Correction Errors in Acoustic Measurements Caused by Temperature Variance

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Introduction

The impulse response and its associated transfer function are the most important properties of linear time invariant acoustic systems. During the impulse response measurement, sometimes plenty of synchronous average has to be implemented to improve the signal-to-noise ratio(SNR). In this case, a little bit temperature-variance could lead to large errors after average. In fact, corresponding to the temperature shift, the impulse response varies with a time-stretching process, and in frequency domain, it appears as resonance frequency shift. This time-stretching process can be compensated by stretching the various-temperature impulse response back to a constant-temperature impulse, and then the correct impulse response can be obtained, and the correct average can be performed.

Time Stretching Model

Since all kinds of impulse responses are determined by the wave equation, the impulse response variance caused by temperature drift can be derived by the wave equation. Eq. 1 is the non-source wave equation of an acoustic system,

$$\begin{cases} \nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = 0 \\ c_0 = \sqrt{\frac{\gamma R T_0}{M}} \\ \left(\frac{\partial p}{\partial \vec{n}} + j\omega\rho_0 Y_n p_n\right)\Big|_{\varphi(\vec{r})} = 0 \end{cases} \tag{1}$$

where T_0 is the absolute temperature, c_0 is the speed of sound in the air. The term $\left(\frac{\partial p}{\partial \vec{n}} + j\omega\rho_0 Y_n p_n\right)\Big|_{\varphi(\vec{r})} = 0$ is the boundary condition. The impulse response $h(t)$ is determined by the wave equation. During the impulse response measurement, assuming that 1, The temperature in the measured acoustic system is uniformly distributed. 2, The temperature does not change within one measurement period. 3, The boundary conditions does not change with the temperature. Then if the temperature changes from T_0 to $T_0+\Delta T$, the speed of sound changes from c_0 to $\zeta \cdot c_0$, where $\zeta = \sqrt{\frac{T_0+\Delta T}{T_0}}$. The wave equation can be rewritten as

$$\begin{cases} \nabla^2 p - \frac{1}{(\zeta\cdot c_0)^2}\frac{\partial^2 p}{\partial t^2} = 0 \\ \left(\frac{\partial p}{\partial \vec{n}} + j\omega\rho_0 Y_n p_n\right)\Big|_{\varphi(\vec{r})} = 0 \end{cases} \tag{2}$$

where $p = p(x, y, z, t)$, Y_n is the admittance of the boundary, and ρ_0 is the density of the air. Because the Y_n , ρ_0 are independent on the temperature, the boundary condition does not change with the temperature and the speed of sound, then making a coordinate system transformation as Eq. 3

$$\begin{bmatrix} x' \\ y' \\ z' \\ t' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \zeta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} \tag{3}$$

The wave equation Eq. 2 can be rewritten as Eq. 4

$$\begin{cases} \nabla'^2 p' - \frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t'^2} = 0 \\ \left(\frac{\partial p}{\partial \vec{n}} + j\omega\rho_0 Y_n p_n\right)\Big|_{\varphi(\vec{r}')} = 0 \end{cases} \tag{4}$$

where $\nabla'^2 = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2}$ and $p' = p(x', y', z', t')$.

Comparing the two wave equations Eq. 4 and Eq. 1, they have the same form, thus the solution of both wave equations should be similar. The only difference is the time-stretching coordinate transformation of Eq. 3. Thus if the impulse response derived from Eq. 1 is $h(t)$, the impulse response from Eq.(4) should be the same as system of Eq. 1, written as $h(t')$. Then transforming $h(t')$ back to (x, y, z, t) coordinate system, the impulse response at temperature $T_0 + \Delta T$ is $h(\zeta \cdot t)$, which varies only with a time-stretching factor ζ . Transforming the impulse response to frequency domain, the transfer function changes from $H(\omega)$ to $\frac{1}{|\zeta|}H(\frac{\omega}{\zeta})$.

Time-Stretching Factor Estimation

In this time-stretching model, if the time-stretching factor ζ is known, the temperature-dependent impulse response can be stretched back to the constant-temperature impulse response. In order to obtain the time-stretching factor ζ, one approach is to directly measure the temperature during the measurement, however, the humidity could also change is speed of sound. Temperature is not the only physical quantity that changes the speed of sound, therefore a more reliable approach is to estimate the the speed of sound shift by maximizing the cross correlation function of the two impulse responses, as shown is Eq. (5)

$$R_{hh}(\zeta_{est}) = \int h(t) h(\frac{\zeta \cdot t}{\zeta_{est}}) \mathbf{d}t \tag{5}$$

where ζ_{est} is the estimated time-stretching factor. When $\zeta_{est} = \zeta$, the cross correlation R_{hh} is maximized. When measuring the impulse response, the excitation signals have to be generated through the loudspeaker, as illustrated by Eq. 6

$$\begin{aligned} x_1(t) &= h(t) \quad * h_{\text{Loudspeaker}}(t) * s(t) \\ x_2(t) &= h(\zeta \cdot t) * h_{\text{Loudspeaker}}(t) * s(t) \end{aligned} \tag{6}$$

$x_1(t)$ and $x_2(t)$ are two measured signals of the same acoustic system. Because of the temperature variance, they contain two different impulse response $h(t)$ and $h(\zeta \cdot t)$. Before estimating the time-stretching factor, the measured signals have to be firstly deconvoluted by excitation signal and the impulse response of the loudspeaker. The impulse response of the loudspeaker have to be precisely measured, otherwise the impulse response of the loudspeaker is also stretched, which could result in unknown errors. As the time-stretching factor is estimated, the various-temperature impulse response is automatically stretched back to a constant temperature impulse response, and the correct average can be performed.

In addition, to calculate the time-stretched impulse response $h(\frac{\zeta \cdot t}{\zeta_{est}})$, the spline interpolation is used. To get better interpolation accuracy, the higher the sampling rate is required [1].

Measurement Results

In order to practically prove this time-stretching model, a small steel chamber is measured. The positions of the loudspeaker and microphone are both fixed. The impulse responses are measured with 44.1 kHz sampling rate. The temperature was arbitrarily changed from 20.4°C to 28.6°C. Totally, 20 impulse responses of various temperatures is recorded.

Because of the temperature variance, the impulse response drifts not only in time domain (Fig. 1), but also in frequency domain (Fig. 2). In time domain ,the later part of the impulse response shows the larger phase shift. In frequency domain, the higher the resonance frequency locates at, the larger frequency shift occurs.

As shown in Fig. 3, directly averaging those 20 time-variant impulse responses will lead to wrong average results, especially at high frequencies. If the different-temperature impulse responses is firstly stretched back to the constant-temperature (20.4°C) impulse response before the synchronous average, the accuracy of the average still hold. But above 9000 Hz, the performance of time-stretching compensation is not as good as the low frequencies. The reason is the 44.1 kHz sampling rate is used in this measurement, and above 9000 Hz, the error of spline interpolation increases very fast. In this case, the higher sampling rate should be used.

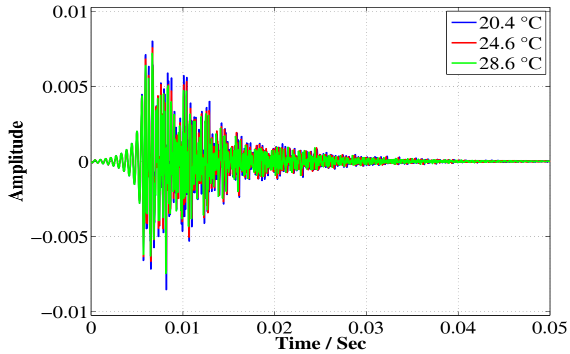

Figure 1: Impulse response over various temperatures, the frequency range is from 2 kHz to 10 kHz

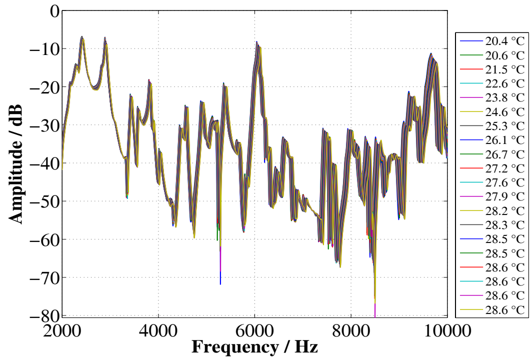

Figure 2: Transfer function over various temperatures

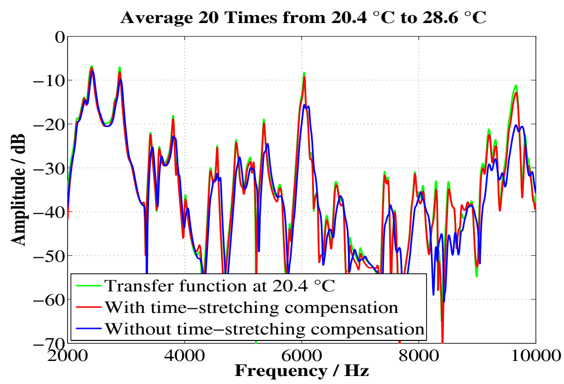

Figure 3: the performance of average over temperature-variant measurement with/without time-stretching compensation

Conclusion

In this paper, a time-stretching model is depicted for the compensation of temperature variance in impulse response measurement. With the correct compensation of temperature drift, people can perform the long-time average in temperature-variant systems.

References

[1] C. De Boor. *A Practical Guide To Splines*, volume 27. Springer Verlag, 2001.