Sensitivity Analysis: Fundamental for Multi-Objective Optimization of Natural-Fibre-Reinforced-Plastic Composites

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Introduction

In this paper the application is natural-fiber-reinforced-plastics (NFRP) composites involved in the construction of civil structures. A new acoustical functionality is gained by designing multifunctional components to meet sound insulation and absorption requirements besides demands on thickness and weight of the components. While absorption quality depends on the dissipation of sound energy, for example effected by porous materials, the insulation is mainly encouraged by solids and increases with higher mass. Therefore the multifunctional components are built of bonded open-porous and hard-pressed layers. The aim of the optimization strategy is to determine the most appropriate layer setup according to individual user needs, i.e. absorption coefficient α, the sound insulation index R, weight G and thickness d. These objective functions are optimized based on measurements or simulations using prediction models to minimize the number of produced samples. The input parameters for this prediction models are sometimes measured with large uncertainties or can only be defined within an area of variation. So in this multi-objective-optimization a question of sensitivity arises; how much the variation of input data lead to changes of the multi-optimization and therefore how reliable are the results. Therefore, a sensitivity analysis is preformed.

Multi-Objective Optimization

The solution of a multi-objective optimization problem consists of the minimization of the combinations of all objective functions under defined constrains.

$$\boldsymbol{min}\ \boldsymbol{F}(\vec{\boldsymbol{x}}) = \{\boldsymbol{f}_1(\boldsymbol{x}_1), \ldots, \boldsymbol{f}_k(\boldsymbol{x}_k)\} \qquad (1)$$

For multi-objective optimization, the combination method is used (equation 2). First all the objective functions $\boldsymbol{f}_i$ are transformed in a common quality range $\tilde{\boldsymbol{f}}_i$. Then it sums the values by using weighting factors to an overall quality range.

$$\boldsymbol{F}(\vec{\boldsymbol{x}}) = \sum_{i=1}^{k} \boldsymbol{w}_i \cdot \left(\boldsymbol{1} - \tilde{\boldsymbol{f}}_i(\vec{x})\right) \quad \text{with} \quad \sum_{i=1}^{k} w_i = 1,\ \boldsymbol{w}_i \geq \boldsymbol{1} \qquad (2)$$

Application on Multi-functional NFRP Components

To provide an effective and systematic adjustment of the layer setup reliable prediction models describing the objective functions $\boldsymbol{f}_i$ of NFRP have to be found. These models have to determine the absorption as well as the insulation characteristics, basic definitions are found in [1]. The hard-pressed plate is handled with models for common elastic plates. Depending on the pore geometry of the porous material, various models are available to predict the sound transmission through these layers, such as Delany-Bazely, Allard-Johnson and Biot models, details in [1]. After comparisons of simulation results with measurements, the Biot model is used here as the predicting model for NFRP components. The input parameters for this model includes Young's modulus E, loss factor η, Poisson ratio ν, air flow resistivity σ, porosity φ, tortuosity α_∞, viscous length Λ and the thermal length Λ'.

The weighted absorption coefficient and the weighted sound reduction index in diffuse sound incidence were determined based on analytical formulas using the transfer matrix method for various combinations of the thicknesses of absorber and insulation component. The elastic parameters of the hard-pressed plate were easily determined by direct measurements, whereas the material properties based on the pore geometry were determined from the inverse parameter identification of the porous absorber model.

Example: Room Acoustic

In a room acoustic application, a combined component is designed as a ceiling. The optimization strategy is applied in the component assembly phase. Here, the material is already defined and the layer thickness should be optimized. Although the insulation is not the primary goal, here insulating properties are still desired. The weight in a ceiling design is a significant factor and should be minimal, while the thickness has a relatively minor role. The major objective function is the weighted absorption coefficient of the combined component. It should reach 0.7. The optimum results are shown in Figure 1; this is computed from the combination of all objective values. The white areas represent infeasible areas because of restriction conditions on the objective functions.

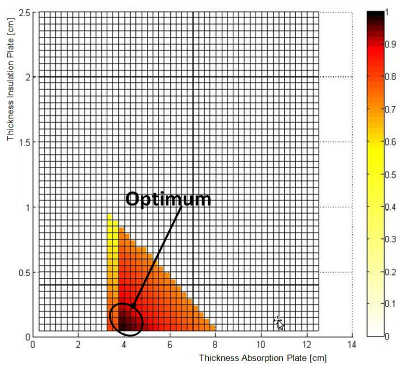

Figure 1: Quality values of the combined component in terms of the thickness.

Notice that for the optimum a set of required input parameters is identified. In order to obtain a reliable and stable optimum region, further investigation on material parameters is needed. However, the effort for obtaining these parameters, which are the input data for the prediction models, varies greatly. While the flow resistivity is comparatively easy to measure, the determination of other parameters such as porosity, tortuosity, and the characteristic lengths of the complex distributed pore geometries can only be done with high effort, normally with help of computer tomography scans.

Sensitivity Analysis

As illustrated in Figure 2, sensitivity analysis techniques are usually performed in the last phase of the optimization cycle for model evaluation, although they can be used as a useful tool in model building. These techniques aim to identify how 'sensitive' the prediction model is to the changes in the value of the input parameters and to the changes in the structure of the model. Our focus lies on methods that are global, quantitative, and model free, capable of testing the robustness and relevance of a model-based analysis in the presence of uncertainties. This is provided by the variance-based methods. Saltelli [2] describes parameter sensitivity analysis as a series of simulation-based model evaluations in which the input parameters are all varied at the same time.

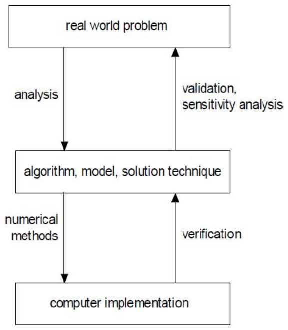

Figure 2: The Optimization Process, adopted from [3].

The Variance Based Method

These methods look at the model as a function of many input parameters, then represents the model in a high dimensional model representation (HDMR). This representation allows us to look at the model as the summation of a certain constant, a summation of the main influence, a summation of the interacting input parameters and so forth [2]. These influences are basis for the sensitivity indices. The first-order sensitivity index is defined as:

$$S_i = \frac{V[E(Y|X_i)]}{V(Y)} \quad (3)$$

It represents the main effect contribution of each input parameter to the variance of the output, see [2]. The total effect index accounts for the total contribution to the output variation due to parameter, i.e. its first order effect plus all higher-order effects due to interactions, therefore it is used to judge on the sensitivity of the model. The total-effect index for X_i is defined as:

$$S_{T_i} = 1 - \frac{V[E(Y|X_{\sim i})]}{V(Y)} \quad (4)$$

Results

A sensitivity analysis for the Biot model is preformed. Since the absorption coefficient varies in a wide frequency range, we divided it into three regions; Lower Range LFR (100-700Hz), Middle Range MFR (700-2000Hz), and Higher Range HFR (2000-6000Hz). Figure 3 shows the results of the sensitivity analysis. Parameter domains were chosen to be narrow based on literature, on experience and on known correlations [4]. Based on the total-order sensitivity index, the most important parameter is the air flow resistivity σ followed by the viscous length Λ in LFR and by tortuosity α_∞ in MFR. Notice that porosity φ and thermal length Λ' have the smallest index in all three ranges, therefore can be considered as non-influential and can be fixed anywhere in their defined restricted domain without affecting the variance of the output α. This can also be said about the E-Modulus and the loss factor. Whereas the viscous length Λ and tortuosity α_∞ show high indices in the HFR and therefore should be set to their true values in order to get robust optimum values.

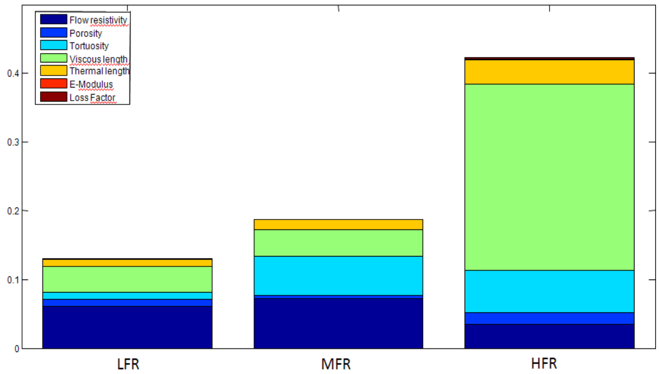

Figure 3: Sensitivity Analysis Results of the Biot prediction model.

Conclusions

The difficulty in investigating the input parameters for the prediction models of NFRP leads to the necessity of performing a sensitivity analysis on the models used in the multi-objective optimization. The identification of the most influential parameters ensures the robustness of the optimum. The parameters that need to be measured and set to their true values depend on the frequency range the multifunctional component is designed for and on the confidence level required for the optimum range.

Acknowledgments

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