Force Field Analysis of Vertical Drainage Pipes Excited by Two-Phase Flow

R. A. Alzugaray, B. A. T. Petersson
Institute of Fluid Mechanics and Engineering Acoustics, TU Berlin, Germany
Email: rialzuga@mach.ut.tu-berlin.de

Introduction

The previously discussed approach for circumferential decomposition and force field determination [1, 2], only one circumferential plane along the cylinder axis has been considered. However, the same methodology can be taken further in order to include also the contributions across the axial direction. The aim of the following steps is to rewrite the expressions of the matrix formulation for a more general excitation case.

Extended velocity field distribution

Again, by assuming a continuous interface along the pipe wall surface with circumferential and axial dependence, the response and excitation coordinates over this contour can be represented by the pairs (θ, z) and (θ_0, z_0), respectively. Thereby, the extended velocity field may be decomposed into orders [3]

$$v(\theta, z) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \hat{V}_{pq}\, \mathrm{e}^{jk_p\theta}\, \mathrm{e}^{jk_q z} \qquad [\mathrm{m/s}] \qquad (1)$$

As can be seen, the extended velocity field needs to be described in terms of two independent orders. For the continuous case, the velocity orders $\hat{V}_{pq}$ can be obtained by

$$\hat{V}_{pq} = \frac{1}{CL} \int_0^C \int_0^L v(\theta, z) \mathrm{e}^{-jk_p\theta}\, \mathrm{e}^{-jk_q z}\, dz\, d\theta \quad [\mathrm{m/s}] \qquad (2)$$

Where both the C and L comprise the interfaces in the circumferential and axial direction, respectively. Moreover, $k_p = 2\pi p/C$ and $k_q = 2\pi q/L$ stand as the interface wavenumbers (with $p,\, q \in \mathbb{Z}$).

Force field calculation

Another way to represent the two-dimensional velocity field $v(\theta, z)$, is in terms of the extended mobility $Y(\theta, z|\theta_0, z_0)$ and force field $F(\theta_0, z_0)$

$$v(\theta, z) = \int_0^C \int_0^L Y(\theta, z|\theta_0, z_0) F(\theta_0, z_0)\, dz_0\, d\theta_0 \quad [\mathrm{m/s}] \qquad (3)$$

Equation (3) can be discretized by sampling M equally spaced planes along the axial interface L and N equidistant points over the circumferential contour C. Hence, when solving the integrals by using the trapezoidal rule, by grouping and summing up equal terms, the matrix formulation is regained and yields

$$\mathbf{v}_{\,\mathrm{cyl}} = \frac{CL}{NM}\, \mathbf{Y}_{\,\mathrm{cyl}}\, \mathbf{F}_{\,\mathrm{cyl}} \qquad [\mathrm{m/s}] \qquad (4)$$

Upon carrying out the matrix inversion, the force field $\mathbf{F}_{\,\mathrm{cyl}}$ can be easily calculated. In addition, this expression resembles the same formula found in the one dimensional decomposition, except of the factor on the front. However – independent of the similarity between both equations – it is worth to point out that the cylindrical mobility matrix $\mathbf{Y}_{\,\mathrm{cyl}}$ represents the physical coupling amongst points within circumferential planes and their interaction along the axial direction. The latter conveys a general approach where the excitation that occurs in a specific plane has an effect upon the whole structure.

In order to handle the combined circumferential and axial interdependence, $\mathbf{Y}_{\,\mathrm{cyl}}$ is furnished by assembling individual mobility matrices for each plane such that describes the direct and transfer coupling between the excitation and response. Equation (5) shows the general expression for $\mathbf{Y}_{\,\mathrm{cyl}}(i|j)$

$$\mathbf{Y}_{\,\mathrm{cyl}} = \begin{bmatrix} \mathbf{Y}_{\,\mathrm{cyl}}\,(1|1) & \mathbf{Y}_{\,\mathrm{cyl}}\,(1|2) & \cdots & \mathbf{Y}_{\,\mathrm{cyl}}\,(1|N) \\ \mathbf{Y}_{\,\mathrm{cyl}}\,(2|1) & \mathbf{Y}_{\,\mathrm{cyl}}\,(2|2) & \cdots & \mathbf{Y}_{\,\mathrm{cyl}}\,(2|N) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}_{\,\mathrm{cyl}}\,(N|1) & \mathbf{Y}_{\,\mathrm{cyl}}\,(N|2) & \cdots & \mathbf{Y}_{\,\mathrm{cyl}}\,(N|N) \end{bmatrix} [\mathrm{m/Ns}] \quad (5)$$

The elements on the main diagonal (for $i = j$) – from now on called cylindrical planar mobility terms – refer to the direct coupling between excitation and response. Each of these matrices contain the point and transfer mobilities $Y(\theta, z|\theta_0, z_0)$ within the plane subjected to the force. As for the off-diagonal elements of the cylindrical mobility matrix (for $i \neq j$) – henceforth the cylindrical cross-planar mobility terms – the transfer coupling between excitation and response is given by the transfer mobilities $Y(\theta, z|\theta_0, z_0)$ at planar positions different from excitation.

In this context and for the sake of better understanding the way how $\mathbf{Y}_{\,\mathrm{cyl}}$ is assembled, Figure (1) shows a graphical example of a cylinder discretized in 3 axial planes. Note that the gray shaded areas correspond to the response planes and the arrows point out the driven plane.

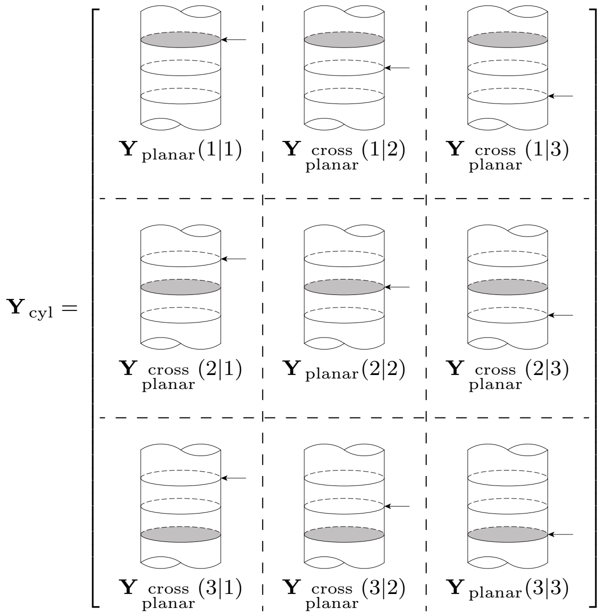

Figure 1: Schematic representation of $\mathbf{Y}_{\text{cyl}}$ for a cylinder discretized in 3 axial planes. The gray-shaded area represents the response plane. $\longleftarrow$, correspond to the excited plane.

Experimental results

Figure (2), shows the experimental back-calculated force field distribution at four different discrete frequencies. The measurements of the cylindrical mobility matrix and the spatial velocity distribution over the pipe wall surface when exerted to a two-phase flow were carried out in a test rig [1] discretized at $M = 12$ and $N = 16$.

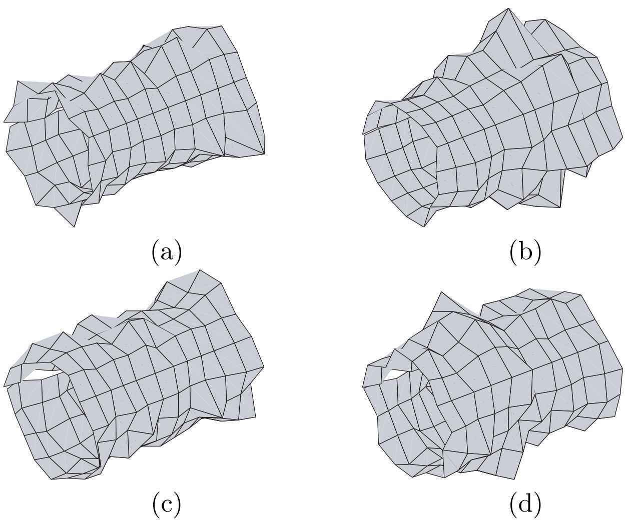

Figure 2: Experimental results for the spatial force field distribution, $F(\theta_0, z_0)$. (a) 200 [Hz], (b) 600 [Hz], (c) 1000 [Hz] and (d) 1400 [Hz].

The amplitudes of the distributions have been normalized with the purpose of comparison. The analysis of such surfaces does not give much information about the vibratory behaviour of the pipe. However, Figure (3) shows the same distributions expanded into orders $\hat{F}_{rs}$, calculated analogous to Equation (2), which is physically more meaningful [3].

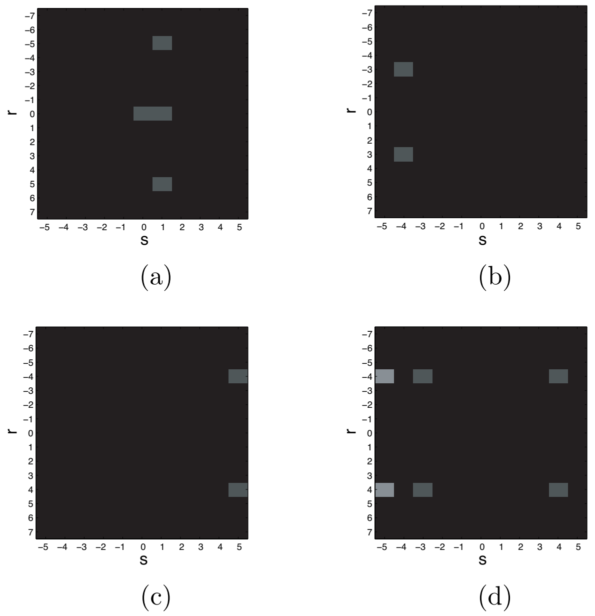

Figure 3: Experimental results for the force orders, $\hat{F}_{rs}$. (a) 200 [Hz], (b) 600 [Hz], (c) 1000 [Hz] and (d) 1400 [Hz].

Concluding remarks

The formulation for the velocity, force field and their respective orders has been presented for the case with circumferential and axial dependence. The latter approach takes advantage of the simplicity of the matrix formulation. However, care must be taken in the matrix inversion process, which is likely to amplify measurements errors. From the experimental results, it is observed that orders are symmetric in the circumferential direction. Further analysis must be done in order to determine the orders that govern the vibratory behaviour of the pipe.

Acknowledgments

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References

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