Efficient Simulation of Acoustic Fluid-Structure Interaction Models by Means of Model Order Reduction

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Introduction

For many engineering applications like vessels, pipes, pumps, turbines, transformers, or ships Fluid-Structure-Interaction (FSI) has to be accounted for. For small amplitude vibrations the elasto-acoustic approach is very popular within the numerical framework of Finite-Element-Method (FEM). One advantage is that both frequency domain and transient solutions are applicable. Another pro is the matrix coupling scheme holding also for strong coupling effects. However extensive computational resources are required for large coupled systems and an option to increase efficiency will be presented in the next sections.

Coupled fluid-structure system of equations

$$\underbrace{\begin{bmatrix} \mathbf{M}_s & \mathbf{0} \\ \rho_f \mathbf{R}^T & \mathbf{M}_f \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} \ddot{\mathbf{U}} \\ \ddot{\mathbf{P}} \end{bmatrix}}_{\ddot{\mathbf{x}}} + \underbrace{\begin{bmatrix} \mathbf{C}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_f \end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix} \dot{\mathbf{U}} \\ \dot{\mathbf{P}} \end{bmatrix}}_{\dot{\mathbf{x}}} + \underbrace{\begin{bmatrix} \mathbf{K}_s & -\mathbf{R} \\ \mathbf{0} & \mathbf{K}_f \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} \mathbf{U} \\ \mathbf{P} \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} \mathbf{F}_s \\ \mathbf{F}_a \end{bmatrix}}_{\mathbf{F}} \tag{1}$$

The 1st line of the FSI system (1) gives the structural equations for the vector of nodal displacements **U**, the 2nd line the fluid (acoustic) equations for the nodal values of sound pressure **P**. This so-called (u,p)-formulation is implemented in ANSYS. Due to the off-diagonal terms in the mass matrix **M** and the stiffness matrix **K** coupling is introduced.

However, note that the system (1) is not symmetric. Thus normal modes aren't available and consequently standard solvers and mode superposition usually applied for efficient solutions in linear dynamics cannot be applied. Instead LU decomposition of the full matrices is required for every frequency step resulting in high computational costs.

Model Order Reduction (MOR)

A relatively new approach for efficient solutions of arbitrary 1st and 2nd order systems is motivated by mathematicians. This model order reduction [2, 3] technique has originally been addressed to system simulation where a high order system is reduced to a low order system to make (control) system simulations feasible. For the present study the focus is set on a fast solution scheme, where even one single solution shall benefit from increasing performance.

The basic idea is the projection of the high order matrices to a low order subspace. By approximating the transfer functions of arbitrary input and output quantities one ends up in the scheme of *implicit moment matching*. *Moment*, because not the transfer function itself but the higher order moments are matched by a Padé approximation. *Implicit*, because this is not done explicitly by computing the coefficients but rather looking for the Krylov-subspace. The latter inherently renders the required properties and may be computed by the Lanczos or Arnoldi algorithm. Refer to [2] and [3] for more details about the mathematical algorithm.

MOR for a harmonic industrial FSI problem

A published industrial application of MOR in frequency domain is the vibration analysis (fatigue) of a Francis turbine runner in water [1] shown in Fig. 1.

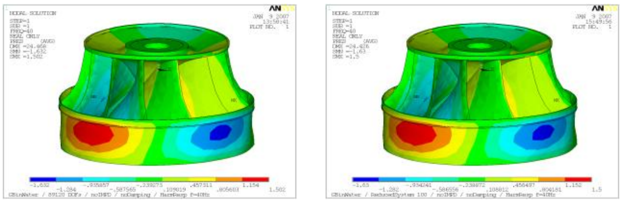

Fig. 1: Real part of pressure distribution at 40 Hz for a Francis turbine runner in water [1]. Observed deviation < 0.2% between full model with 90000 Degrees of Freedom (DOFs) (left) and Krylov-subspace based reduced model with 100 DOFs (right).

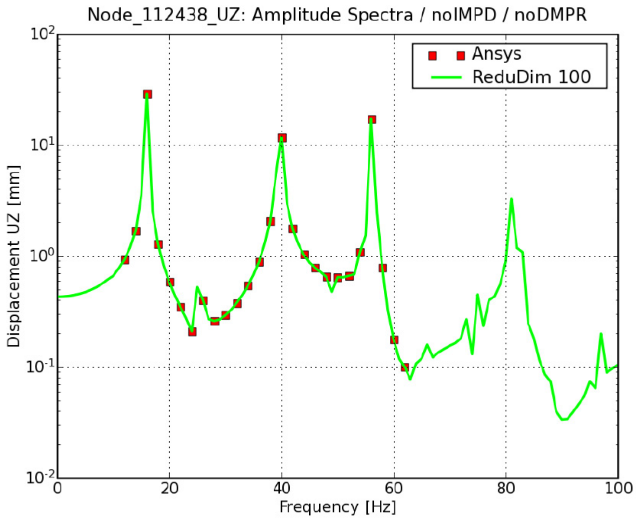

Fig. 2: Amplitude spectrum for axial displacement at center trailing edge [1]. ANSYS model with full matrices (marker points) versus reduced ANSYS & MOR model (solid line).

The accuracy of the approach is apparent in the contour plot of the computed pressure distribution (Fig. 1) for the full ANSYS model and the reduced MOR system as well as for the frequency response of displacement amplitude (Fig. 2). The efficiency of this approach is confirmed by the fact that the frequency sweep of full model takes 2h, whereas just 2min are required for the reduced model.

MOR for a transient uncoupled acoustic test case

Neglecting FSI in the following a simple transient, uncoupled acoustic 2D test case is investigated where a short-time (T_P=0.5ms) pressure perturbation is excited at the left end of a duct (l=4m). For this problem a propagating plane wave is expected. FE simulation based on a full transient ANSYS solution (FLUID30 elements) support this (see Fig. 3) fact of wave propagation without notable dispersion.

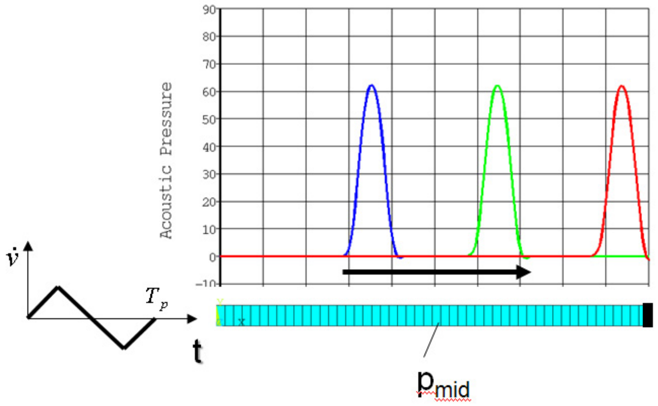

Fig. 3: Acoustic duct excited by a source at the left end. At the right end a) a rigid b) an absorbent boundary condition (b.c.) is applied. The spatial pressure distribution p(x) in the duct is shown at three instances of time showing the propagating wave without dispersion (ANSYS full matrices).

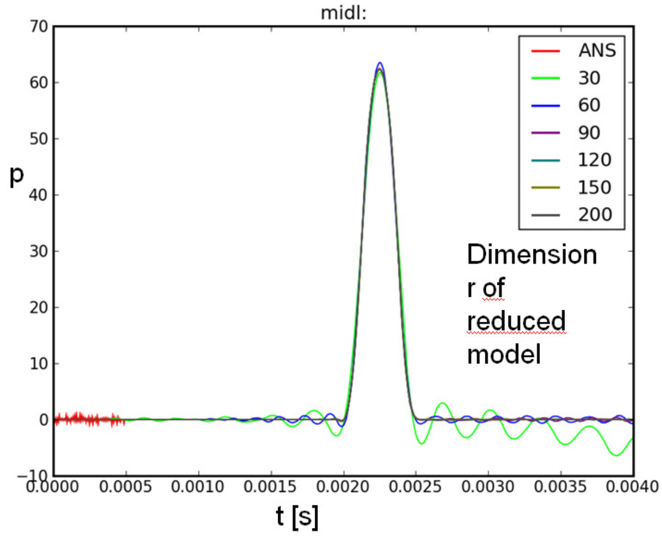

Fig. 4: Pressure signal $p_{mid}(t)$ simulated at the center of the duct (see **Fig. 3**) with rigid, reflecting b.c. at the right end for different orders r=30..200 of the reduced model (MOR) in comparison to the full transient ANSYS solution.

Applying MOR for this system with a rigid, reflecting boundary condition the short-time excitation is a challenge due to resulting high frequency components that have to be resolved in the transfer function (implicit moment matching) by the reduced order model. In contrast to Fig. 3 (full transient simulation) the transient pressure simulation in Fig. 4 by means of MOR clearly shows numerical dispersion for low orders (r=30). However those artificial, non-physical oscillations decrease with increasing order r.

This convergence behavior is becoming worse for an absorbent b.c. at the right end. In that case non-proportional damping is introduced and despite the fact that an order r=200 has been tested (rel. high order compared to 800 DOFs of full model) the solution still shows numerical dispersion. The current MOR algorithms are prepared for proportional damping (C=αM+βK) and localized, non-proportional damping requires modification in the applied Arnoldi algorithm [2], [3].

Outlook

It has been shown that MOR is an attractive technique for efficient simulations of large coupled systems. Due to its flexibility MOR is not just applicable to 2nd order systems but also to other problems (e.g. thermal). For harmonic coupled analysis (low frequency) the performance of MOR has been demonstrated for an industrial case. For transient analysis with high frequency components and/or non-proportional damping mechanisms further development has to be spent to improve the applied Arnoldi algorithm [2] for computation of the Krylov-subspace.

MOR, however, is not the only means to reduce solution times of coupled FSI systems. Revisiting the unsymmetric system of eq. (1) another option is the extension of the standard mode superposition scheme by applying left and right eigenvectors.

Alternatively an additional DOF φ can be introduced in order to get symmetric governing equations [4] (neglecting damping for the moment)

$$\begin{bmatrix} \mathbf{M}_s & \mathbf{0} & \rho_f \mathbf{R} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_f \\ \rho_f \mathbf{R}^T & \mathbf{M}_f & -\rho_f \mathbf{K}_f \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}} \\ \ddot{\mathbf{P}} \\ \ddot{\mathbf{\Phi}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_s & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{1}{\rho_f}\mathbf{M}_f & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{P} \\ \mathbf{\Phi} \end{bmatrix} = \mathbf{0}. \quad (2)$$

The fluid displacement potential φ (or its nodal vector **Φ**), is related to the sound pressure by $p=\rho_f \Omega^2 \varphi$. Besides the fact of speed ups in solution time (e.g. an observed speed-up factor 70 for a harmonic frequency sweep of a small coupled model) it is attractive for engineering community to have access to modal analysis and all subsequent spectral or transient analysis types based on modal techniques (power spectral density, response spectrum, harmonic/transient mode superposition). Test cases simulated in ANSYS 12.1 (beta feature) show promising results with respect to both functionality and efficiency, e.g. for nuclear contained fluid analysis [4].

Literatur

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