On high modal density estimation and auralization

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Introduction

The analysis of NVH issues (noise, vibration, harshness) benefits significantly from auralization technique for example in Transfer Path Analysis (TPA). Usually the source-receiver transfer functions are measured in experiments, but due to the complexity of real mechanical structures, they are subject to high modal densities and overlap. Therefore different modes can not be distinguished any more in the mid and high frequency regime although all modes are expected to contribute with broad-band excitation. The question in this context is, how auralization of complex structures may be affected by high modal densities or overlap factors. In a case study, the modal density is estimated by means of a transfer function on a rectangular steel plate. It is found that a lower number of modes is sufficient to model the system accurately especially in the high frequency part of the transfer function.

Theoretical background

The computation of the modal density of a thin rectangular plate is derived from an autoregressive-moving average model (ARMA), which will be introduced along with conventional and the proposed ARMA subband method.

Autoregressive moving average model

The autoregressive moving average model generally states, that any correlated, linear stochastic process can be constructed with an appropriate filter structure and gaussian distributed white noise as input. The ARMA process $y(n)$ is characterized by a difference equation in the form of weighted linear combinations of input and previous output samples. By z-transform of the process, one obtains the characteristic transfer function of any transmission system according

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_L z^{-L}}{1 + a_1 z^{-1} + \cdots + a_M z^{-M}} \quad . \qquad (1)$$

The polynomial roots of nominator and denominator provide the zeros and poles in the z-plane. Hence, any linear time-invariant system can be fully described by calculation of its filter coefficients a_i and b_j. This way, ARMA models provide better modelling capabilities at the expense of higher complexity since no closed form solution for the non-linear system of equations in (1) exists.

Conventional parametric methods

Several different methods are available to identify the coefficients of a pole-zero filter efficiently. They solve equation (1) for a given impulse response and assume a Dirac impulse as system input for example. However, coefficients are computed for nominator and denominator separately on order to deal with a linear equation error minimization problem only [3].

Basically, the algorithms differ from whether they are processed in time or frequency domain and how many impulse response samples will be computed exactly for the overdetermined system of equations. Limited thereby, this yields differing numerical solutions, also dependent on the input data and other boundary conditions like noise for example. Typical time domain algorithms are Prony's method, sometimes also used in structural modal analysis [2, p.190] or the Steiglitz-McBride method [4].

Subband filtering approach

In contrast to processing fullband impulse responses, a subband approach is proposed to analyse and model the impulse response in different frequency subbands. For this purpose a perfect reconstruction filter bank is applied, which has the unique property of alias cancellation through neighbouring subbands to reassemble the input signal with zero error. These kind of filter banks are also known as polyphase filter banks, e.g. cosine modulated filter banks [5, p.377 ff.]. For the investigations, a perfect reconstruction filterbank with 10 bands is designed from a FIR low-pass prototype filter of order 100, having a stopband attenuation of over 80 dB. The 3 dB bandwidths are chosen equally with $0.1 f_{nyquist}$, the realization of the analysis filter bank is depicted in figure 1.

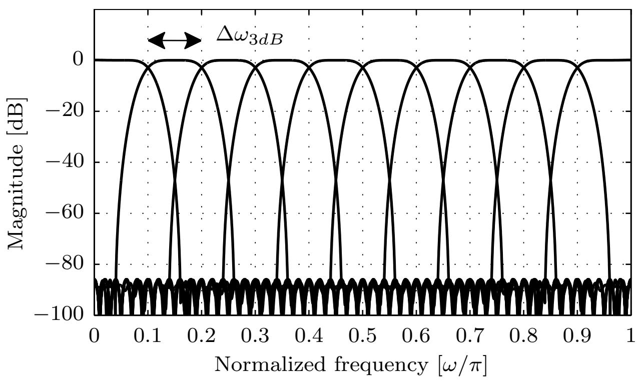

Figure 1: Ten bands analysis filter bank designed from a FIR low-pass prototype filter of order 100

The first processing step is to filter the original impulse response with the analysis filter bank in order to split the time signal for different frequency bands. Next the subband signals are downsampled by factor 10, which equals the number of bands and is called critical sampling, and processed by the ARMA estimator. The corresponding model orders are chosen individually per frequency band. Finally the ARMA subband output is upsampled again and filtered with the synthesis filter bank accordingly to reconstruct the original but parameterized impulse response.This subband approach provides the advantage of dedicated modelling capabilites combined with reduced computational load in each subband and less numerical problems with parameter identification.

Modal density estimation

The modal density estimation is carried out in a case study of a cross mobility frequency response function of a simple rectangular steel plate. The transfer function up to 5 kHz is calculated analytical for a highly damped, simply supported plate and compared to the estimation models. For the analytical solution of the wave equation in case of bending waves on a plate as two-dimensional continuum, the reader is referred to [1, p.77] for example.

Modelling performance

In a first step, the cross mobility transfer function on the rectangular steel plate is modelled with conventional approaches over the whole frequency range to validate the applicability of these methods. The result in terms of magnitude error between analytical and modelled transfer function is plotted in figure 2. A good modelling accuracy can be achieved for frequencies above 100 Hz with a model order of $L{=}M{=}800$ for Prony and Steiglitz-McBride method.
Furthermore both parametric models are evaluated in case of noisy transfer functions since these are corrupted with noise to some extent in real-world measurement scenarios. For gaussian distributed additive and multiplicative noise in the impulse response, the normalized squared error increases, but not as much as expected (approximately by only 1 dB), as summarized in table 1.

Since it turned out that the output of the Steiglitz-McBride method is not necessarily stable due to the exact computational approach, it was discarded for the subband ARMA method. The reconstructed cross mobility transfer function of the rectangular plate is shown in figure 3 for the Prony subband approach only. Magnitude and phase response can be approximated with high accuracy at a cumulated model order of 336 degrees of freedom. Deviations do exist especially in regions with low magnitude contribution. However, the normalized error is very small with -22,93 dB.

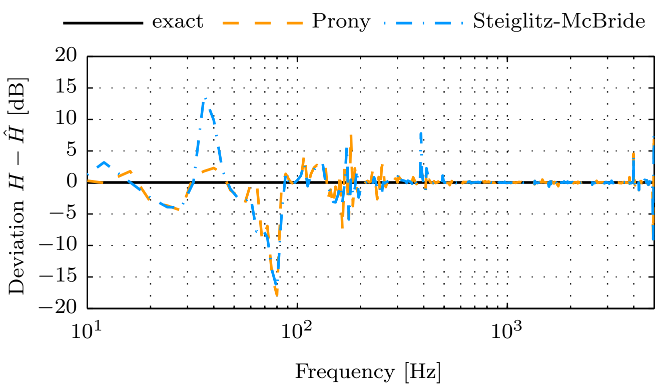

Figure 2: Magnitude error of the entire frequency band approach and two parametric methods of order $L{=}M{=}800$

Table 1: Normalized error $||H - \hat{H}||_2^2 \,/\, ||H||_2^2$ for two different SNR and noise situations (model order $L{=}M{=}800$)

Method	Normalized squared error [dB]		
	SNR=∞ ($\emptyset$)	SNR=20dB (+)	SNR=20dB ($*$)
Prony	-15,12	-14,65	-13,99
StMcB	-6,49	-6,05	-5,79

A closer look on the magnitude error between original and parametric frequency response function, see figure 4, reveals larger errors compared to the fullband approach, mainly in the regions of the crossover frequencies. Error peaks can also be detected at low (subband) frequencies or in regions with low magnitude contribution. Low frequency errors may occur due to limited modelling precision especially of Prony's method for later samples in the impulse response tail. However, overall modelling performance can still be considered effective.

Mode count estimation

The modal density of the frequency response function is evaluated by taking all poles per frequency band into account. The results of the introduced ARMA approaches is compared to the analytical solution and to a theoretical value. The model order of the subband approach are summarised in table 2. One can see that for a constant filter bandwidth, the model order decreases significantly with some minor fluctuations for higher bands. The

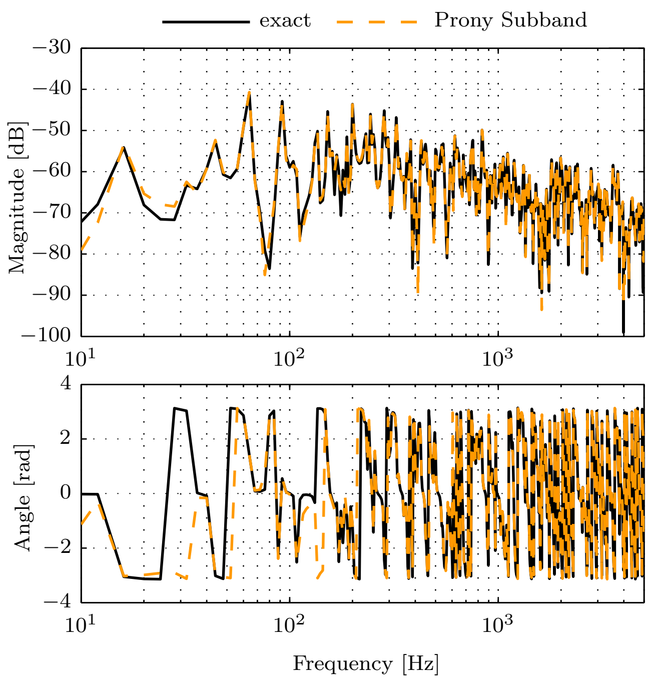

Figure 3: Reconstructed transfer function from the subband approach and Prony's method, cumulated order 336

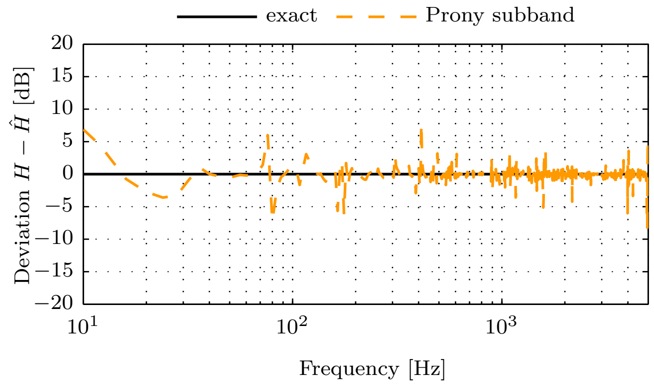

Figure 4: Magnitude error of the proposed subband approach with Prony's method, cumulated order 336

theoretical modal density of a thin rectangular plate can be computed from its geometrical properties surface area S, thickness d and its material constants density ρ and flexural bending stiffness B with

$$n(\omega) = \frac{S}{4\pi}\sqrt{\frac{\rho d}{B}} \; . \qquad (2)$$

Actually the modal density is a function of frequency and depends on the plate boundary conditions and thus 2 is more accurate for higher frequencies.

Typical representations for modal densities are calculated in third-octave bands. The values obtained from the different approaches for the rectangular steel plate are depicted in figure 5. The analytical modal solution converges towards the value calculated from equation 2 as expected, whereas the modal densities obtained from the ARMA approaches converge to lower values.

Discussion

An interesting fact is, that the calculated modal densities from the parametric approaches coincide with the analytical solution for very low frequencies. Since the modal damping, which is proportional to modal overlap factor, increases with frequency, unique modes can not be detected any more. This obviously reduces the required model order in each independent subband and seems to be plausible as modal contributions get broader with higher damping and therefore frequency.

Conclusion

In a case study of a rectangular thin steel plate it is found that a structure with high modal density and hence high modal overlap can be modelled with good accuracy in magnitude and phase and a fewer number of modes than

Table 2: Filter bank orders with Prony's method

Method	Subband poles and zeros				
	I II	III IV	V VI	VII VIII	IX X
Prony	80 70	30 30	26 30	20 16	18 16

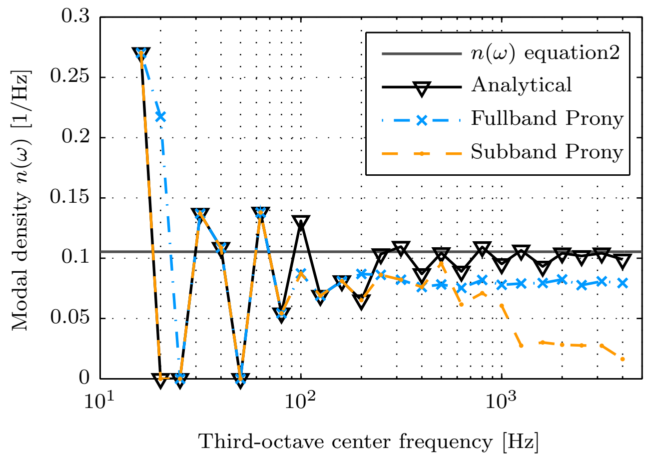

Figure 5: Third-octave exact and estimated modal densities

physically present. This was achieved with parametric approaches processed in the subband domain of a perfect reconstruction filter bank applied to the impulse response of the system. From an auralization point of view these results suggest that there is no necessity to capture every mode and spectral detail in the mid and high frequency regime.

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