Application of the Interval Arithmetic in the Reverberation Time Uncertainty Assessment

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Introduction

The presentation of using the interval arithmetic formalism for the analysis of uncertainty calculation of the reverberation time in closed spaces is the aim of the paper. The existing calculation methods of the reverberation time depend on:

- geometrical parameters of the analysed room (it means: length, width, height),
- kind of materials of surrounding surfaces (sound absorption characteristics),
- distribution and acoustic characteristics of the reverberating surfaces,
- characteristics of interior transmission (including a sound absorption coefficient),
- sound source location.

In real computational problems of designing rooms of the qualified acoustics the difficulties in the explicit determination of these parameter values occur, which translates into the designing decisions. As a consequence the usefulness of numerical values of the analysed parameters are only approximate. Thus, in the decision-making process, it is necessary to assess the uncertainty of parameters in various models and in effect the interval of the possible variability of the reverberation time estimation.

There are several formal models for the reverberation time assessment appropriate to the estimation task. The idea of applying the Moore's interval arithmetic [1,2] for the determination of uncertainty assessment of the reverberation time calculated by equations of: Sabine, Eyring-Norris, Fitzroy and Millington-Sette [5,6], at assuming the determined uncertainty of their computational parameters, was formulated by the authors. The proposed formalism of the interval arithmetic provides the opportunity of estimating the interval, within which the result will be contained (presented by the authors in their previous paper [4]).

Computational formalism of the interval arithmetic

The characteristic feature of the interval arithmetic are operations on intervals not on numbers. These intervals determine the uncertainty of measurements or calculations and are fixed in such a way as to warrant that the real (measured) value will be contained within the given interval. The method of error assessments by means of intervals was initiated in 1950, however barely in the sixtieths it was named the interval arithmetic in the papers of Moore [1,2].

Intervals are determined as closed, limited collections of real numbers, e.g. (1):

$$\hat{x} = [\underline{x}, \overline{x}] = \{x \in R : \underline{x} \le x \le \overline{x}\} \tag{1}$$

where $\underline{x}$ is the lower limit, while $\overline{x}$ determines the upper limit.

Applications of the interval arithmetic for modelling the reverberation time

The reverberation time measurements performed by Franco Cotana [3] in the Magna Hall of the Perugia University by various statistic methods, taking into account uncertainties of input parameters, were the bases of the reverberation time determination.

The sound absorption coefficient is important in the acoustics of rooms. The most often it is experimentally determined, which means that the measurement uncertainty can have a significant influence on the computational results. The influence on the estimation results, apart from this coefficient, can also have errors: in measuring the space cubature, in the estimation of the sound absorption coefficient of air – which depends on a temperature, pressure, humidity etc. In order to take into consideration the uncertainty of the above listed parameters, the authors applied the interval arithmetic formalism for the estimation of the reverberation time. Input parameters as well as the result, are presented in forms of intervals within which the real (measured) value is contained - at each computational stage.

The reverberation time T_{Sab} was determined from the Sabine equation (2) (with taking into account the sound absorption of air).

$$T_{Sab} = \frac{0{,}161V}{4mV + S\alpha_{Sab}} \tag{2}$$

$$\alpha_{Sab} = \frac{1}{S}\sum_{i=1}^{n}\alpha_i S_i\ ,\ S = \sum_{i=1}^{n} S_i \tag{3}$$

where:

V – room volume [m^3],
S – total room surface [m^2],
m – sound absorption coefficient of air [Np/m],
α_{Sab} – average sound absorption coefficient.

The following data of the Magna Hall were used in calculations: V = 1200 m^3, m = 2.7·10^{-3} Np/m. Sound absorption coefficients for individual surfaces α_i and coordinates of points determining surface S_i are given in paper of Cotana [3]. Input parameters V, m, S, α_i, together with their uncertainty, are presented by means of interval numbers. The uncertainty of input parameters in the interval from 0% to 5% was assumed at the reverberation time estimation.

Next, the reverberation time was estimated by means of the Eyring and Norris equation (4), with a logarithmic dependency of the sound absorption coefficient.

$$T_{E\&N} = \frac{0{,}16V}{4mV + S\alpha_{E\&N}}, \quad \alpha_{E\&N} = -\ln(1-\alpha_{Sab}) \qquad (4)$$

As the next model the Millighton and Sette equation was used (5).

$$T_{M\&S} = \frac{0{,}16V}{4mV + \alpha_{M\&S}}, \quad \alpha_{M\&S} = -\sum_{i=1}^{n} S_i \ln(1-\alpha_i) \qquad (5)$$

The Fitzroy's model (6) assumes that the reverberation time in a room depends on the direction. It can be estimated by means of this model, when sound absorption coefficients: α_x, α_y, α_z (8), of surface x, y and z are determined first.

$$T_{Fitz} = \left(\frac{S_x}{S}\right)T_x + \left(\frac{S_y}{S}\right)T_y + \left(\frac{S_z}{S}\right)T_z \qquad (6)$$

$$T_x = \frac{0{,}16V}{S\alpha_{Fitz,x}}, T_y = \frac{0{,}16V}{S\alpha_{Fitz,y}}, T_z = \frac{0{,}16V}{S\alpha_{Fitz,z}} \qquad (7)$$

$$\alpha_{Fitz,x} = -\ln(1-\alpha_x), \alpha_{Fitz,y} = -\ln(1-\alpha_y), \quad \alpha_{Fitz,z} = -\ln(1-\alpha_z) \qquad (8)$$

The following dependencies of the reverberation time on the uncertainty of input parameters was obtained (Figure 1).

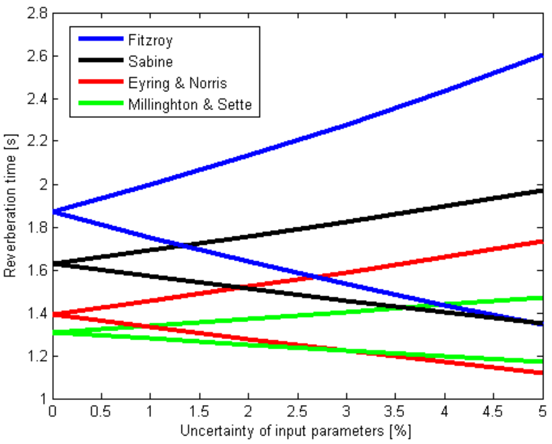

Figure 1: Reverberation time obtained by various statistic methods for a frequency of 250 Hz, at the uncertainty of parameters from 0% to 5%.

The list of the results obtained by various statistic methods as well as their percentage uncertainty ε_{Sab}, $\varepsilon_{E\&N}$, $\varepsilon_{M\&S}$, ε_{Fitz} in dependence on the percentage uncertainty of the input parameters are presented in Table 1.

As can be seen in the above list, the percentage uncertainty of estimates increases with the number of calculations needed for the reverberation time determination. When the Fitzroy's model is applied the uncertainty is higher due to the number of mathematic operations.

Table 1: Modelling results in dependence on uncertainties of parameters: V, m, S_i, α_i

Uncertainty of V, m, S_i, α_i [%]	T_{Sab} [s]	ε_{Sab} [%]	$T_{E\&N}$ [s]	$\varepsilon_{E\&N}$ [%]
1	[1.57, 1.7]	3.6	[1.33, 1.46]	4.2
2	[1.51, 1.76]	7	[1.28, 1.52]	8s
3	[1.46, 1.83]	10	[1.22, 1.59]	11.6
4	[1.40, 1.9]	13	[1.17, 1.66]	14.8
5	[1.35, 1.97]	15	[1.12, 1.73]	17.7
Uncertainty of V, m, S_i, α_i [%]	$T_{M\&S}$ [s]	$\varepsilon_{M\&S}$ [%]	T_{Fitz} [s]	ε_{Fitz} [%]
1	[1.28, 1.34]	2.3	[1.75, 2]	6.2
2	[1.25, 1.37]	4.4	[1.64, 2.13]	11.6
3	[1.22, 1.40]	6.5	[1.53, 2.28]	16.4
4	[1.2, 1.44]	8.4	[1.43, 2.43]	20.5
5	[1.17, 1.47]	10.3	[1.34, 2.6]	24.1

Final remarks

The way in which the application of the interval arithmetic formalism can be helpful in the uncertainty estimation in the reverberation time assessment in the room of the qualified acoustics, was illustrated in this paper. The selection of the proposed model formalism can be justified, from one side - by the functionality of non-probabilistic uncertainty descriptions (model parameters), adopted by the expert performing calculations of the acoustic design of the analysed room, while from the other side – by the applicability of the proven computational algorithms. Both factors, in the proposed formalisation, occur together in relation to uncertain variables in the total calculation process.

The results - presented in the hereby paper - can be useful in undertaking decisions on the basis of not fully known parameters of the room being designed. The interesting direction of further investigations related to the proposed formalism, is their extension into other uncertainty analyses of acoustic calculations.

Literature

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