A new preconditioner for the iterative solution of the systems describing the vibro-acoustics of multi-layered panels

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Introduction

Multi-layered panels are widely used in industrial applications of noise insulations. A typical example is the fuselage of an aircraft, which is composed of an outer aluminium shell and inner trim panels. In general, each layer of the panel can be made of different materials, such as elastic, porous, and fibrous materials. On the one hand, the innumerable combinations of the layers with different material parameters enrich the vibro-acoustic properties of the panel. On the other hand, it turns the optimization of such a panel to an exhausting task. It has been shown that numerical methods like finite element method (FEM) can be effectively used to predict the vibro-acoustic behaviours of the system. However, the efficiency of the calculation depends greatly on the system matrix, which is unfortunately often very ill-conditioned due to the mixture of various material models. In order to apply iterative solvers based on the Krylov subspace methods, a robust preconditioner is essential for the FEM calculations of multi-layered panels. In the current work, a special preconditioning technique is developed for solving a multi-layered system by means of the Schur complement.

FEM model of a multi-layered panel

The most outstanding geometrical nature of the multi-layered panel is that the thickness of each layer is very thin. Therefore, plate formulations of the different material models can be used in the FEM calculation. In general, plate elements have a less number of the degrees of freedom. Also the discretization with the plate elements is much simpler than with the volume elements due to the absence of the thickness dimension.

An elastic plate can be commonly formulated by either Kirchhoff or Mindlin-Reissner plate theory. The respective FE formulations are described in many textbooks. Porous material has more complex mechanical features, since it contains both structural and fluid phases. Based on the most well known Biot's theory of poroelasticity, a plate formulation of a porous layer has been implemented in the current model [1]. Furthermore, the authors have developed a plate formulation for simulating the acoustical behavior of a thin layer of fluid. This fluid plate formulation is not only applicable to an air layer, but also to various fibrous blankets, which can be regarded as a viscous fluid layer [2].

Since all the layers are formulated as plates in the current model, only a single two-dimensional mesh is required for any number of layers, which can be accordingly modeled by the respective variables on the mesh.

Preconditioning the system of a multi-layered panel

For the sake of clarity, the preconditioning technique is described along with an example, where the sound transmission loss of a multi-layered panel is calculated. As shown in fig. (1), the panel consists of five layers, two elastic plates made of aluminium, a porous blanket, a fibrous blanket, and an air gap. The material properties are listed in table (1). According to the standard measuring procedure of the sound transmission loss (see ISO 140-3), the panel is excited on one side by the acoustical diffuse field. On the other side of the panel, a block of acoustic volume elements is attached, which fulfils a non-reflecting boundary condition by using the infinite element method [3].

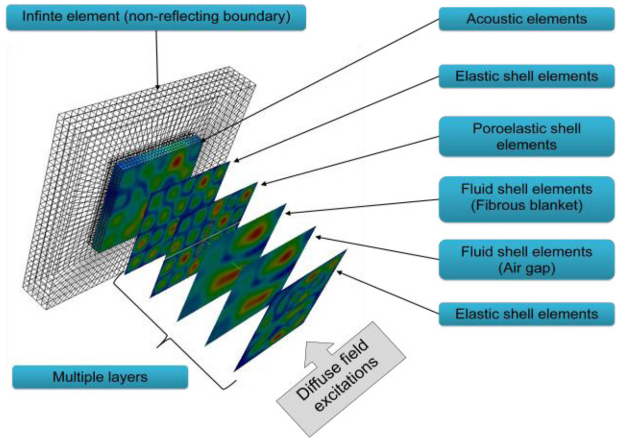

Figure 1. A FEM model of calculating the sound transmission loss of a multi-layered panel.

The sparse pattern of the system matrix is shown in fig. (2). It is noticeable that the stiffness matrices of the acoustical domain and the layers are marked by the square boxes and the coupling matrices of the subdomains by the dot lines, respectively. The most outstanding feature of the matrix pattern is that each subdomain is only coupled with its direct neighbors. By using the Schur complement, the system matrix A can be factorized by [4]

$$A = \sum \begin{bmatrix} A_i & E \\ F & A_{i+1} \end{bmatrix} = \sum \begin{bmatrix} L_{A_i} & 0 \\ FU_{A_i}^{-1} & L_S \end{bmatrix} \begin{bmatrix} U_{A_i} & L_{A_i}^{-1}E \\ 0 & U_S \end{bmatrix}, \quad (1)$$

where $\boldsymbol{A_i}$ is the stiffness matrix of a subsystem, $\boldsymbol{i}$ the index of the subsystem, $\boldsymbol{E}$ and $\boldsymbol{F}$ are the corresponding coupling matrices of $\boldsymbol{A_i}$ and $\boldsymbol{A_{i+1}}$, $\boldsymbol{L_{A_i}}$

Table 1: Material parameters of the exemplar multi-layered panel, ρ is the density, *h* the thickness, γ the poisson ration, *E* Young's modulus, σ the porosity, τ the fluid resistance, and c the speed of sound.

	Aluminium	**Porous blanket**	**Fibrous blanket**	**Air**
ρ [kg/m^3]	2770	5.6	7	1.2
h [mm]	2	10	25.4	35
γ	0.33	0.43	-	-
E [N/m^2]	7.24E10	1.28E4	-	-
σ	-	0.995	-	-
Τ [Ns/m^4]	-	4.63E4	-	-
c [m/s]	-	-	120	343

and $\boldsymbol{U}_{A_i}$ the lower and upper triangular matrices of $\boldsymbol{A}_i$, $\boldsymbol{L}_S$ and $\boldsymbol{L}_S$ the lower and upper triangular matrices of the Schur complement $\mathbf{S} = \boldsymbol{A}_{i+1} - \mathbf{F}\mathbf{U}_{A_i}^{-1}\mathbf{L}_{A_i}^{-1}\mathbf{E}$. Nevertheless, the calculation of the Schurcomplement can be computationally expansive. The preconditioner requires only an approximation to the inverse of the system matrix. Instead of the Schur complement, using the factorization of $\mathbf{C}$ still provides the results with enough precision. This means that the triangular form of the global system matrix can be calculated more efficient and it is much simpler to parallelize. Therefore, equation (1) can be rewritten as

$$A \approx \sum \begin{bmatrix} L_{A_i} & 0 \\ F_i U_{A_i}^{-1} & L_{A_{i+1}} \end{bmatrix} \begin{bmatrix} U_{A_i} & L_{A_i}^{-1} E_i \\ 0 & U_{A_{i+1}} \end{bmatrix}. \qquad (2)$$

Since each subsystem is treated separately, one can apply either complete or incomplete factorization (ILU) depending on the subsystem [5]. For example, applying ILU with the drop tolerance 0.001 to the acoustical subsystem and the fluid plates, LU to the elastic plates and porous plates, the condition number can be greatly reduced as shown in fig. (3). Therefore, standard GMRES can be utilized to solve the system. The result is compared with a direct solver and the measurement in fig. (4). It shows an excellent agreement.

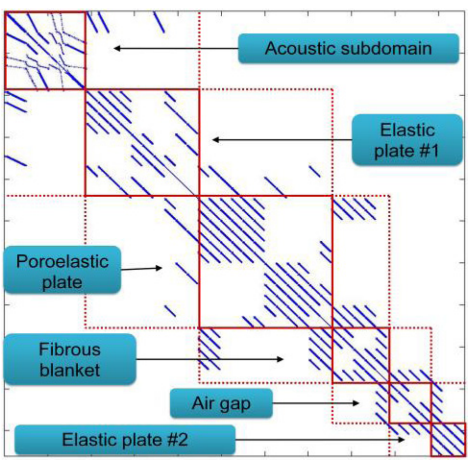

Figure 2. Sparse pattern of the exemplary system matrix

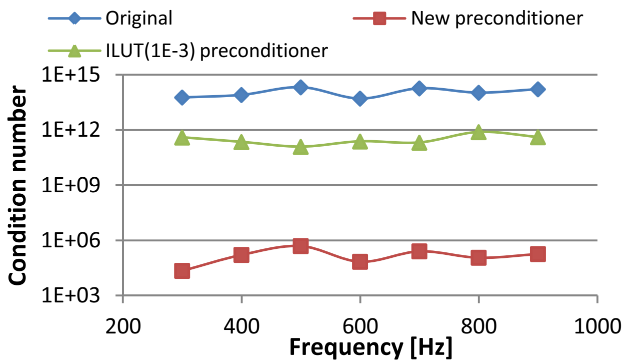

Figure 3. Comparison of the condition numbers of the exemplary system matrix with different preconditioners

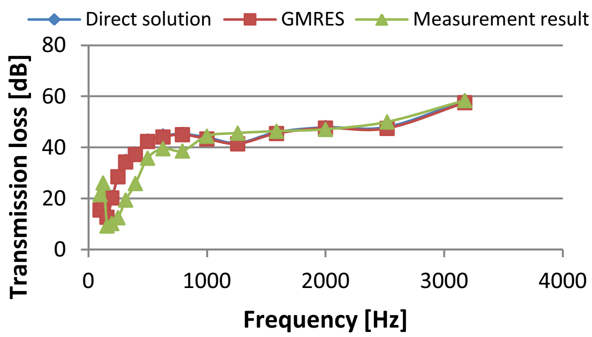

Figure 4. Validation of the calculated sound transmission loss by both direct solver and GMRES with the new preconditioner

Conclusion

With the help of the new preconditioner, complex multi-layered panel systems can be solved rather efficiently by iterative solvers based on the Krylov subspace.

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