Room Acoustical Models based on strongly coupled Substructures

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Introduction

In the scope of this work a FSI method for room acoustical simulations is presented.

Acoustic cavities are modeled with realistic boundary conditions using the Component Mode Synthesis (CMS) to compute the steady state response. To enable more complex geometries and bigger models, the Spectral Finite Element Method (SEM) is used for modeling the acoustic fluid [2], [3]. Boundary conditions are introduced via wavenumber dependent impedances, where compound absorbers, containing porous material and elastic parts, are modeled using the Theory of Porous Media (TPM) and Integral Transform Methods (ITM) [1], whereas elastic elements, which can be mounted also inside the cavity, are considered in a modal based Craig Bampton approach [4].

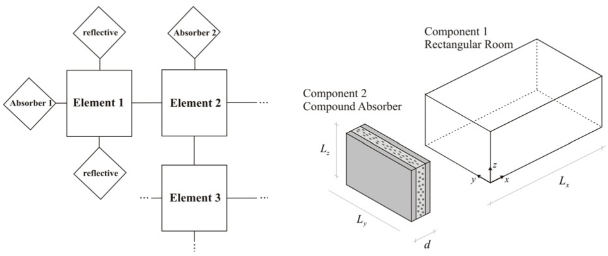

Figure 1: System divided into subsystems (left) and one subsystem as a 3d rectangular structure consisting of two components (right)

Coupling Fluid and Substructures

The method is explained in detail using the simple 3d rectangular system, shown in Fig. 1 (right). A harmonic analysis is carried out under an oscillating load with a circular frequency of excitation Ω. The state variables in the sound field are approximated in the scope of a Ritz approach (1).

$$\begin{aligned} p_L^*(\mathbf{x},t) &= p_L(\mathbf{x})e^{i\Omega t} \qquad \mathbf{v}_L^*(\mathbf{x},t) = \mathbf{v}_L(\mathbf{x})e^{i\Omega t} \\ p_L(\mathbf{x}) &\approx \sum_j A_j p_j^N(\mathbf{x}) + \sum_k B_k p_k^C(\mathbf{x}) \\ \mathbf{v}_L(\mathbf{x}) &\approx \sum_j A_j \mathbf{v}_j^N(\mathbf{x}) + \sum_k B_k \mathbf{v}_k^C(\mathbf{x}) \end{aligned} \tag{1}$$

The velocity potential Φ is introduced to describe the sound field:

$$\begin{aligned} \Phi^*(\mathbf{x},t) &= \Phi(\mathbf{x})e^{i\Omega t} \\ \Delta\Phi^*(\mathbf{x},t) &= \frac{1}{c_L^2}\frac{\partial^2\Phi^*(\mathbf{x},t)}{\partial t^2} \\ \Delta\Phi(\mathbf{x}) &= -k^2\Phi(\mathbf{x}), \qquad k^2 = \frac{\Omega^2}{c_L^2} \end{aligned} \tag{2}$$

Out of the velocity potential two sets of basis functions are computed, which is typical for a Component Mode Synthesis: normal modes (N) and constraint modes (C).

$$\begin{aligned} p_j^N(\mathbf{x}) &= -\frac{\rho_L c_L^2}{i\Omega}\Delta\Phi_j \qquad & p_k^C(\mathbf{x}) &= -\frac{\rho_L c_L^2}{i\Omega}\Delta\Phi_k \\ \mathbf{v}_j^N(\mathbf{x}) &= \nabla\Phi_j & \mathbf{v}_k^C(\mathbf{x}) &= \nabla\Phi_k \end{aligned} \tag{3}$$

The resulting sound field has to satisfy the equations shown in (4), where Hamilton's Principle is applied to fulfill Newton's law.

$$-\nabla p^* = \rho_L\frac{\partial \mathbf{v}^*}{\partial t}, \qquad \frac{\partial p^*}{\partial t} = -\rho_L c_L^2 \nabla^T \mathbf{v}_L^* \tag{4}$$

The Lagrangian function $L_L = T_L - U_L$ for the acoustic fluid results from the kinetic energy T_L and the potential energy U_L:

$$\begin{aligned} T_L &= \frac{\rho_L}{2}\int_V |\mathbf{v}_L(\mathbf{x})|^2 dV \\ U_L &= \frac{1}{2\rho_L c_L^2}\int_V |p_L(\mathbf{x})|^2 dV \end{aligned} \tag{5}$$

The harmonically oscillating load is considered by its virtual work δW_{Load}^{nc}:

$$\delta W_{Load}^{nc} = \int_A p_{Load}\,\mathbf{n}_A(\mathbf{x})\,\delta\mathbf{w}_L(\mathbf{x})\,dA \tag{6}$$

Boundaries are incorporated by their Lagrangian function L_{BC} and by the virtual work of the related non conservative damping forces δW_{BC}^{nc},

$$\int_{t_1}^{t_2} \delta\left(L_L + L_{BC}(Z)\right) + \delta W_{BC}^{nc}(Z) + \delta W_{Load}^{nc}\,dt = 0 \tag{7}$$

which are gained out of impedances Z of the adjancent structures. The normal and constraint modes of the velocity potential Φ are calculated out of a weak formulation of the Helmholtz equation with the help of the Spectral Finite Element Method [3], obeying the related boundary conditions, which results in a homogeneous Helmholtz equation for the normal modes $\Phi_j^N(\mathbf{x})$ and an inhomogeneous Helmholtz equation for the constraint modes $\Phi_k^C(\mathbf{x})$, where a specific wave pattern is imposed for the velocity at the interface to the connected substructures.

$$\left(\mathbf{K} - \frac{\omega^2}{c_L^2}\mathbf{M}\right)\hat{\phi} = \mathbf{F}\,, \tag{8}$$

Absorptive Substructures

Plate like compound absorbers, consisting of porous and elastic layers are modeled efficiently with the help of Integral Transform Methods (ITM), where the porous material is described with the Theory of Porous Media (TPM). Wavenumber and frequency dependent impedances are derived in order to provide the coupling in the scope of Hamilton's principle. A detailed description is given in [1] and [2].

The coupled systems of differential equations, which are exemplarily given for the porous material in equations (9) are solved in the Fourier domain after applying a Helmholtz decomposition under consideration of the boundary conditions at the interfaces between the individual layers.

$$\begin{aligned} -n^S \operatorname{grad} p + (\tilde{\lambda}^S + \mu^S) \operatorname{grad}\operatorname{div} \mathbf{u}_S & + \\ +\mu^S \operatorname{div}\operatorname{grad} \mathbf{u}_S S_G(\mathbf{v}_G - \mathbf{v}_S) &= \rho^S \mathbf{a}_S \\ -n^G \operatorname{grad} p - S_G(\mathbf{v}_G - \mathbf{v}_S) &= \rho^G \\ \frac{n^G}{R\,T}\frac{\partial p}{\partial t} + \rho_0^{GR} n^G \operatorname{div}(\mathbf{v}_G) + \rho_0^{GR} n^S \operatorname{div}(\mathbf{v}_S) &= 0 \end{aligned} \tag{9}$$

In order to provide the coupling with the acoustic volume, the Lagrangian $L_{BC}(Z)$ of the absorber and the virtual work of the non conservative damping forces δW_{BC}^{nc}, which are gained out of the impedance Z of the substructure, have to be included in equation (7). Therefore finally impedances $Z(k_x, k_y, \Omega)$, which consider the angle of inclination of the sound waves due to the wavenumbers k_x and k_y, are derived for the compound absorber.

Interior Substructures

Interior structures can't be modeled with the help of the ITM approach. They are considered in the scope of a Craig Bampton method using Sprectral Finite Elements (SEM) and modal based attachment modes at the interfaces.

Coupling of a fluid with a resonator or a reflector is done by establishing the equilibrium inside of each component.

$$\begin{aligned} M_L \ddot{\tilde{p}} + K_L \tilde{p} &= f_L + f_w \\ M_S \ddot{\tilde{w}} + K_S \tilde{w} &= f_S + f_p \end{aligned} \tag{10}$$

The interaction forces f_w and f_p cause the additional mass and stiffness matrices M_{SL} and K_{SL} in the equation of motion of the coupled system:

$$\begin{bmatrix} M_{SS} & 0 \\ M_{SL} & M_{LL} \end{bmatrix} \begin{bmatrix} \ddot{\tilde{w}} \\ \ddot{\tilde{p}} \end{bmatrix} + \begin{bmatrix} K_{SS} & K_{SL} \\ 0 & K_{LL} \end{bmatrix} \begin{bmatrix} \tilde{w} \\ \tilde{p} \end{bmatrix} = \begin{bmatrix} f_S \\ f_L \end{bmatrix} \tag{11}$$

In (11) the subscripts S and L stand for the solid and for the fluid. F_L and F_S denote the external loading acting on the fluid and on the solid.

Due to the large number of unknowns in the finite element model for the fully coupled system the modal analysis is very expensive for large structures. The Craig Bampton approach permits a computation of the the fully coupled system out of the subsystem solutions.

In the full system, substructures are represented with generalized coordinates. In the modified Craig Bampton method [4], quasi-static global attachment modes are used at the interface between two subsystems.

$$\begin{bmatrix} \Psi_{t\bar{a}} \\ \Psi_{ta} \end{bmatrix} = \begin{bmatrix} K_{\bar{a}\bar{a}} & K_{\bar{a}a} \\ K_{\bar{a}a}^T & K_{aa} \end{bmatrix}^{-1} \begin{bmatrix} 0_{t\bar{a}} \\ \Phi_{ta} \end{bmatrix} \tag{12}$$

In equation (12), a stands for the the interface nodes and $\bar{a}$ for not-interface nodes. In this modified approach, the number of attachment modes depends on the frequency range of interest and it is significantly smaller than the number of attachment modes in standard Craig Bampton method. Finally the impedance Z, which is used to formulate the Lagrangian $L_{BC}(Z)$ in equation (7) for coupling the substructures, is evaluated for the subsystem.

Example

The example represents a rectangular geometry with dimensions $L_x = 3m$ and $L_y = 1m$, which is coupled with an absorptive boundary condition. Plane waves, harmonically oscillating in time are excited in the volume. The steady state response is computed in a frequency range up to 300 Hz and the results are compared to a room with reflective walls. In *System 1* a totally reflective boundary is modeled at $x = 0$. In *System 2* a layer of melamine foam with a thickness of $10cm$ is placed at $x = 0$. For both systems the load is positioned at $x = 3m$.

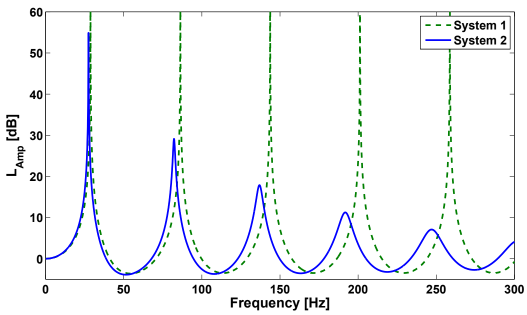

Figure 2: FSI example - Influence of the porous absorber

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