A Calculation Model for Anisotropic Reverberation with Specular and Diffuse Reflections

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Introduction

Both Sabine's formula and Eyring's formula for the reverberation time assume that the sound field within an acoustic enclosure is diffuse, i.e. homogeneous and isotropic. However these assumptions are never fulfilled exactly. Calculation models like ray tracing or even based on wave acoustics are very time consuming. Therefore a calculation model between those both extremes is suggested. From the two assumtions of homogeneity and isotropy only the homogeneity is maintained and the sound field is regarded to be anisotropic (see Fig. 1). The Anisotropic Reverberation Model (ARM) to be represented here first time allows to compute reverberation times as a function of scattering coefficients. Its basic idea is an energy interchange between directional ranges, similar as with the radiosity method [1].

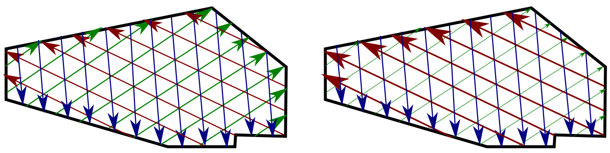

Figure 1: The assumption of an isotropic (left) and an anisotropic (right) sound field.

The Set of Directional Ranges

To distinguish sound energies traveling in different directional ranges, ARM uses - instead of one scalar value for the sound energy - a sound energy vector $\mathbf{x}$, whose components x_j correspond to the amounts of sound energies travelling in $j = 1 \dots N$ distinct directional ranges, that are equally distributed over the whole solid angle range 4π. On a unit sphere each direction $\vec{r}_j$ represents a small area on this sphere with almost equal area and diameter. To avoid different densities of directions in polar and equatorial regions of the unit sphere, the set of directions is based on the vertices of a icosahedron where the faces are subdivided in smaller triangles (see Fig. 2). The mathematics of sphere tesselations can be found in [2] chapter 40.

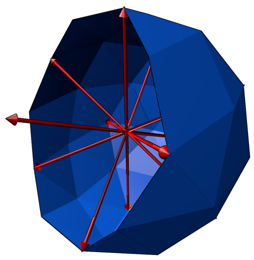

Figure 2: A set of distinct directions based on the icosahedron geometry.

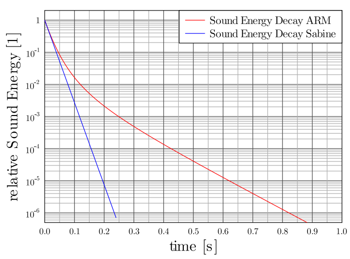

Figure 3: The decay of the sound energy can be modelled with a "hanging" curve.

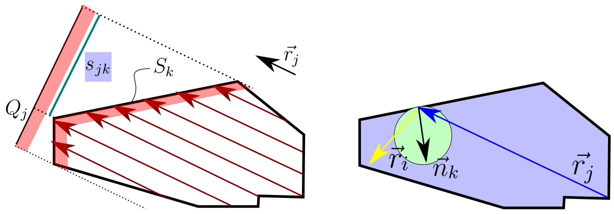

Figure 4: Some terms used in formulas. Q_j is the projected area of the room with respect to direction $\vec{r}_j$.

The ARM-Energy-Transfer-Matrix

The acoustic enclosure is described as a closed set of $k = 1 \dots K$ walls each with a geometric surface S_k and a normal vector $\vec{n}_k$ pointing to the inside with unit length. Sound energies of a specific direction impinge on walls and are reflected according to the inclination of the wall, its absorption coefficient α and its scatter coefficient σ. The unit vectors for the quantized directional ranges are named $\vec{r}_i$ or $\vec{r}_j$, j for the incident and i for reflected directions (see Fig. 4). The part of the energy of one specific direction $\vec{r}_j$ that reaches any wall within a small time interval dt is called $e_j = \frac{Q_j c}{V}$. This energy is distributed on different walls each with the fraction $s_{jk} = \frac{S_k}{Q_j} \max(-\vec{r}_j \vec{n}_k, 0)$ (see Fig. 4). Only the not absorbed part $(1 - \alpha_k)$ of the incident energy is reflected either specular $(1 - \sigma_k)$ or diffuse (σ_k) [3]. A diffuse reflection distributes the sound energy into other directions according to Lambert's cosine law $d_{ki} = \frac{1}{D_k} \max(\vec{r}_i \vec{n}_k, 0)$ where D_k is a normalization constant. In case of the specular reflection the coefficient m_{ikj} determines the new direction for the sound energy:

$$m_{ikj} = \begin{cases} 1 & \text{if } \vec{r}_i \text{ is the k-mirror direction of } \vec{r}_j \\ 0 & \text{other cases} \end{cases} \quad (1)$$

With that, the core equation describing the energy transfer from the j-th directional range over the wall k to the directional range i reads:

$$a_{ikj} = e_j s_{jk} (1 - \alpha_k)[\sigma_k d_{ki} + (1 - \sigma_k) m_{ikj}] \quad (2)$$

Finally, an element of the ARM-Energy-Transfer-Matrix a_{ij} is calculated according to the sum over all involved walls:

$$a_{ij} = \sum_k a_{ikj} \tag{3}$$

The multiplication of the Matrix $\mathbf{A} = [a_{ij}]$ with the vector of the sound energies $\mathbf{x}$ yields the change of the sound energies:

$$\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} \tag{4}$$

This linear system of differential equations can either be solved iteratively e. g. with the forward Euler method:

$$\mathbf{x}(t + \Delta t) = \mathbf{x} + \Delta t \mathbf{A} \mathbf{x} \tag{5}$$

or using eigenvalues λ_i and eigenvectors $\mathbf{v_i}$ by determining the coefficients c_i to match the initial conditions [4]:

$$\mathbf{x}(t) = \sum_i c_i e^{\lambda_i t} \mathbf{v_i} \tag{6}$$

where the $\mathbf{v_i}$ characterize the directional energy distribution and the $\lambda_i = -\frac{1}{\tau_i}$ describe the corresponding partial reverberation constants τ_i. A typical anisotropic distribution of sound energy is dominated by one directional energy distribution with the longest reverberation time.

The Dominant Eigenvector

The solution with eigenvectors in eq. (6) suggests, that the anisotropic distribution of sound energy is dominated by the eigenvalue with the greatest real part. The corresponding eigenvector determines the distribution of sound energy to different directions. A visualisation of these sound energies as in Fig. 5 shows the dominant sound directions together with the room geometry and may help to asses the effects of the sound field.

An ARM Application Example

Some room acoustical properties can be calculated with the ARM and not with Sabine's formula. The examples shown here are based on a rectangular room of size $15.2m \times 8m \times 4m$ using materials with absorption coefficient $\alpha = 0.9$ and scatter coefficient $\sigma = 0.9$ on all walls except on the small front and back walls where $\alpha = 0.1$ and $\sigma = 0.1$ is used. This scenario with opposing walls with specular reflections causing dominating energy flow just in one direction is designed to model the effect of flutter echoes. As initial condition for the sound energies an isotropic distribution was used.

Flutter echoes

An example for a sound energy decay is shown in Fig. 3. As typical for non diffuse sound fields, it shows different slopes at the beginning and at the end. The reverberation time of $0.82s$ calculated by ARM is clearly longer than Sabine's value $0.234s$. To avoid flutter echoes one of the small walls is inclined by angle θ varying from $0°$ to $15°$. The resulting reverberation times are shown in Fig. 6 left. To model the effect of small differences of the inclination it is necessary to use directions that differ in the same magnitude from each other. Therefore in all examples a set of 1280 distinct directions was used. In other cases smaller number of directions also gives good

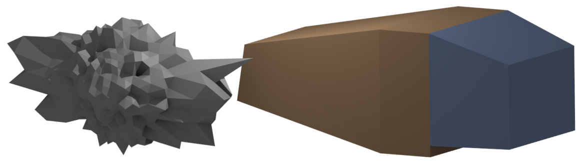

Figure 5: A visualisation of the eigenvector (left) gives information on the anisotropy of the sound field in the room (right).

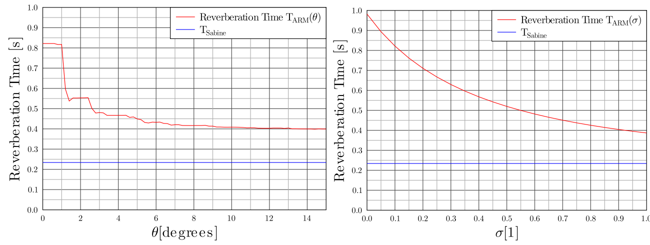

Figure 6: Both wall inclination (left) and scatter coefficient (right) have an effect on the reverberation time.

results. Another possibility to fight flutter echoes is to emphasize diffuse instead of specular reflections. To demonstrate this effect the scatter coefficient of the front and back walls is varied from 0 to 1 as shown in Fig. 6 right. The reverberation time decreases with increasing scatter coefficient as it is expected for a vanishing flutter echo.

Conclusion and Outlook

The presented Anisotropic Reverberation Model (ARM) is capable to model room acoustical properties with less effort than, e.g. the ray tracing model. Effects of inclined walls and scatter coefficients are taken into account. So the consequences of flutter echoes can be calculated. In contrast to methods like ray tracing it is not necessary to define positions of sound sources or receivers. The criteria to find an appropriate set of directional ranges and some interpolation methods are still to be developed. Quantitative comparisons to other room acoustical models need to be evaluated both for synthetic examples with special features and for practical examples. It should be investigated, to what extend also other parameters than just the reverberation time can be computed by ARM.

References

[1] Stephenson, U.: Quantized Pyramidal Beam Tracing - a new algorithm for room acoustics and noise immision prognosis. Acta Acustica united with Acustica 82 (1996), 517-525

[2] Freeden, W. and Nashed, M. and Sonar, T.: Handbook of Geomathematics, Springer, Heidelberg, 2010

[3] Vorländer, M.: Auralization, Springer, Heidelberg, 2008

[4] Meyer, C. D.: Matrix Analysis and Applied Linear Algebra, SIAM, Philadelphia, 2004