Combining Ray Tracing with Higher Order Diffraction Based on the Uncertainty Relation

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Introduction

In both room and city acoustics, ray tracing (RT) and beam tracing (BT) simulation methods are widely used, however, diffraction simulation is still lacking. Stephenson's Uncertainty Based Diffraction (UBD) model [1] is an energetic approach, that has been validated quite well at the single screen and the slit as reference cases [2]. In this paper, an analytical formulation of the UBD model for second order diffraction is derived and evaluated.

The Sound Particle Diffraction Model

Inspired by the uncertainty relation, the diffraction effect should be the stronger the closer the by-pass-distance of a particle to an edge a. The first concept is a Diffraction Angle Probability Density Function (DAPDF) derived from the spatial Fourier transform of the transfer function of a slit (Fraunhofer diffraction), smoothed over a wide frequency band and simplified to

$$D(\varepsilon, a, \varepsilon_1) = \frac{D_0(a, \varepsilon_1)}{1 + 2 \cdot u^2} \qquad u = 2 \cdot b(a, \varepsilon_1) \cdot \varepsilon \quad , \quad (1)$$

where $b(a, \varepsilon_1)$ is the apparent slit width, ε is the total deflection angle (see Fig. 1, left) and $D_0(a, \varepsilon_1)$ is a normalization factor. For only one edge passed simultaneously, the *effective* slit width $b(a, \varepsilon_1)$ reads

$$b(a, \varepsilon_1) = 6 \cdot a \cdot cos(\varepsilon_1) \qquad (2)$$

The projection factor $cos(\varepsilon_1)$ is an extension found recently to fulfill the reciprocity principle [3]. All distances are here in units of wavelengths λ. In the implemented version, the sound particles are split up into secondary ones at each diffraction event when hitting 'transparent walls' over the edges.

Finally, a transmission degree T defined as the received intensity with the obstacle relative to that in free field is wanted (or its level $L = 10 \cdot \log_{10} T$). In 2D (as the investigation is restricted to) the proportion T is the same in 3D. For single diffraction, the sound particle detection formula reads:

$$T = \frac{2 \cdot \pi \cdot R}{N} \cdot \sum_{i=1}^{N_0} \sum_{j=1}^{S_0} \frac{w_{i,j} \cdot e_{i,j}}{S_D}, \qquad (3)$$

where N, N_0 and S_0 are the numbers of emitted, diffracted and each time detected sound particles, R is the direct distance source-receiver, the $w_{i,j}$ are the inner crossing distances of the particles in the detector (surface S_D) [4] and the $e_{i,j}$ are the energy fractions of particles, which are integrals of the DAPDFs $\int_{\Delta\varepsilon} D(\varepsilon, a, \varepsilon_1)$ over small angle ranges $\Delta\varepsilon$, the sound particle represents.

Transition to an Integral Formulation

For a deeper analysis, the numerical error due to the finite number of particles should be avoided. As can be shown [3], with the transition to infinitely small receiver diameters and an infinite number of particles the sum term $\sum_{j=1}^{S_0} \frac{w_{i,j} \cdot \int_{\Delta\varepsilon} D(\varepsilon,a,\varepsilon_1)}{S_D}$ can be replaced by $\frac{\int_{\Delta\varepsilon} D(\varepsilon,a,\varepsilon_1)}{r_2 \cdot \Delta\varepsilon} \approx \frac{D(\varepsilon,a,\varepsilon_1)}{r_2}$

With $\frac{2\pi}{N} = d\varepsilon_1$ and $d\varepsilon_1 = \frac{cos(\varepsilon_1) da}{r_1}$ one gets the integral

$$T = R \int_0^\infty \frac{D(\varepsilon, b, \varepsilon_1)\, cos(\varepsilon_1)}{r_1 r_2} \cdot da \qquad (4)$$

where $r_{1,2}$ and $\varepsilon_{1,2}$ are the distances and angles from the diffraction points to the source and the receiver.

Higher Order Diffraction

Now the sound particles are split up at each edge recursively.

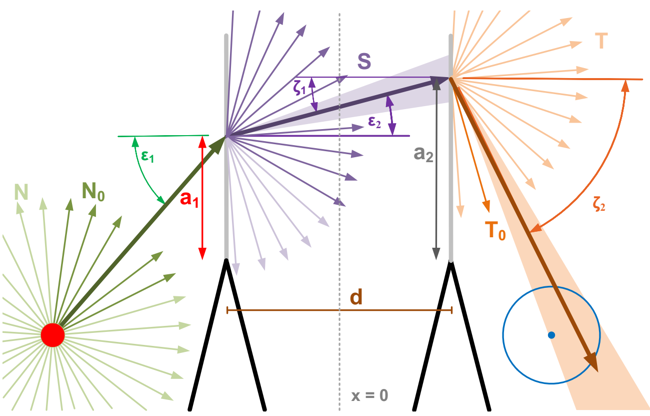

Figure 1: Recursive split-up of sound particles by double diffraction. Two edges (=$z-axes$) in a distance of d at $(-\frac{d}{2}, 0)$ and $(+\frac{d}{2}, 0)$; source and receiver at $(r_{S,R}, \varphi_{S,R})$ counted from the left and right edge respectively; $a_{1,2}$ are the by-pass-distances (for $r_{1,2,3}$, $\varepsilon_{1,2}$ and $\zeta_{1,2}$ see the text)

Thus, the total transmission degree T, can be written as

$$T = \frac{2 \cdot \pi \cdot R}{N} \cdot \sum_{i=1}^{N_0} \sum_{j=1}^{S_0} e_{i,j} \cdot \sum_{k=1}^{T_0} \frac{w_{i,j,k} \cdot f_{i,j,k}}{S_D}. \qquad (5)$$

For double diffraction, the second sum is now over S_0 of S secondary particles hitting the second transparent wall, and the third over T_0 of T tertiary particles crossing the receiver detector, r_2 and $\varepsilon_2 = -\zeta_1$ stand for the distance and angle in between and r_3 and ζ_2 to the receiver. The energy fractions $e_{i,j}$ and from these the fractions $f_{i,j,k}$ are integrals of the DAPDFs over respective angle ranges (filled surfaces in Fig. 1). With considerations and transformations as above from equ. 5 the following equation can be derived:

$$T = R \int_0^\infty\int_0^\infty \frac{D(\varepsilon, a_1, \varepsilon_1)\, D(\zeta, a_2, \zeta_1)\, cos(\zeta_1) cos(\varepsilon_1)}{r_1 r_2 r_3} da_2 da_1$$

where $\varepsilon = \varepsilon_1 + \varepsilon_2$, $\varepsilon_2 = -\zeta_1$ and $\zeta = \zeta_1 + \zeta_2$ (negative angles indicate $y < 0$, i.e. 'shadow zones')

Validation

As reference models, both a wave theoretical and a deterministic model are chosen. For the wave theoretical model, the edge diffraction toolbox [5] using Svensson's secondary source model (SSM) [6] is used. The deterministic detour law of Maekawa (MDL) [7] is modified by a **equivalent** thin screen for double diffraction [8]. The differences between the levels of the propossed UBD model and the reference models $\Delta L_{T,MDL} = L_{T,UBD} - L_{T,MDL}$ and $\Delta L_{T,SSM} = L_{T,UBD} - L_{T,SSM}$ are computed for different setups.

First, a setup with $r_S = r_R = d = 10\lambda$ and $\varphi_S = 0°$ was chosen for different φ_R (see Fig. 2)

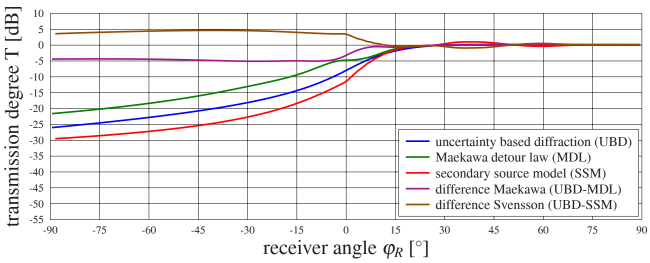

Figure 2: Comparison of different transmission levels $L_T[dB]$ for double diffraction with $r_S = r_R = d = 10\lambda$ and $\varphi_S = 0°$ as a function of φ_R

As for all setups, all three methods match the $L_T = 0dB$ in the view zone ($\varphi_R > 0°$) quite exact, but only the wave based SSM is capable to represent the interference effect ($L_T > 0dB$,here at $\varphi_R \approx 35°$). In the shadow zone the UBD overestimates the diffracted energy ($\Delta L_{T,SSM} > 0$) while the MDL overestimates the diffracted energy ($\Delta L_{T,MDL} < 0$).

In the first test, all distances (in relative wavelengths) are changed simultaneously ($r_S = r_R = d$). This can be reinterpreted as a constant distance with changing frequency. For very low distances (0.1λ), the UBD matches perfectly the SSM, whereas for high distances (100λ) the difference increases linearly up to $\Delta L_{T,SSM} = 5dB$, however, this is less harmful in regions of $L_{T,SSM} = -40dB$.

For constant source and receiver distances $r_S = r_R$, but variable distances d, the agreements between UBD and SSM are better ($\Delta L_{T,SSM} < 3dB$) in a wide distance range. The convergence of $d \to 0$ to single diffraction was shown with a difference of $\Delta L_T < 2dB$ (with the SSM it was not computable due to singularities).

Finally, a shift of the source into the shadow zone (lower than the edge) showed acceptable agreement in a wide range. Fig. 3 shows one of many comparisons for sources deep in the shadow zone. The difference between the UBD

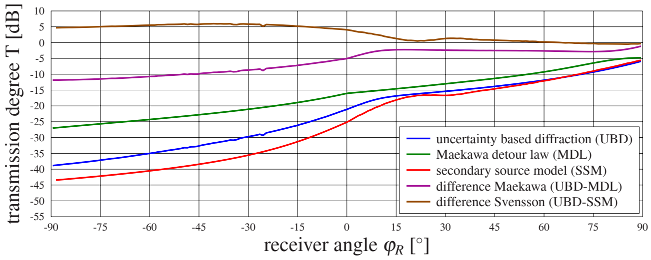

Figure 3: Comparison of different transmission levels $L_T[dB]$ for double diffraction with $r_S = r_R = d = 10\lambda$ and $\varphi_S = -45°$ as a function of φ_R

and the SSM, however, is quite constant. This is a benefit of the earlier found cosine factor to yield reciprocity. Without that, an additional difference of $10dB$ had occurred[3].

Conclusion

Double diffraction uses a double integral over the product of <u>two</u> DAPDFs. A generalization to higher order diffraction is possible analytically with double diffraction. The differences for one diffraction between the models seem to add up. The accuracy of the UBD model (compared to the SSM) is by far better than the MDL.

Literatur

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