}$$

where $\underline{k}_{zmn} = \sqrt{\underline{k}_a^2 - \left(\frac{m\pi}{L_x}\right)^2 - \left(\frac{n\pi}{L_y}\right)^2}$. L_x and L_y are the cross-sectional dimensions of the room or cavity. $\underline{k}_a$ is the acoustic wavenumber in air.

Thin plates

For acoustically thin plates, the transverse displacement of the plates $\underline{w}_i$ $(i = 1 \ldots N)$ fulfils Kirchhoff's thin plate bending wave equation:

$$\nabla^4\underline{w}_i - \underline{k}_{B,i}^4\underline{w}_i = \left[\underline{p}_{a,i} - \underline{p}_{a,i+1}\right]_{z=z_{pi}} / \underline{B}_i', \tag{2}$$

where $\underline{k}_{B,i}$ is the bending wave number and $\underline{B}_i'$ the plate bending stiffness.

Assuming simply supported plates, following wave function expansion is used for the transverse displacement:

$$\underline{w}(x,y) = \sum_{p=1}^{P}\sum_{q=1}^{Q}\underline{C}_{pq}\sin\left(\frac{p\pi}{L_{px}}x\right)\sin\left(\frac{q\pi}{L_{py}}y\right). \tag{3}$$

Multilayered structures

Multilayered structures containing elastic and porous materials are incorporated in the WBM by means of the TMM. In this case, the transverse displacement of the structure depends on the z-coordinate. Therefore, separate expansion functions are used for the displacements at emitting and receiving side:

$$\underline{w}_{e/r}(x,y) = \sum_{p=1}^{P}\sum_{q=1}^{Q}\underline{C}_{e/r,pq}\sin\left(\frac{p\pi}{L_{px}}x\right)\sin\left(\frac{q\pi}{L_{py}}y\right). \tag{4}$$

The transfer function $\underline{H}_v = \underline{w}_e/\underline{w}_r$ gives the relation between displacement at emitting and receiving side. The transverse displacement of each structure is determined by following impedance equation:

$$\underline{Z}_{p,i}\ j\omega\underline{w}_{r,i} = \left[\underline{p}_{a,i} - \underline{p}_{a,i+1}\right]_{z=z_{pi}}. \tag{5}$$

The dynamic properties $\underline{H}_v$ and $\underline{Z}_p$ of the multilayered structures are calculated with the transfer matrix method [1], assuming infinite layers.

In the limit for acoustically thin plates, $\underline{w}_e \equiv \underline{w}_r$ or $\underline{H}_v \equiv 1$ and Eq. (5) is equivalent to the thin plate bending wave equation (2).

Numerical example: thick plate

The Kirchhoff plate bending wave equation assumes acoustically thin plates, neglecting shear deformation and rotational inertia. A typically used limit is $\lambda_B > 6h$, that is the thickness is smaller than a sixth of the bending wavelength. The TMM takes into account the full wave propagation through each layer, making thick plate simulations possible. Figure 2 shows that the thin plate assumption is valid for a 8 mm glazing. For the 15 cm sandlimestone wall, the thin plate frequency limit is around 1000 Hz, but there are differences between the WBM for thin and thick plates in the whole frequency range considered. Overall, the thin plate theory overestimates transmission loss.

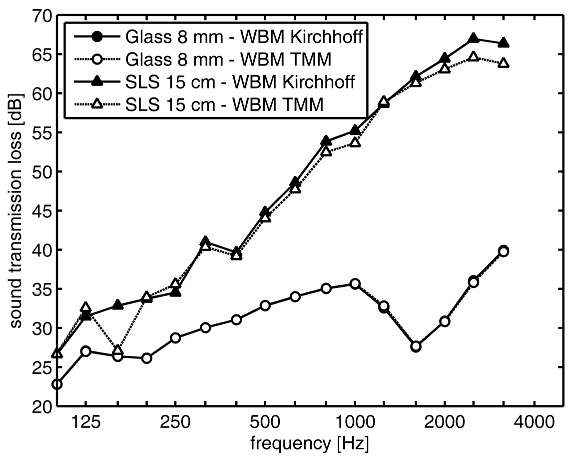

Figure 2: 8 mm glazing (1.25 m x 1.50 m) and 15 cm sandlimestone (3.05 m x 2.95 m), material properties Table 1.

Experimental validation example

The sound transmission loss of a sandwich panel, with dimensions 1.25 m x 1.50 m and a total thickness of 150 mm, was measured in the transmission chambers of the K.U.Leuven. The panel consists of a core of expanded polystyrene (EPS) with a 4 mm fiberboard plate glued on each side. The sound transmission is predicted with the WBM. In the first simulation, the EPS core is modeled as a locally reacting material with stiffness s'':

$$\underline{s}'' = \frac{E_{EPS}}{d_{EPS}}(1 + j\eta_{EPS}). \tag{6}$$

	ρ [$\frac{\text{kg}}{\text{m}^3}$]	E [MPa]	η [-]	ν [-]
Glass	2500	62000	0.05	0.24
SLS	1830	16000	0.015	0.20
FB	850	3500	0.01	0.46
EPS	15	13.1	0.10	0.10

Table 1: Material data used in simulations. (SLS = sandlimestone, FB = fiberboard)

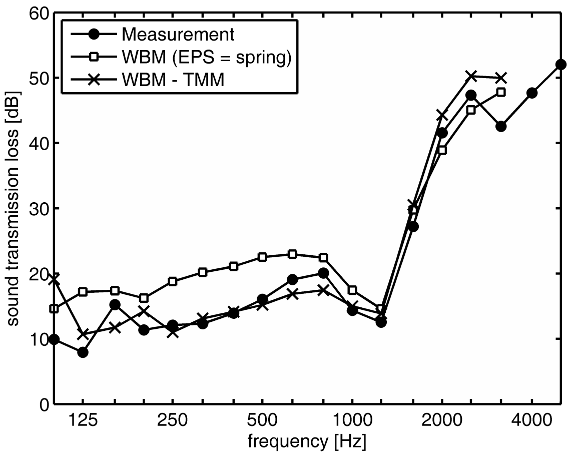

Figure 3: EPS sandwich panel, material properties Table 1.

In the extended WBM with integration of TMM, the EPS core is modeled as an elastic layer.

Figure 3 shows that the extended WBM gives better prediction results for the sound transmission loss. When the EPS core is modeled as a locally reacting spring, the mass-spring-mass resonance dip around 1000-1250 Hz is well predicted. However, in the mid-frequency range till 1000 Hz, the sound transmission loss is overestimated. In this region, sound transmission is dominated by shear wave coincidence in the EPS core, which is not taken into account in the first WBM. The same is true for the thickness resonance dip (of longitudinal waves in the EPS) around 3150 Hz. The extended WBM is able to predict these coincidence phenomena.

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References

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