The Reverberation Time as a Function of the Surface Scattering Coefficients

Uwe M. Stephenson

HafenCity University, 22297 Hamburg, Germany, Email: post@umstephenson.de

Motivation

In contrast to common reverberation theory, the reverberation times RT in non-diffuse sound fields depend on the room shape, the distribution of the absorption and especially the **scattering coefficients σ.** With parallel plane walls of low absorption and scattering, as in shoe-box-rooms, flutter echoes may occur with a RT much longer than according to Sabine. The opposite is the case with focusing effects onto absorbing parts of the surface (often the audience) as in domes or semi-circular rooms. In all these cases, scattering and direction effects onto absorbing or non-absorbing parts of the surface play the crucial role [1]. However, the RT can up to now only be computed numerically or for totally diffusely reflecting walls. So, the aim is to find approximating formulae or at least a semi-analytical approach for RT correction factors as a direct function of σ. For this purpose, this study is restricted to 2D. Two opposite cases are investigated analytically and for reference numerically: A) a rectangular 'room' and B) a semi-circular room.

Reverberation in the diffuse sound field

The approach here is to consider a step wise energy decay as with the Eyring theory [2]. With that, the RT constant is the proportion of the mean free path length $\Lambda = 4V/S$ and an average absorption exponent α'_m

$$\tau_{ey} = \frac{\Lambda}{c\,\alpha_m'} \qquad (1)$$

where $\alpha'_m = -\ln(1-\alpha_m)$, $\alpha_m = \sum \alpha_i\, S_i/S$ is the mean absorption degree a 'representative' 'sound particle' sees in a 'diffuse sound field' (V=volume, S= surface , S_i= single surfaces, α_i = absorption degrees, c= sound velocity). The time for a 60dB decay is generally $T = 6\ln(10)\,\tau$.

In 2D (with U= circumference, S= ground area) the mean free path length is $\Lambda = \pi \cdot S\,/\,U$ (2)

The 'average absorption degree' is lengths-weighted:

$$\alpha_m = \frac{\sum \alpha_i b_i}{U} = \frac{B}{U} \qquad (3)$$

(edge lengths b_i, B= eff. absorption length). So, in 2D, the Sabine RT is $T_{sab} = 6 \cdot \ln(10)\frac{\Lambda}{c\,\alpha_m} \approx 0.128\frac{S}{B}$ (4)

The Kuttruff reverberation time depends also on the variation of the absorption degrees and is even smaller than the Eyring value [3]: $T_{sab} >\approx T_{Ey} >\approx T_{Kutt}$

A) Reverberation in a rectangular room

To enhance the chance for a semi-analytical investigation, the problem is simplified as much as possible: a 'room' of length L ('floor 'and 'ceiling' totally absorbing) and height H (front- and end walls totally reflecting and scattering with σ). Parameters are: the **proportion *q=H/L*** (typically< 1 or <<1) and σ (fig.1). $\sigma = 1$ means Lambert reflection with the probability density

$$\frac{dp}{d\vartheta} = cos(\vartheta)/2 \text{ , normalized for 2D.} \qquad (5)$$

The source is in the middle. Inserting the surface S= qL^2 , the circumference $U = 2L(1+q)$ and the mean absorption degree $\alpha = 1/(1+q)$ and exp. $\alpha_m' = ln(1 + 1/q)$ to equ.1-4 yields for reference the Sabine reverberation time

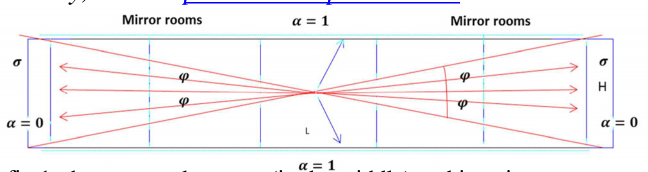


fig.1: the rectangular room (in the middle) and its mirror rooms only in longitudinal direction as only front walls reflecting

$$Tsab = 0.064 \cdot q \cdot L \qquad (6)$$

and (better) the Eyring $T_{ey} = \frac{Tsab}{(1+q)\cdot ln(1+1/q)}$ (7)

and the even smaller Kuttruff value (8)

$T_{Kutt} = T_{Ey} \cdot \alpha_m'/\alpha_m''$ with $\alpha_m'' = \alpha_m' + q/(1+q^2)$.

The aim is to compute the energy decays (start with 1) in the whole room. The idea is to classify the sound energies into:

- the geometrical reflected 'ur-'radiation from the source ('*s*');
- the (after the 1. refl.) always diffusely refl. energy ('*d*') and
- the, after once diffusely, *j* times geom. refl. energies ('*gj*').

It is assumed that the main energy stream is horizontal, so the mean free path length is *L* instead of Λ, while diffuse radiation fractions are partly absorbed by the long 'side' walls. In an iteration, the energy fractions are considered immediately after the *k*-th front-wall-reflections. So, some 'transition coefficients' *U (Uss, Usd, Udd,* Udg^a *)* have to be estimated first. The idea to estimate the (rest of the) ur-radiation $E_s(k)$ is: consider the angle fraction $4\varphi/(2\pi)$ determined by *H/2* in a distance of *(k-1/2)* mirror room lengths (fig.1). Then:

$E_S(t) = \frac{2}{\pi}\arctan\left(\frac{H}{2ct}\right)$ or $E_S(k) = \frac{2}{\pi}\arctan\left(\frac{q}{2k-1}\right) \sim \frac{1}{k}$ (9)

The transition coefficients $k \rightarrow k+1$ without scattering are:

$$U_0(k) = E_S(k+1)/E_S(k) \approx 1 - 2/(2k+1) \qquad (10)$$

This decay is not exponential!

Assuming Lambert diffuse reflection, the fraction of energy reaching a 'side' wall from a front wall is (as known from the 'radiosity' method or the Kuttruff integral [3]) determined by the integral $g_{HL} = 1/H \int_0^H \int_0^L \frac{cos\vartheta \cdot cos\vartheta' \cdot dxdy}{2r}$ (11)

$cos\vartheta = x/r$, $cos\vartheta' = y/r$ and $r = \sqrt{x^2+y^2}$ (fig.2). The

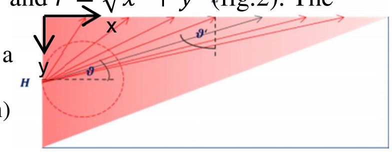


fig.2: Diffuse energy interchange between a front and a side wall (dashed circle indicates Lambert reflection)

transition coeff. 'diffuse-absorption' for both sides is (j=0):

$$U_{dga}(j) = 2g_{HL} = \frac{1}{q}\left(1 + \sqrt{j^2+q^2} - \sqrt{(j+1)^2+q^2}\right) (12).$$

$\alpha_{eff}' = -\ln(1 - U_{dga}(0))$ inserted in eq.1 instead α_m' yields for $\sigma = 1$ a RT T_{diff} even longer than T_{sab} and >> T_{Kutt} !

The transition coeff. from a diffuse reflection to absorption after *j* geometric reflections (i.e. 'in the *j*th mirror room') can be computed in a similar way (equ. 12 for j>0).

The transition coefficients are now:

- ur-geo- ur-geo: $U_{SS}(k) = U_0(k) \cdot (1-\sigma)$
- ur-geo-diffuse: $U_{SD}(k) = U_0(k) \cdot \sigma$
- diffuse- diffuse: $U_{dd} = \left(1 - U_{dga}(0)\right)\sigma$
- diffuse- first time geo: $U_{dg} = \left(1 - U_{dga}(0)\right)(1-\sigma)$
- j. geo – j+1. geo: $U_{gg}(j) = \left(1 - V_{dga}(j)\right)(1-\sigma)$
- j. geo- diffuse : $U_{gd}(j) = \left(1 - V_{dga}(j)\right)\sigma$

where $V_{dga}(j) = U_{dga}(j)/\sum_0^{j-1} U_{dga}(i) \approx 1/(j+1)$ (13)

Correspondingly, the energies in the different classes win or lose energy in every step; each iteration step can be performed by a matrix multiplication involving these coeff. : The reverberation time of the total process is computed from the decay of the summed-up energies. Here only one result can be shown for the typical case q=0.5 and $\sigma = 0.1$:

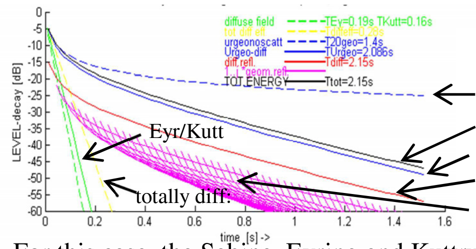


fig.3: level decays in a rectangular room: urgeo without scatt. total energy urgeo with scattering diffuse geo j.order after diff.

For this case, the Sabine, Eyring and Kuttruff RT are very low (<0.2s, $T_{diff} = 0.28s$) while the RT for the geom. refl. energy (depending on the threshold where it is counted, here at -20dB) is very much higher (1.4s, for 30dB without scattering 9.36s!). Astonishingly, the RT for the urgeo, diffuse and total energies are very similar in all cases of q as they mutually depend from each other (here 2.15s). After a very few reflections, they decay 'in parallel' just on different levels depending on σ. The RT in such a rectangular room drastically increase with decreasing scattering coefficients of the front walls. For $\sigma \to 1$ the RT approach T_{diff} but are still higher than according Sabine. Some num. results:

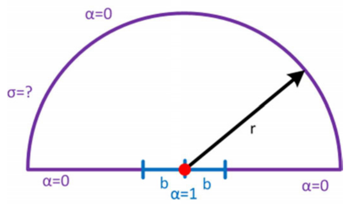
σ	0.05	0.1	0.25	0.5	1
RTtot	3.1	1.8	0.85	0.49	0.28

For all q, the RT can be estimated by $T_{tot} \approx T_{diff}/\sigma^{0.8}$

B) Reverberation in a semi-circular room

Also here the problem is simplified as much as possible: there are only two geometric parameter: the radius r and the width $2b<<r$ of a small piece of floor around the centre (parameter ***q=b/r***) which is totally absorbing; all other 'surfaces' are totally reflecting , with the degree scattering σ (fig.4)

Fig.4. The simplified model of the semi-circular room



The source is close over the ground in the centre. If a 'sound particle' hits the ground, it is either absorbed in the range $|x| < b$ else reflected but re-emitted from the centre.
The approach is here to ask: Which 'effective absorption coefficient' α_{eff} does a 'sound particle' 'see' on the floor if it is reflected from the ceiling according the scattering coefficient σ? This is then inserted into the Eyring equ. 1
For reference again, inserting the surface $S = \pi r^2/2$ and the equiv. absorption length $B = 2b$ into equ. 4 yields for the Sabine rev.time $T_{sab} = 0.100 \cdot r^2/b$ (14)
As the first step of the analytical investigation, an approximate probability distribution for the semi-diffuse reflection $\frac{dp}{d\gamma}$ is derived from the Lambert law (equ. 5). For vertical incidence and an interpolation between the geometrically and diffusely reflected vector [4] the mixed reflection angle can be approximated very well by $\gamma \approx \sigma \cdot \vartheta$ (15)
Thus, the probability density for the mixed reflection law is (see fig.5): $p' = \frac{dp}{d\gamma} = cos(\gamma/\sigma)/(2\sigma)$ (16)

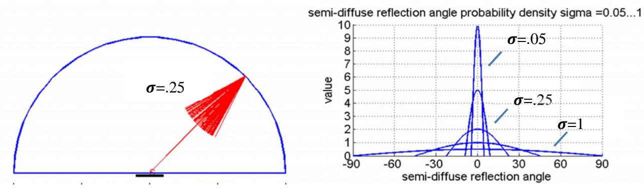
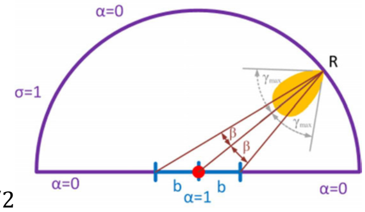


fig.5: Sound particles scattered at the ceiling (with σ=0.25) / approximate probability distribution for the semi-diffuse reflection

Fig.6. The absorbing part of the floor 2b is seen from a reflection point *R* over an angle of 2β; the yellow bubble at *R* indicates the reduced Lambert probability within the angle range of $\pm\gamma_{max} = \sigma \cdot \pi/2$



It can be (even for *q<0.5* by better than 10%) approximated (fig.6) that from a point *R* at an angle $\varphi = -\frac{\pi}{2} \ldots +\frac{\pi}{2}$ on the ceiling the absorbing part *2b* is seen over an angle of twice

$$\beta \approx b \cdot cos\varphi/r \quad (17)$$

on average over all φ: $\beta_m \approx 2/\pi \cdot q$ (18)

The probability that the absorbing part of the floor is hit is the integral over the reflection distribution:

$$p_a = \int_{-\beta_m}^{+\beta_m} p' d\gamma = sin(\beta_m/\sigma) \approx \frac{2}{\pi} \cdot \frac{q}{\sigma} = \alpha_{eff} \quad (19)$$

This is (for *q/ σ <<1*) at the same time the mean abs. degree which 'a sound particle sees' after a ceiling reflection (may be >1). The relevant 2 path lengths (from the centre to the ceiling and back) are about $\Lambda \approx 2r$. This and α_{eff} inserted in equ. 4 yields : $T_{semi} \approx 0.128\sigma \cdot \frac{r^2}{b} = 1.28 \cdot \sigma \cdot T_{sab}$ (20)
The reverberation time approaches zero with totally geometric reflections of the ceiling and approaches the Sabine value for totally diffuse reflections – both not surprising.
A sound particle simulation (3000 particles) was started for comparison. The reverberation time T_{sp} was computed from a linear regression analysis of the level decay in the range 0…-30dB**.** Fig.7 shows a result of a comparison.

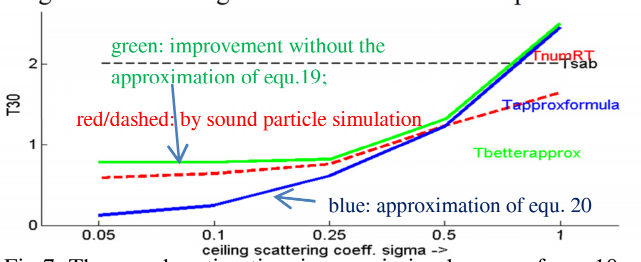


Fig.7: The reverberation time in a semi-circular room for r=10m, b/r=0.5 as a function of the scattering coefficient of the ceiling σ: even for a wide absorbing part ($T_{sab} = 2s$) the agreement with the approximation of equ. 20 is quite good, at least in tendency.

Conclusion

Such formulae remain an estimation, a study for special cases.

References

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[2] Stephenson, U.M.: A Rigorous Definition of the Term "Diffuse Sound Field" – and why the Sabine Reverberation Formula is Different from the Eyring Formula; in: Fortschritte der Akustik, DAGA 2012, Hrsg. DPG-GmbH, Bad Honnef, 2012

[3] Kuttruff, H.: Room Acoustics, Elsevier Science Publishers Ltd., Barking, England, 3rd ed. (1991)

[4] Stephenson, U.; Eine Schallteilchen-Computer-Simulation zur Berechnung der für die Hörsamkeit in Konzertsälen maßgebenden Parameter. ACUSTICA 59 (1985), p. 1-2