Analytical Uncertainty Analysis of an Acoustic System with Tonal Excitation

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Abstract

In this contribution the problem of tonal excitation signals with acoustic modal systems with uncertain boundary conditions is studied. The system used is a rectangular enclosure with interior sound source and receiver. We determine the uncertainty of the output signal by introducing variances of complex boundary impedances and variances in sensors positioning. The first example focuses on the uncertainties in numerical predictions, and the second deals with practical constraints in any kind of measurement. The input signals are tonal signals with added background white noise and also a sample of electric engine noise during a run-up. The system reacts with respect to these excitations, and the uncertainty of the output signal is calculated and discussed for applications in condition monitoring and sound design by discussing the variances of level and loudness.

Introduction

Statistical analysis of acoustic system parameters beneficiates engineering to observe and interpret simulation and measurement results with a defined level of confidence. While with broadband excitation signals variances in a modal system do not affect the output level significantly, this may completely change when tonal signals are used. The reason is that the output with a broadband excitation mainly depends on the integral modal response, whereas a tonal excitation depends on the actual slope of the mode. Slight variations in frequency show great variations in the output. The same phenomenon occurs when the modal response is slightly shifted due to variances in modal damping (boundary conditions) or source and receiver placement.

In this paper first an acoustic system with two input uncertainty parameters: boundary conditions and sensors positioning is introduced. Then the system is excited with two tonal sound sources. The level of confidence of the output result is simulated on the basis of the variances of the input uncertainty parameters. Finally, as a first step the output analysis is performed by investigating the level and psychoacoustic loudness.

Acoustic System with Uncertainty

A rectangular box-like enclosure with dimensions of $0.8 \times 0.5 \times 0.3\ m^3$ according to [1] is used as acoustic modal system. The sound source and the receiver are placed in interior opposite corners of the enclosure. The transfer function between sound source and receiver is analytically defined as [3]:

$$TF(\omega) = -\frac{4\pi c^2}{V} \sum_i \frac{\psi_i(r_s)\psi_i(r_r)}{(\omega^2 - \omega_i{}^2 - j\delta_i\omega_i)K_i} \quad (1)$$

where $\psi_i(r_s)$ is i^{th} eigenfunction at source position (r_s) and $\psi_i(r_r)$ is i^{th} eigenfunction at receiver position (r_r) in Cartesian coordinates. The two input sound sources are defined with the following calculations:

1- A harmonic series of pure tones with added background white noise:

$$S_1(f) = 4\delta(f-150) + \delta(f-300) + 0.75\delta(f-450) + 0.5\delta(f-600) + white\ noise \quad (2)$$

2- A simulated electric machine noise during run-up [2]:

$$\underline{S}_2 = Ae^{-j2\pi\mu\left(n_1+\frac{n_2-n_1}{T_{run}}t\right)t} \quad (3)$$

where n_1, n_2, μ, T_{run} and A are start speed, end speed, frequency order, run-up time and amplitude of the engine during run–up respectively. The amplitude of the engine noise can be estimated as [2]:

$$A = \sqrt{8\pi c\rho_0\gamma_0 \left(\frac{2\pi R_r n}{c}\right)^{5.5} S} \quad (4)$$

where n, R_r, and S are mechanical speed, rotor radius and radiating surface, respectively.

System Analysis and Results

The transfer function of the acoustic system with defined uncertainty sources up to 1 kHz is shown in Figure 1.

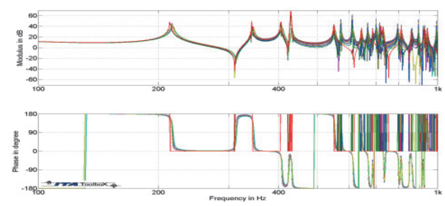

Figure 1. transfer function of acoustic system

The shapes of the peaks at eigenfrequencies are slightly deformed due to uncertainties. Each color belongs to specific uncertainty input parameters (sensor positioning and boundary variations).

Uncertainty input parameters

The following variations in complex boundary conditions are introduced:

$$\left.\begin{aligned} \bar{b} &= 1.4227 \cdot 10^5 \\ \sigma_b &= 1.3877 \cdot 10^5 \end{aligned}\right\} \Rightarrow \quad (5)$$

$$C_b = \bar{b} \pm \sigma_b = [1.4227 \cdot 10^5 \pm 1.3877 \cdot 10^5$$

with $\bar{b}$ and σ_b are the mean absolute value of boundary conditions and standard deviation of complex boundaries. Using following information confidence interval belongs to boundary variations is obtained and defined by C_b.

The second source of uncertainty in this study is variation in sensor positioning which is shown in Figure 2. The variance of the source positioning from the mean value is 0.12 m, and for the receiver positioning is 0.07.

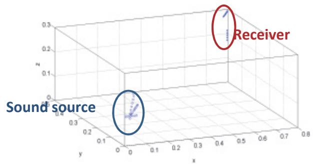

Figure 2. Sensors positions

Uncertainty Output Analysis

The deviation analysis of the output response due to the pure tones with background white noise as well as engine noise is shown in Figure 3. As expected the largest variations occur at the slopes of the modes. The deviation analysis of output signals return the same characteristics, which is due to the concentration of uncertainty sources on the acoustic transfer function rather than input signals. In Figure 3 each color represents deviation of output with respect to each uncertainty input parameters.

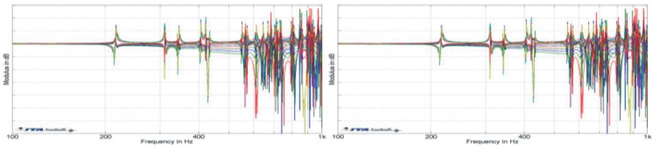

Figure 3. Deviation of output due to pure tones with background white noise (left), electric machine noise (right)

The mean value of the sound pressure at receiver with variety uncertainty is obtained and confidence interval using following equation is calculated:

$$\Delta L = 20\log(1 \pm \frac{\sigma_p}{\bar{p}}) \qquad (6)$$

Figure 4 shows the confidence interval of the sound pressure level. Standard deviations of sound pressure level from mean value is shown with blue and green color, where blue belongs to mean plus standard deviation (STD) of output and green to minus STD.

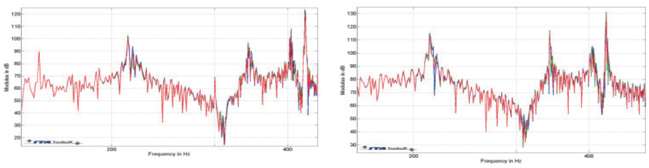

Figure 4. Confidence interval of the tonal excitation (lefts), electric machine noise (rights)

Application-dependent Analysis

This study can further be discussed with two scenarios of applications. The first application is a conditions monitoring problem where the level of the output is the decision criterion for fault detection. In case of a tonal excitation the fault detection suffers from uncertainties and the confidence interval of the level analysis.

Level Analysis

The A-weighted level of the output signal for both excitation signals are shown in Figure 5. Each color line belongs to particular uncertainty parameters.

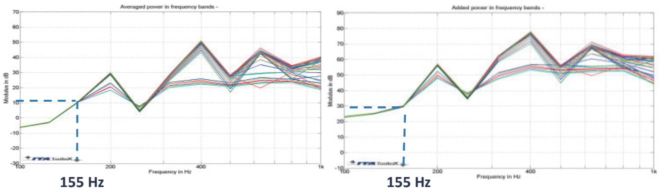

Figure 5. The A-weighted in third octave band with tonal excitation (left), and engine noise (right)

The level differences become more significant in higher frequencies starting from 155 Hz.

Loudness Analysis

In the application example of sound design project by using numerical simulations, the output signal may be analyzed for its loudness. Thus it is considered the variance of the loudness. Figure 6 shows the specific loudness of the output signal in sone per Bark for the two uncertainty input parameters (blue, red and green lines belong to mean plus STD, mean value and mean value minus STD, respectively).

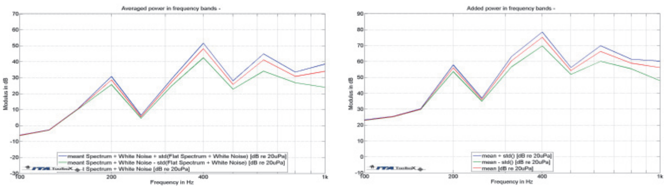

Figure 6. Confidence interval of the level analysis with tonal excitation (left), and engine noise (right)

As another example, the confidence interval of the loudness is obtained and illustrated in the following figure (color lines denote the same pattern of Figure 6).

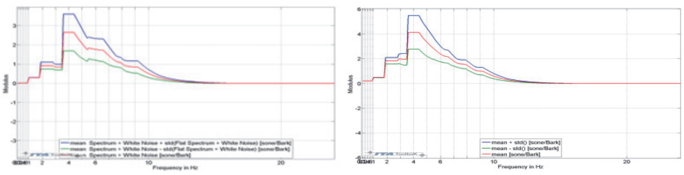

Figure 7. Confidence interval of loudness analysis with tonal excitation (left) and engine noise (right)

Conclusion

A simple acoustic modal system is introduced which consists of a rectangular enclosure with interior sound source and receiver to estimate the influence of variation in boundary condition and sensors positioning on the receiver signal. The excitation signals are a tonal noise with background white noise and an engine noise during run-up. It was obtained the confidence intervals of the analysis by addressing the standard deviations of the output signal and the variations of further analysis methods to obtain level and loudness. This study will be continued aiming at modelling the functional relation between output variances and input variances.

References

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