

Ambisonic panning with constant energy constraint

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Introduction

Since audio playback techniques have extended to a pair of loudspeakers for stereo, strategies for making sounds appear from adjustable locations has been a driving idea in mixing of audio signals. Mixing consoles have been equipped with so-called panning knobs that distribute single audio signals accordingly to a pair of loudspeakers, or even five pairs in surround.

We lengthly discussed Ambisonics as a surround playback technique at various levels of complexity in [1] and revisited its simple interpretation as a panning technique for spherically surrounding loudspeaker arrangements, cf. [2]. Thereby, we introduced energy preserving Ambisonics. This contribution neatly and compactly recapitulates the main finding of how to enforce a constant-energy constraint using Ambisonics, from a more general perspective.

Within this article, we define the Cartesian direction vector as $\boldsymbol{\theta} = [\cos(\varphi) \sin(\vartheta), \sin(\varphi) \sin(\vartheta), \cos(\vartheta)]^T$ with φ and ϑ being the azimuth and zenith angle, respectively.

Amplitude panning

Amplitude panning distributes an audio signal $s(t)$ to $l = 1, 2, \dots, L$ loudspeakers at the directions $\boldsymbol{\theta}_l$ using the panning functions $g_l(\boldsymbol{\theta}_s)$

$$x_l(t) = g_l(\boldsymbol{\theta}_s) s(t). \quad (1)$$

Suitable panning functions create the impression of a single sound event at the direction $\boldsymbol{\theta}_s$.

Gathered in a vector $\mathbf{g} = [g_1(\boldsymbol{\theta}_s), \dots, g_L(\boldsymbol{\theta}_s)]^T$, panning functions are usually constrained to constant “energy” $\|\mathbf{g}\|^2 = 1$. This is easily achieved from any unnormalized $\tilde{\mathbf{g}}$ by

$$\mathbf{g} = \frac{\tilde{\mathbf{g}}}{\|\tilde{\mathbf{g}}\|} \quad (2)$$

what largely ensures a direction independent loudness and works well for vector base amplitude panning, cf. [3] and also for Ambisonic panning.

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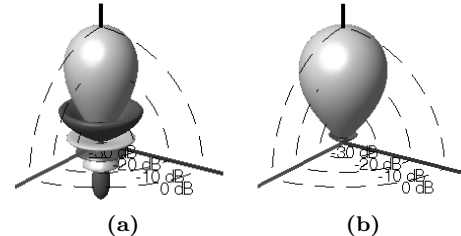


Figure 1: Panning function of the order $N = 5$ plotted as balloon diagram (a) without and (b) with smoothing.

Ambisonic panning

The simplest form of Ambisonic panning functions of the (maximum) order N is

$$\tilde{g}_l(\boldsymbol{\theta}_s) = \frac{4\pi}{L} \sum_{n=0}^N \sum_{m=-n}^n w_n Y_n^m(\boldsymbol{\theta}_l) Y_n^m(\boldsymbol{\theta}_s), \quad (3)$$

where w_n are optional order weights and $Y_n^m(\boldsymbol{\theta})$ are the real-valued spherical harmonics (SHs) defined as

$$Y_n^m(\boldsymbol{\theta}) = N_n^{|m|} P_n^{|m|}(\cos \vartheta) \begin{cases} \cos(m\varphi), & \text{for } m \geq 0 \\ \sin(m\varphi), & \text{for } m < 0. \end{cases}$$

In the above equation, $P_n^{|m|}$ denotes the associated Legendre function, and $N_n^{|m|}$ is the scalar energy-normalization of the SHs.

Fig. 1 depicts the panning function of order $N = 5$ for a loudspeaker located at the zenith, (a) without weights w_n and (b) with weighting resulting in an attenuation of its side lobes. There are different weighting functions¹ available. However, weighting it is not considered here for simplicity.

Eq. (3) is reformulated to a vector product $\tilde{g}_l(\boldsymbol{\theta}_s) = \frac{4\pi}{L} \mathbf{y}_N^T(\boldsymbol{\theta}_l) \mathbf{y}_N(\boldsymbol{\theta}_s)$, with $\mathbf{y}_N(\boldsymbol{\theta}) := [Y_n^m(\boldsymbol{\theta})]_{q=1 \dots (N+1)^2}$ using the linear index $q = n^2 + n + m + 1$. Similarly, the vector-valued panning function is expressed as

$$\tilde{\mathbf{g}} = \mathbf{Y}_N^T \mathbf{y}_N(\boldsymbol{\theta}_s) \quad (4)$$

with the matrix $\mathbf{Y}_N := \frac{4\pi}{L} [\mathbf{y}_N(\boldsymbol{\theta}_l)]_{l=1 \dots L}$. The Ambisonic panning functions are smooth N^{th} order spherical functions, see Fig. 1, but generally not normalized for arbitrary loudspeaker setups. Normalization according to Eq. (2) tends to increase the order of the panning function, thus the variability with regard to the panning direction $\boldsymbol{\theta}_s$. This is avoided by the approach presented in the following section.

¹Literature on Ambisonics mainly discusses two types of weighting: *max-r_E* and *in-phase*, cf. [2, 4].

Constant energy

The vector norm $\|\mathbf{y}_N(\boldsymbol{\theta}_s)\| = \frac{N+1}{\sqrt{4\pi}}$ of the vector in Eq. (4) is constant. Consequently, a constant energy constraint is met if \mathbf{Y}_N is an orthogonal matrix, i.e. $\mathbf{Y}_N \mathbf{Y}_N^T = \mathbf{Y}_N^T \mathbf{Y}_N = \mathbf{I}$. However, this is only the case when using four loudspeakers arranged on the vertices of a tetrahedron and $N = 1$.

Generally, the singular value decomposition (SVD) reveals energy scaling factors in \mathbf{Y}_N , as it factorizes the matrix

$$\mathbf{Y}_N = \mathbf{U} \mathbf{S} \mathbf{V}^T, \quad (5)$$

into $\mathbf{U}_{(N+1)^2 \times (N+1)^2}$ and $\mathbf{V}_{L \times L}$, two orthogonal matrices, and $\mathbf{S} = \text{diag}_{(N+1)^2 \times L}\{\mathbf{s}\}$, a diagonal matrix containing the singular values. The energy is

$$\|\hat{\mathbf{g}}\|^2 = \mathbf{y}_N^T(\boldsymbol{\theta}_s) \mathbf{U} \text{diag}\{\mathbf{s}\}^2 \mathbf{U}^T \mathbf{y}_N(\boldsymbol{\theta}_s), \quad (6)$$

and it is obviously only scaled by $\text{diag}\{\mathbf{s}\}^2$. Constant energy $\|\mathbf{g}\|^2 = 1$ would be conveniently achieved by Eq. (2) or an approach that manages to enforce $\text{diag}\{\mathbf{s}\}^2 = \frac{4\pi}{(N+1)^2} \mathbf{I}$.

As a solution we formulated a new Ambisonic panning function in [1]

$$\mathbf{g} = \frac{\sqrt{4\pi}}{N+1} \mathbf{V} \mathbf{U}^T \mathbf{y}_N(\boldsymbol{\theta}_s), \quad (7)$$

If $L \neq (N+1)^2$, the column size of the bigger matrix (either \mathbf{U} or \mathbf{V}) must be truncated. If there are at least $(N+1)^2$ loudspeakers, the constant energy constraint $\|\mathbf{g}\|^2 = 1$ is fulfilled for every direction $\boldsymbol{\theta}_s$.

Ambisonic panning for partially surrounding loudspeakers, cf. [5], uses spherical Slepian functions instead of SHs, cf. [6]. The constant energy approach described above can be applied similarly for panning with these functions.

Example

Ambisonic panning with constant energy is exemplarily shown for 17 loudspeakers arranged on the grid of an 18-nodes equal area partitioning [7]. The 9th node is left free to disturb the uniformity of the setup.

Fig. 2(a) shows $\|\mathbf{g}(\boldsymbol{\theta}_s)\|^2$ according to Eq.(4) for an Ambisonic order of $N = 3$; the loudspeaker positions are indicated by the black crosses. The energy decreases significantly if the panning direction is close to the position of the omitted node. Constrained Ambisonic panning functions are depicted in Fig. 3 for the five horizontal loudspeakers of the exemplary setup, for panning on the equator. The directly normalized functions of Fig. 3(a) are obviously of higher order than 3 and exhibit a strong variation around 150° , the location of the omitted node. By contrast, Fig. 3(b) shows the smooth 3rd order panning of Eq. (7).

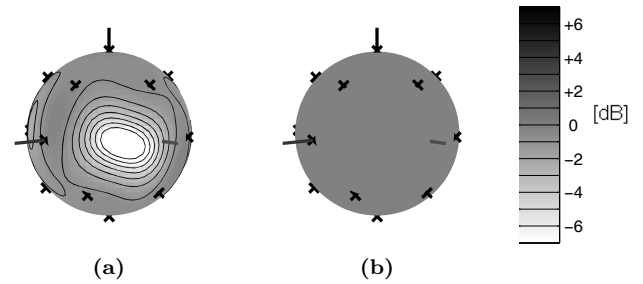


Figure 2: Energy of the panning function $\|\mathbf{g}\|^2$ depending on the panning direction $\boldsymbol{\theta}_s$, (a) for Ambisonic panning and (b) including a constant energy constraint.

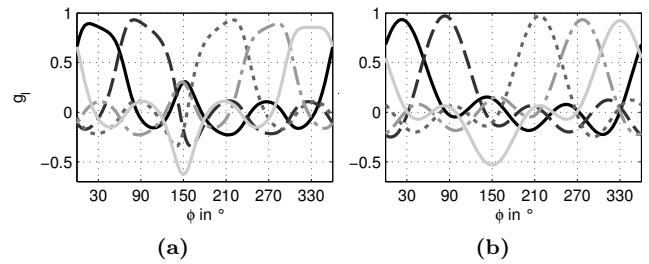


Figure 3: Ambisonic panning functions for a horizontal angle φ for the horizontal loudspeakers of the example arrangement: (a) according to Eq. (2) and (b) according to Eq. (7).

Conclusion

We have briefly summarized our new Ambisonic panning function with an constant energy constraint. In contrast to simple normalization, the new approach yields smooth N^{th} order panning functions. Because of its smoothness, it yields good results for sounds moving in space, in particular.

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