A Modified Overlap-Add Filter Bank With Reduced Delay

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Introduction

Filter bank structures are often realized by a sliding window short-time Fourier transform (STFT) as analysis stage. Depending on the method that is used for the signal synthesis, these filter banks are categorized as overlap-add (OLA) or overlap-save (OLS) filter banks. Since both methods have their own advantages and drawbacks, we propose a trade-off between them that helps to reduce the signal delay. A design procedure for analysis and synthesis window is shown, which has a special emphasis on simple computation. Thus, the windows can be calculated during program initialization when the desired FFT order or the frame shift are not known in advance. It is shown that the proposed windows outperform the standard approach of using Hann-windows in terms of echo canceling quality and perfect reconstruction.

The proposed filter bank structure has been successfully applied and tested in an in-car communication system and a signal enhancement system for firefighter breathing protection masks. In both cases, it is important to keep the system delay as low as possible in order to avoid that the loudspeaker playback is perceived as an echo. Similar demands hold for hearing aids applications.

Complex Modulated Filter Banks

Most often, filter banks are viewed as a set of N parallel band pass filters. After the input signal $x(n)$ has been passed through these filters, downsampling by factor $R \leq N$ can be applied to the band limited signals to obtain the subband signals[1] $X(\mu, k)$ for frequency bin μ and frame $k = \lfloor n/R \rfloor$. If the bandpass filters are derived from prototype lowpass filters $v(n)$ by complex modulation, this structure can be implemented efficiently as a DFT filter bank by using the fast Fourier transform (FFT) when N is chosen as a power of two. This implementation can also be seen as an STFT with a sliding window that is evaluated every R samples:

$$X(\mu, k) = \mathrm{DFT}_N\big\{v(l)\, x(kR + l)\big\}. \qquad (1)$$

Here, $l = 0, \ldots, N-1$ is the local time index within the analyzed signal part. Only the first $N/2+1$ frequency bins have to be stored and processed for real valued input signals, while the remaining ones are the complex conjugate and can be recreated before the synthesis operation.

Signal synthesis is done in a straightforward manner by first computing the inverse DFT of each processed spectrum $Y(\mu, k)$ and overlapping the time domain signals after weighting with the synthesis window $w(n)$. This window $w(n)$ corresponds to an anti-imaging filter that has to be applied after upsampling the subband signals in the concept of parallel bandpass filters in order to suppress repetitions of the signal spectrum. From a time domain point of view, it interpolates between the samples after filling the downsampled signals with zeros. The effective window length is M, meaning that $w(n) = 0 \; \forall\, n \notin \{0, \ldots, M-1\}$. For producing a single output sample, this overlap-add procedure can be expressed by

$$y(kR{+}r) = \sum_{\kappa=0}^{K-1} w(\kappa R{+}r) \sum_{\mu=0}^{N-1} Y(\mu, k{+}K{-}\kappa) e^{j\frac{2\pi}{N}\mu(\kappa R+r)}, \qquad (2)$$

where $K = \lceil M/R \rceil$. The local time index $r = 0, \ldots, R-1$ describes the position of the output sample within the current output frame.

Delay considerations

Since sound cards and DSP systems usually process input samples block wise, output samples will be aggregated until $r = R-1$ before writing them to the output buffer. This means that a delay of K frames is introduced by the synthesis through Eq. (2). The delay for collecting R input samples in the analysis (1) is then included. Note that Eq. (1) covers only the case where the window length is equal to the DFT order N. For polyphase implementations where the window length is a multiple of the DFT order, additional delay could be introduced depending on the shape of $v(n)$.

For DFT filter banks, most often the synthesis window length is chosen as the DFT order $M = N$ for obtaining good anti-imaging properties by having a large filter order. However, this causes a large delay according to Eq. (2). An OLS filter bank results from setting $M = R$. In order to avoid drawbacks from the OLS approach (e.g., the need for projection filters) and still reducing the delay, we propose a filter length of $R < M < N$ resulting in somewhat degraded anti-imaging properties.

Approaches to Filter Design

A standard approach in the design of prototype lowpass filters is to use raised-cosine (Hann) windows. For downsampling ratios $N/R = 2^\beta$, $\beta \in \mathbb{N}$ and $M = N$, they fulfill the condition

$$\sum_{\kappa=0}^{K-1} v(\kappa R + r)\, w(\kappa R + r) \stackrel{!}{=} g \qquad (3)$$

for perfect reconstruction, meaning that the filter bank without processing of the subband signals introduces only

[1] The subbands can also be seen as time-aligned spectra if all filter bank channels are considered at a certain time instance.

a delay and a gain g. Several methods have been proposed for iteratively optimizing these filters. A mathematical formulation of the so-called *in-band aliasing* and the *total aliasing* is given in [1]. Minimization of these error criteria is done subject to the constraint of near-perfect reconstruction, meaning that amplitude distortions are allowed to a certain extend. Similar approaches are reported in [2] and [3] which mainly differ in details about the error function for optimization. Different filter lengths $M \neq N$ are generally allowed, but only taken into account when the filter group delay is included in the error function. This is not the case for [4] which has an emphasis on designing pairs of analysis/synthesis windows that can be switched to trade-off time and frequency resolution depending on the input signal. Once again, different criteria related to the filter stop band attenuation are optimized in an iterative way, limiting the usage of this method to offline filter design. A different approach for $N = M$ is followed in [5] where a raised-cosine transfer function serves as a prototype.

Non-Iterative Filter Design

Given an analysis window $v(n)$, a matching synthesis window ensuring perfect reconstruction for arbitrary values of N and R can be obtained by the transformation

$$\mathbf{w} = \mathcal{W}\{\mathbf{v}\} = \left[\mathbf{V}^{\mathrm{T}}\mathbf{V}\right]^{-1}\mathbf{V}^{\mathrm{T}}\mathbf{c}. \tag{4}$$

The $N \times 1$ vectors $\mathbf{v} = [v(0), \ldots, v(N-1)]^{\mathrm{T}}$ and $\mathbf{w}$ contain the filter coefficients, $\mathbf{c}$ is a vector of R ones, and the matrix $\mathbf{V} = [\mathbf{V}_0, \ldots, \mathbf{V}_K]$ contains the matrices

$$\mathbf{V}_\kappa = \begin{bmatrix} v(\kappa R) & 0 & \ldots & 0 \\ 0 & v(\kappa R+1) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & v(\kappa R+R-1) \end{bmatrix} \tag{5}$$

with coefficients $v(n)$ on the diagonals [6]. This solution with the pseudo-inverse matrix yields the matching coefficients with minimum norm but, however, there are other solutions to this problem. If the method (4) is applied for shortened windows with $M < N$ an additional smoothing window $h(n)$ should be introduced to avoid discontinuous results:

$$w(n) = \begin{cases} \frac{h(n)\mathcal{W}\{h(n)\}}{\max\{v(n),\varepsilon\}}, & \text{for } n = 0, \ldots, M-1 \\ 0, & \text{else.} \end{cases} \tag{6}$$

The maximum operation with the variable ε avoids divisions by zero or very small values which can occur at the edges of $v(n)$. Note that for values $\varepsilon \neq 0$ condition (3) is violated. However, near-perfect reconstruction can be achieved which is sufficient in many applications [4]. The remaining free parameters in this design scheme of Eq. (6) are the analysis window $v(n)$ and the intermediate smoothing window $h(n)$. Both have been derived by setting different parameters in the prototype window

$$g(n) = \begin{cases} \left[\cos^2\left(\frac{\pi n}{2N_1} + \frac{\pi}{2}\right)\right]^{\alpha_1}, & \text{for } 0 \leq n < N_1 \\ \left[\cos^2\left(\frac{\pi(n-N_1+N_2)}{2N_2} + \frac{\pi}{2}\right)\right]^{\alpha_2}, & \text{for } N_1 \leq n < N, \end{cases} \tag{7}$$

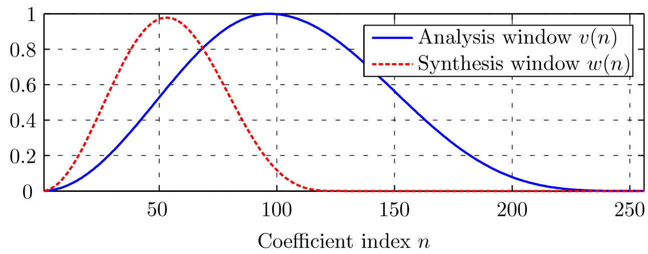

Figure 1: Window pair designed for a low-delay speech enhancement system.

that consists of two raised-cosine parts. The length $N_1 = N - N_2$ can be set to produce asymmetric windows, the powers α_1 and α_2 allow stronger tapering towards the left and right end. For a design with $M = N/2$ we propose to set $N_1 = (N - M/2)/2$, $\alpha_1 = 1$ and $\alpha_2 = 2$ for the analysis window. For the smoothing window $h(n)$ we propose $N_1 = N_2 = M/2$, $\alpha_1 = 1$ and $\alpha_2 = K/10$.

Design example

Fig. 1 shows a window pair that has been designed for a low-delay speech enhancement system with the parameters $N = 256$, $M = N/2$ and $R = 32$. The sampling rate is $f_s = 22050$ Hz, so the resulting filter bank delay is $M/f_s = 5.8$ ms. Tests with an echo canceler within this systems showed echo reduction of about 35 dB and thus sufficient suppression of aliasing distortions. Compared to a filter bank with Hann windows of these lengths, perfect reconstruction is achieved and the echo cancellation could be improved by approximately 5 dB.

References

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