Some further experiments with the beam diffraction model based on the uncertainty relation - is it valid also with double diffraction?

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Motivation

In room and city acoustics (noise immission prognosis), ray or beam tracing methods (RT/BT) are well approved. A version of RT is the statistical sound particle method with its detector technique [1]. BT is an efficient deterministic straight forward implementation of the mirror image source method MISM. The aim is still an efficient recursive introduction of diffraction, also for higher orders, but without explosion of the number of rays and computation time. Therefore the transition from RT to BT [2] and a convex subdivision [3] of the room seems necessary to allow an overlap of beams and hence a re-unification as it is proposed by the method of Quantized Pyramidal Beam Tracing (QPBT) [4]. The author´s early energetic approach to diffraction based on the uncertainty relation (UR) [5] (to be published first time in-depth in 2010 in ACUSTICA [6]) has been improved in recent years in several steps [7], now utilizing BT, tested for many additional configurations. Reference cases were the semi-infinite screen and the slit (two edges) as a self-consistency-test. At least Maekawa's 'classical' 'detour-model' [8] should be fulfilled. Later Svensson´s exact wave-theoretical secondary edge source model [9] became the reference model. Last year, some discrepancies occurred, mainly with the non-fulfilment of the reciprocity principle in some cases. To overcome this, now some improved versions of the two basic functions have been tested: the 'Diffraction angle probability density function' (DAPDF) and the 'Edge Diffraction Strength'(EDS). A new DAPDF could be derived from wave theory. Further more, the applicability of the model to double diffraction has been investigated numerically, a) at a slit, but now with finite source and receiver distances, b) at two edges in cascade, forming a 'thick' obstacle. This short paper is as a continuation of the last year's paper [7] and only reports some results. Nevertheless, the basic features of the UR-based diffraction model shall be repeated.

The Sound Particle Diffraction Model

The basic hypotheses are: diffraction is an edge effect, however with RT, rays never hit edges exactly, they pass only nearby, thus, the classical wave theoretical diffraction approaches can not be utilized. The particle model and the energetic superposition shall be retained. Inspired by the UR (the by-pass-distance as an 'uncertainty') the diffraction probability ***D*** should be the stronger the closer the by-pass-distance ***a***. The diffraction pattern, the DAPDF is (as known from the Fraunhofer diffraction for parallel incidence) derived from the spatial Fourier transform of the transfer function of a slit $\propto \sin^2 u / u^2$ with $u = \pi \cdot b_{eff} \cdot \sin\varepsilon$ (0), smoothed over a wide frequency band and simplified:

$$D(u) = N_0 / \left(1 + 2u^2\right) \text{ with } u = 2 \cdot b_{eff} \cdot \varepsilon \qquad (1)$$

where b_{eff} is the apparent slit width in wavelengths, ε is the deflection angle (see fig. 1) and N_0 *is a* normalization factor.

To develop a module which is applicable also to several edges passed nearby simultaneously, EDS of several edges may be added up to a total $TEDS = \sum EDS_i$ (2)

An 'effective slit width' is then $b_{eff} = 1/TEDS$ (3)

By self-consistency-considerations, it turns out that

$$EDS(a) = 1/(6 \cdot a) \qquad (4)$$

So, with only one edge, a by-passing particle would 'see' a relative slit-width of $b_{eff}=6a$ (all distances in units of wavelengths λ).

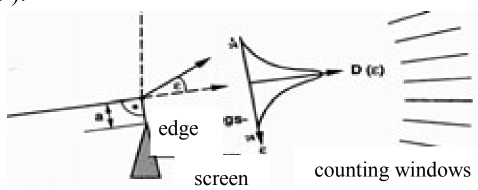

Fig.1: The sound particle diffraction model: Each moment a particle passes an edge of a screen at a distance *a* (below), it 'sees' a slit. According to the uncertainty relation a certain EDS causes the particle to be diffracted according to the DAPDF=D(ε). On the right some angle windows used to count the diffracted particles and to add up their energies to the transmission degrees. All the shifted DAPDFs of the different rays add up to the screen transmission function (as e.g. in fig. 3).

Method of evaluation

For a systematic analysis, 2dim. RT- and BT were evaluated for sources S and receivers R at finite distances r_s and r_r of 1,3,10,30,100 λ and 15 angles φ_r (and later also φ_s) -84…+84° in steps of 12°, in total 375 combinations at the screen (fig.2) as well as at the slit (of width ***b*** between two edges at ***–b/2*** and ***+b/2*** on the y-axis).

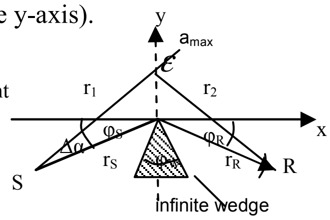

Fig.2: Geometrical definitions at the screen; dotted: the 'transparent wall' (aperture) at which the particles are diffracted

For all these parameters, the transmission degree T was determined (intensity with diffraction versus free field, for the slit, the DAP itself [7]). At the first go, the agreements with the reference functions were very good for almost all cases, now also for finite distances (standard deviation 0.66dB, curves similar as in fig. 3). In 1986, this happened even for many cases of the slit, however, there were up to 3 dB too high levels at angles 'deep in the shadow' compared with the slit function itself. To reduce that, another EDS was tested with an exponentially decreasing strength

$$EDSE(a) = 1/\left(3 \cdot a + e^a\right) \qquad (4b)$$

Also, to spare computation time, the EDS-functions may be limited, such that they are zero for $a>a_{max}=7\lambda$.

With this, at the slit the agreements become much better: max. deviation 1dB, std. dev. 0.5dB.

After transition to the more efficient beam tracing [7], the agreements with the Svensson reference model were again very good (only 0.39dB).

To exclude any numerical error due to the finite number of beams in further optimizations, a (numerical) beam integration was introduced. Thereby, the screen transmission is simply computable by an integral over the by-pass-distance

$$T = R \cdot \int_0^{a\max} \frac{d(\varepsilon(a), b_{eff}(a)) \cdot \cos(\varphi_s)}{r_1(a) \cdot r_2(a) \cdot \cos(\Delta\alpha(a))} da \qquad (5)$$

where ***d*** is the DAPDF involving the EDS ***b**$_{eff}$**(a), R*** is the direct source-receiver distance, ***r**$_{1,2}$* are the radii to source and receiver from the bending point and $\Delta\alpha$ is the angle at the source (see fig.2.)

The problem with the reciprocity principle

Equ. 5 is ***not*** symmetric with respect to an interchange of source/ receiver. As turned out in [7], reciprocity, an important condition for the correctness of the model, is not fulfilled. (It seemed so before, but only for $\varphi s = 0$, as if only $\varphi s + \varphi r \approx \varepsilon$ were relevant.) If, however, also φs and φr are interchanged, severe deviations occurred in cases of high negative values of φs. So, equ. 5 should be made symmetric by introducing a $\cos(\varphi r)$ factor in the nominator, or the DAPDF should be completed approximately by a $\cos(\varepsilon)$ factor in the nominator of equ. 1 with $u = \pi \cdot b \cdot \sin\varepsilon$ from now on.

Attempts at optimizations of the DAPDF

The classical textbook derivation of the Fraunhofer formula (0) is only an approximation for small angles and for a plane perpendicular incident wave [10]. A more thorough derivation, starting with the Kirchhoff-Helmholtz-Integral for the aperture revealed that a factor $f = [\cos(\varphi s) + \cos(\varphi r)]/2$ occurs, the pressure at the receiver is $p \propto f \cdot b \cdot \left(\frac{\sin(u)}{u}\right) \cdot e^{ikr_r} / r_r$ (6).

To get the energetic transmission ***T***, ***f*** has to be squared, and in reality only $\varphi_s + \varphi_r = \varepsilon$ is relevant such that $\varphi_s = \varphi_r = \varepsilon/2$ and the characteristic factor $f^2 = \cos^2(\varepsilon/2) = (1 + \cos\varepsilon)$ occurs. With that the reciprocity is better fulfilled. The following DAPDFs were tested, here displayed with the results from sound particle diffraction simulations at the screen, 1. with the EDSE, 2. with the EDS, 3. at the slit with the EDSE e.g. with b=10, standard deviations compared with Svensson.

Tab.1. | comp.ref.	screen EDSE	EDS	slit [dB]
1) $D1(b_{eff}, \varepsilon) = D(u)$	1.3	0.7	0.5
2) $D2(b_{eff}, \varepsilon) = D(u) \cdot \cos\varepsilon$	3	1.8	0.7
3) $D3(b_{eff}, \varepsilon) = D(u) \cdot (1 + \cos\varepsilon)/2$	1.8	0.8	0.5

So, ***D$_3$***, seems to be the optimum DAPDF. fig. 3 shows one of the screen results.

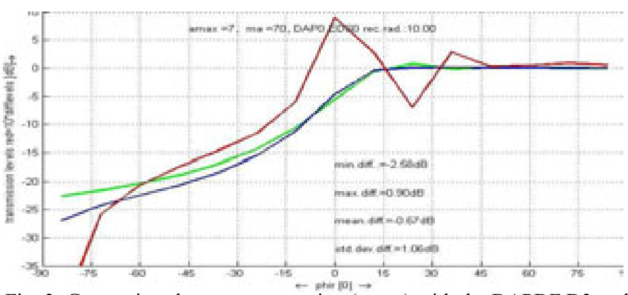

Fig. 3: Comparison between ray tracing (green) with the DAPDF D3 and Svensson's reference method (blue). The transmission degree in dB is given as function of the receiver angle, to the left the 'shadow' region; red curve: deviation* 10; a_{max}=7 λ, source and receiver distance: 10 λ, $\varphi_s = 0$.

Now first time experiments for the slit with finite source and receiver distances were performed with even better results:

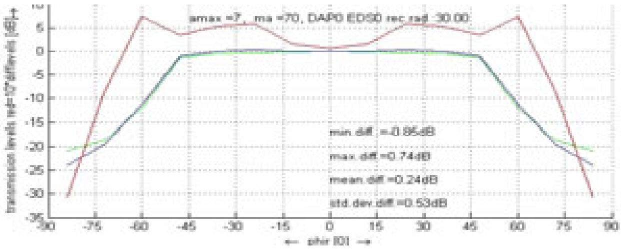

Fig. 4: Same kind of comparison as with fig.3, but for a slit; example for width b=30 λ and source and receiver distance: 30 λ, DAPDF= D3.

Some experiments with double diffraction

By convex sub-division, unintentional (not real) double-diffraction might occur.

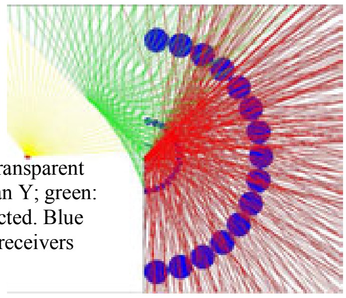

Fig.5: Double diffraction at 2 'transparent walls' forming with the screen an Y; green: rays 1.order, red: 2. order diffracted. Blue circles: particle detectors at the receivers

In many cases (even for 90° split-off-walls as in fig. 5), only small errors < 1dB (compared with single diffraction in the middle).

Conclusion

An improved DAPDF better fulfilling the reciprocity principle was found yielding good agreements with the reference models for screen and slit, also for finite distances, mostly better 1dB. Unintentional double diffraction at two close edges is not harmful. But better DAPDFs, second and higher order diffraction, and a generalization to 3 dimensions still have to be tested. It seems, the UR can be used as an efficient approximation in acoustics for diffraction, even for higher orders. For the other results with double diffraction address the author. Thanks to A. Pohl for the RT-experiments.

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