On bubble stability in a standing sound field

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Introduction

In ultrasonic cleaning and in sonochemistry bubbles and their motion (oscillation, translation) play a dominant role. The present contribution discusses the stability of a single spherical bubble in a standing sound field, emphasizing the influence of the static pressure on bubble stability.

Bubble model

A gas filled spherical bubble isolated in an infinite, compressible and viscous liquid oscillates under the action of a sinusoidal sound wave. The model is based on the Keller-Miksis model [1] simplified by omitting the time delay [2]:

$$\begin{aligned}&\left(1-\frac{\dot{R}}{C_0}\right)R\ddot{R}+\frac{3}{2}\dot{R}^2\left(1-\frac{\dot{R}}{3C_0}\right)\\&=\left(1+\frac{\dot{R}}{C_0}\right)\frac{p_l}{\rho}+\frac{R}{\rho C_0}\frac{\mathrm{d}p_l}{\mathrm{d}t},\end{aligned}\tag{1}$$

$$\begin{aligned}p_l &= \left(p_{\text{stat}}-p_{\text{v}}+\frac{2\sigma}{R_{\text{n}}}\right)\left(\frac{R_{\text{n}}}{R}\right)^{3\gamma}\\&\quad -p_{\text{stat}}+p_{\text{v}}-\frac{2\sigma}{R}-\frac{4\mu}{R}\dot{R}+p_{\text{ac}}(t),\end{aligned}\tag{2}$$

$$p_{\text{ac}}(t) = -p_{\text{a}}\sin(\omega t).$$

The equation is solved numerically for the constants: liquid density $\rho = 0.998$ kg/m^3, surface tension $\sigma = 0.07275$ N/m, viscosity $\mu = 0.001$ Pa·s, vapor pressure (water) $p_{\text{v}} = 2.330$ kPa, static pressure $p_{\text{stat}} = 100$ kPa to 20 000 kPa, polytropic exponent $\gamma = 1.67$, sound velocity in the liquid $C_0 = 1482$ m/s. The variables in the equation are the radius of the bubble R and time t, an overdot means the derivative with respect to time t, thus $\dot{R}$ is the bubble wall velocity and $\ddot{R}$ is the bubble wall acceleration. R_{n} is the bubble radius at rest. The driving sound field $p_{\text{ac}}(t)$ is assumed to be sinusoidal with pressure amplitude p_{a} and circular frequency $\omega = 2\pi\nu_{\text{a}}$, ν_{a} being the driving frequency.

Stability

The positional, shape and diffusion stability will be considered as defined in the subsequent sections. When all three stability requirements are fulfilled, the bubble is said to be stable for the given parameters.

Positional stability

In a sound field with its pressure gradients a bubble experiences forces across the bubble surface. In a standing sound field the net force on the bubble is directed along the gradient of the sound field and reads

$$\vec{F}_{\text{B}} = -\langle \nabla p_{\text{ac}}\, V(t)\rangle_\tau \quad \text{with} \quad p_{\text{ac}} = p_{\text{a}}(\vec{x})\,\sin(\omega t)\,. \tag{3}$$

$\vec{F}_B$ is the force averaged over a time span τ of the bubble oscillation, the bubble entering via its volume $V(t) = 4\pi/3\,R^3(t)$. The bubble is located at $\vec{x}$ where it encounters the pressure amplitude $p_{\text{a}}(\vec{x})$ with the actual sound pressure varying sinusoidally in time. In the diagrams below, the points in the parameter plane $(R_{\text{n}}, p_{\text{a}})$ are calculated, where $\vec{F}_{\text{B}}$ switches sign [3] giving the positional stability border, where bubbles are driven away from the pressure antinode upon increasing the driving pressure amplitude (green-red border).

Shape stability

To test the shape stability a perturbation in the form of spherical harmonics is introduced and it is calculated, whether the time dependent amplitude coefficients, a_l, of the harmonics decay (parametric stability) or grow (parametric instability). The lowest aspherical mode, numbered $l = 2$, is the easiest to grow (for the equations to solve, see [4]). In the diagrams below, the shape instability border is calculated in the plane $(R_{\text{n}}, p_{\text{a}})$ for different gas concentrations of the liquid, c_∞, and for different static pressures, p_{stat}. This leads to the upper white area of shape instability.

Diffusional stability

A bubble exchanges gas molecules across its interface with the surrounding liquid. Because of surface tension the gas pressure inside a bubble is higher than the partial gas pressure in the liquid and thus a bubble dissolves. If, however, a bubble oscillates nonlinearly in a sound field, the mass flow may be reversed, an effect called rectified diffusion. The mass flow leads to an alteration of the bubble radius at rest, R_{n}. No diffusion on average is given by $\mathrm{d}R_{\text{n}}/\mathrm{d}t = 0$ and leads to the expression (see [4] for more details):

$$\frac{c_\infty}{c_{\text{sat}}} - \left(1+\frac{2\sigma}{R_{\text{n}}p_{\text{stat}}}\right)\frac{\left\langle (R/R_{\text{n}})^{4-3\gamma}\right\rangle_{T_{\text{osc}}}}{\left\langle (R/R_{\text{n}})^{4}\right\rangle_{T_{\text{osc}}}} = 0\,, \tag{4}$$

where c_∞ is the actual given gas concentration in the liquid far away from the bubble and c_{sat} is the saturation gas concentration of the liquid. An artificial inert gas with $c_{\text{sat}} = 0.6$ mol/m^3 is taken. The notation $\langle\cdot\rangle$ means averaging over time and the averaging time is one period of the oscillation of the bubble, T_{osc}. In the diagrams below, the diffusional equilibrium is calculated in the $(R_{\text{n}}, p_{\text{a}})$ plane leading to the lower green-white or lower red-white border.

Bubble habitat

The region in parameter space, where positional and shape stability criteria are fulfilled for a not dissolving

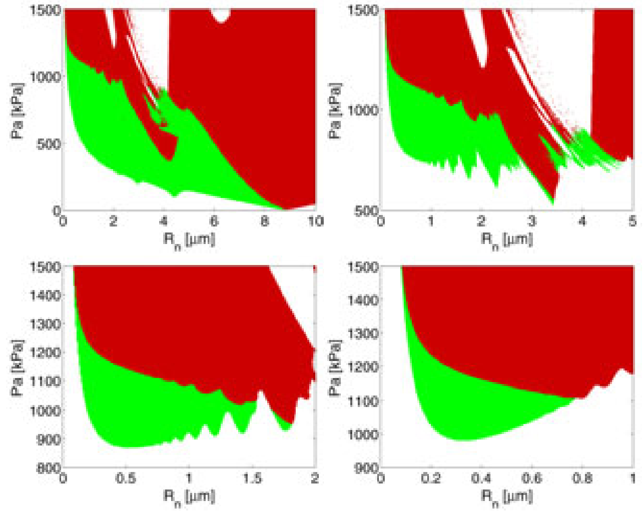

Figure 1: Region of shape and positionally stable bubble oscillations for not dissolving bubbles (bubble habitat, green area) in a standing sound field of $\nu_a = 1$ MHz for the static pressure $p_{stat} = 600$ kPa at four different gas concentrations, given by $c_\infty = c_{sat}$ (upper left diagram), $0.1\,c_{sat}$, $0.01\,c_{sat}$, and $0.001\,c_{sat}$ (lower right diagram). In the red area the oscillations are stable with respect to spherical shape, but unstable with respect to position in the pressure antinode. In the upper white areas the bubbles are shape unstable. In the white area below the lower bound bubbles start to dissolve. Linear resonance radius $R_{n0} = 8.81$ μm.

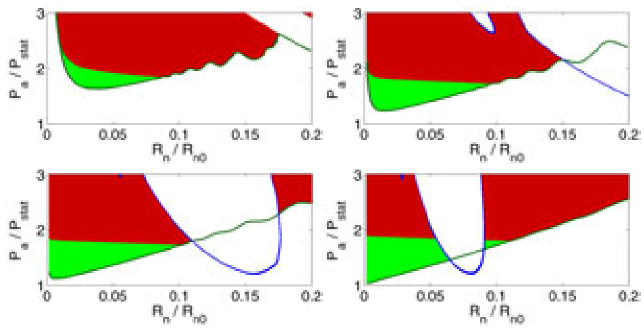

Figure 2: Region of shape and positionally stable bubble oscillations for not dissolving bubbles (bubble habitat, green area) in a standing sound field of $\nu_a = 1$ MHz at the the gas concentration of $c_\infty = 0.001\,c_{sat}$ for the four static pressures $p_{stat} = 600$ kPa ($R_{n0} = 8.81$ μm), 2 MPa ($R_{n0} = 15.95$ μm), 5 MPa ($R_{n0} = 25.2$ μm), and 20 MPa ($R_{n0} = 50.4$ μm). Area color code as in Fig. 1. The dark green lines are the boundary for diffusional stability, the blue lines mark the boundaries for shape stability.

bubble, is called the bubble habitat. In Fig. 1, this region is plotted in green color in the parameter plane (R_n, p_a) for a bubble driven at 1 MHz at the elevated static pressure of $p_{stat} = 600$ kPa for four different gas concentrations c_∞ as given in the figure caption. At constant static pressure the bubble habitat shrinks with decreasing gas concentration in the liquid, c_∞, down from saturation gas pressure, c_{sat}, the dissolution boundary first getting rugged ($c_\infty = 0.1\,c_{sat}$), then smooth again ($c_\infty = 0.001\,c_{sat}$). The range of the bubble sizes of the habitat shrinks from more than 8 μm down to less than 1 μm at $c_\infty = 0.001\,c_{sat}$.

In Fig. 2, the habitat (green area) is plotted at the reduced gas concentration $c_\infty = 0.001\,c_{sat}$ for four different static pressures as given in the figure caption. The parameter plane $(R_n/R_{n0}, p_a/p_{stat})$ has been introduced with the bubble radius at rest, R_n, normalized with the linear resonance radius, R_{n0}, and the driving pressure amplitude, p_a, normalized with the static pressure, p_{stat}. The linear resonance radius, R_{n0}, is calculated according to the equation

$$\nu_0 = \frac{1}{2\pi R_{n0}\sqrt{\rho}}\sqrt{3\kappa(p_{stat} + \frac{2\sigma}{R_{n0}} - p_v) - \frac{2\sigma}{R_{n0}} - \frac{4\mu^2}{\rho R_{n0}^2}}. \tag{5}$$

with $\nu_0 = \nu_a = 1$ MHz. When the static pressure is increased at constant gas concentration, the bubble habitat grows and extends to increasingly larger bubbles, as the linear resonance radius, R_{n0}, grows with increasing static pressure. The maximum bubble sizes in the habitat rise from less than 1 μm at $p_{stat} = 600$ kPa to more than 5 μm at $p_{stat} = 20$ MPa. In the normalized parameter plane the habitat attains a triangular form in the limit of high static pressures with normalized driving pressures between one and two and normalized radii up to 0.1. This region may be intersected by the loss of shape stability.

Conclusion

With decreasing gas concentration in the liquid the bubble habitat shrinks and extends to ever smaller bubble radii. With increasing static pressure the bubble habitat expands and extends to ever larger bubble radii. An increased static pressure thus can compensate a concentration decrease to very low gas concentrations that usually is necessary to ensure diffusional stability [5]. With larger bubbles, however, shape instability finally reaches the habitat region again (Fig. 2, lower right diagram).

References

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