Synchronization of nonlinear, acoustical oscillators

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Introduction

The Synchronization (german *Mitnahme-Effekt*) of two organ pipes has been previously studied exemplarily [2], in great detail, the synchronization of an organ pipe by an external loudspeaker was investigated [1]. As a result, the organ pipe has been modeled by a quite simple self-sustained oscillator and the corresponding Arnold-tongue was reproduced with good coincidence of model and measurement. Here, we present new results regarding the coupling of two interacting organ pipes. As a new validation and modeling strategy, we use detailed numerical simulation of the compressible Navier-Stokes equations. This way, some anomalies, found by experimental investigation [3] shall be explained based on fluid dynamics.

Synchronization

Synchronization occurs if a self-sustained oscillator on his limit cycle interacts with an external periodic driving force or another oscillator of the same type. Typically, external period or the period of the second oscillator is slightly detuned, when uncoupled. The *coupled* system, however, shows exactly one period. The phenomenon is as well called frequency locking, mode locking or take-away effect. In Fig. 1, the phase portrait of the paradigmatic van der Pol oscillator is shown, known in musical acoustics by its relation to the Rayleigh oscillator, for two different initial conditions. One recognizes the limit cycle which occurs due to the interplay of linear driving and nonlinear saturation.

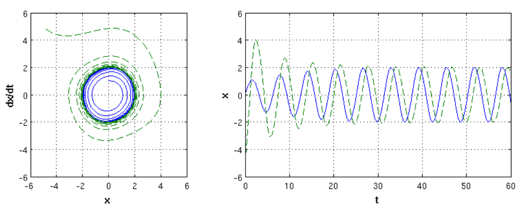

Figure 1: Self-sustained oscillations (van der Pol system). Different initial conditions (green and blue) yield a stable limit cycle after transients.

If two such systems with (uncoupled case) frequency ω_0 and ω are coupled, new frequencies of the common system are observed, here denoted by Ω and ω. When one plots the difference of the frequencies observed in the coupled system, $\Omega - \omega$ versus the detuning $\omega_0 - \omega$, a so-called "synchronization plateau" occurs in a certain range of detuning for a given coupling strength ε. The baffling fact is that in this plateau the two coupled oscillators show one common and *identical* frequency. Further, the oscillators have a "locked" phase difference of $\Delta\phi$, which varies in the range of $[\phi_0, \phi + \pi]$ in the plateau region, cf. Fig. 2 (a). If in addition the coupling is varied, the width of the plateau varies forming the so-called Arnold-tongue, cf. Fig. 2 (b). In principle, it should be possible to conclude inversely from the Arnold tongue some system properties, in particular the coupling.

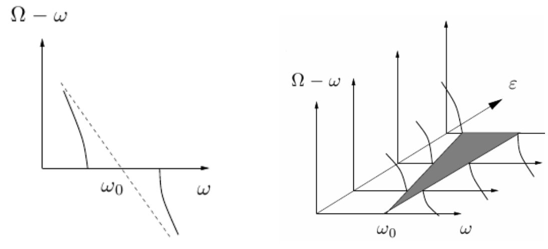

Figure 2: (a) Synchronization plateau about ω_0. (b) Arnold tongue for varying coupling strength.

Experiment and Results

Recently [3], we carried out a systematic series using two closed end organ pipes, which were identical, but for their tuning. Controllable detuning was achieved by using an automatized setup where one pipe was manipulated by a step-controlled piston on its top. The fixed frequency was at 720 Hz, the other was varied in steps of 0.5 Hz. The coupling was varied by changing the distance, d, between the pipes to 1, 10, 30, 50, 75, 100, 200, 300 and 400 mm, measured in the far field. As expected, synchronization occurs, exemplarily shown in Fig. 3a for the unscaled spectrum (using the true frequencies, and not the differences) for different distances. The color coding corresponds to the amplitude of the coupled system, measured at the microphone for $d = 1$ mm. From left to right, the 2nd pipe is detuned. For better visualization, only the range about the 1st harmonics is shown. With decreasing detuning, the frequency of the 2nd pipe is lowered, or "taken away". The sidebands, caused by the nonlinear interference of the frequencies are attracted correspondingly. At a detuning Δf of ca. -3 Hz the 1st pipe is adjusted by the 2nd (and vice versa). From now on, the pipes oscillate synchronuosly over a range of ca. 6 Hz. In total, the pipes "feel" each other over a range of more than 10 Hz. For too large detuning, synchronization cannot be maintained and the pipes' interaction is not strong enough to compensate the detuning. The sequence Fig. 3b - 3d shows the narrowing of the synchronization plateau with distance for three distances.

From a systematic investigation, we conclude the coupling function, which does not seem to behave like the $1/r^2$-law, expected from the decay of energy in the far field of a monopole source. Instead, some more logarithmically looking function is found, details are presented elsewhere [3].

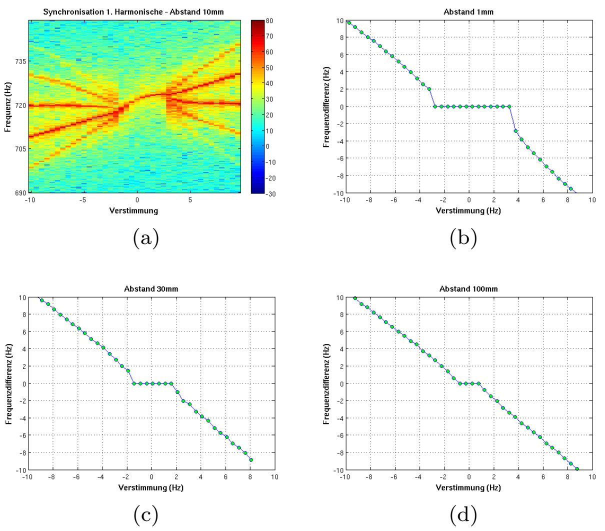

Figure 3: (a) Color coded spectra about the 1st harmonics with detunig steps of 0.5 Hz. (b)-(d) The synchronization plateaus for the distances 1, 30 and 100 mm.

The wavelength of the fixed pipe is $\lambda_0 = 476$ mm. Successless attempts to interprete this result motivated us to start investigation by numerical means, using computational fluid dynamics/aeroacoustics (CFD/CAA), since recent works indicate a quite complex interplay of fluid mechaincal and aero-acoustical processes, which are basically not understood.

Computational Fluid Dynamics / Computational Aeroacoustics (CFD/CAA)

In order to clarify the questions regarding the functioning of the organ pipe as a self-sustained oscillator and the coupling issue, the compressible Navier-Stokes-Equations were computed fully resolved on a fine grid in 2D. The simulations were carried out with the C++-Library OpenFOAM 2.0 [4] on a 6-core PC. In Figs. 4a - 4c some snapshots are shown for turbulent kinetic energy, pressure and the absolute value of the velocity. Simulation times were 100 und 200 ms, respectively.

We identified preliminary processes which contribute to the tone generation. The *jet* is the basic generator as it produces at its Reynolds number of about 10^3 to 10^4 typical vortices which radiate sound. Since the jet is fed back periodically by the pressure perturbations in the resonator, quickly, the corresponding frequency is amplified strongly in the jet and other frequencies are basically suppressed. This in turn results in a periodic oscillation of the jet (often called the "air sheet") at the upper labium. The coupling of the inflowing mass into the resonator plays a role, as well as the feedback described

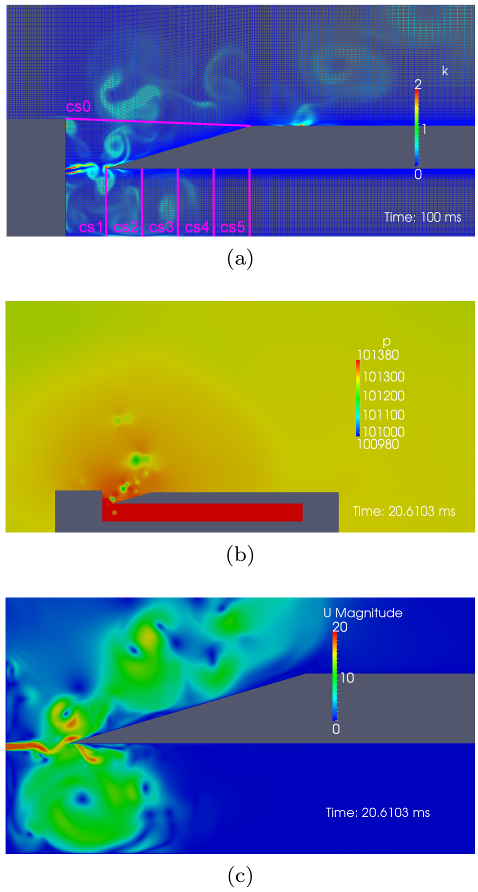

Figure 4: (a) Calculation grid with defined cross-sections for the analysis, here for the turbulent kinetic energy k. (b) pressure p. (c) absolute value of velocity $|\mathbf{U}|$.

above. To complete the picture as self-sustained oscillator, one still has to identify the nonlinear losses: they are the waves radiated away and the energy dissipated in the resonator. By using this simple model, we achieved a good coincidence of numerics and experiment, reported elsewhere. In essence the coupling function can now be concluded by a full simulation of two oscillators. This is ongoing work.

References

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[4] OpenFoam 2.0, URL: `http://www.openfoam.com/`