Acoustic Centering for High-Order Source Directivities

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Introduction

The modeling of sound-source directivity patterns is relevant to musical-instruments acoustics, design of musical synthesizers, room acoustical measurements and simulation and virtual acoustics [1, 2]. Spherical microphone arrays facilitate the representation of the directivity pattern using spherical harmonics, which form a convenient domain for spatial interpolation, analysis, and storage format of sound radiation [3]. However, misalignment of the acoustic source relative to the physical center of the array may result in large deviation of the recorded phase and spatial aliasing errors [4]. In order to suppress this effect, an alignment algorithm which minimizes the energy of the higher order spherical harmonics coefficients has been recently proposed and reported to deliver satisfying results when employed on low-order directivity patterns [5]. However, if the source has a directivity of an order which is higher than allowed by the sampling array, the objective function of the alignment algorithm fails to converge to the correct alignment value. This work presents the analysis of acoustic center alignment for higher order directivity patterns and proposes an algorithm which performs better under these conditions in terms of convexity of the objective function and convergence to a point near the physical center of the physical sound radiation.

Source Directivity Pattern

The pressure function along a surface of a sphere of radius r is given in the spherical harmonics domain by

$$p\left(k,r,\Omega\right)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}p_{nm}\left(k,r\right)Y_n^m\left(\Omega\right) \tag{1}$$

where $p\left(k,r,\Omega\right)$ is the pressure at wave number k at distance r from the origin at spatial angle $\Omega=(\theta,\phi)$. The orthogonal functions $Y_n^m\left(\cdot\right)$ are referred to as spherical harmonics and coefficients p_{nm} are given by the spherical Fourier transform. The solution to the exterior problem [6] indicates that $p\left(k,r,\cdot\right)$ is the directivity pattern of a radiating source and that

$$p_{nm}\left(k,r\right)=c_{nm}\left(k\right)h_n\left(kr\right) \tag{2}$$

where $h_n\left(\cdot\right)$ is the spherical Hankel function of the first kind of order n [6].

Acoustic Centering

In the general case, the acoustic center of the source is not aligned with the physical center of the microphone array. This results in directivity-pattern distortion. Moreover, an order-limited pressure function is no longer order-limited with regard to a translated origin such that [5]

$$\begin{aligned} p\left(k,r,\Omega\right) &= \sum_{n=0}^{N}\sum_{m=-n}^{n}c_{nm}\left(k\right)h_n\left(kr\right)Y_n^m\left(\Omega\right) \\ &= \sum_{n=0}^{\infty}\sum_{m=-n}^{n}c'_{nm}\left(k\right)h_n\left(kr'\right)Y_n^m\left(\Omega'\right) \end{aligned} \tag{3}$$

where r' and Ω' represent the same location in a coordinate system with a translated origin. The high-order coefficients in the translated pressure function may result in spatial aliasing, Therefore it is required to perform acoustic centering, such that energy in higher-order coefficients is minimized. Previous acoustic-centering algorithms [5] attempted directly to minimize $|c_{nm}|$ of higher orders. This however may lead to non-convex objective functions in higher frequencies which are associated with high-order directivity patterns [5]. For this reason a new acoustic-centering algorithm is proposed, such that the symmetry of the radiation pattern is maintained.

The objective function of the proposed acoustic-centering algorithm is given in the spatial domain by

$$J=-\frac{\frac{1}{2\pi}\int_0^{\pi}\left|\int_0^{2\pi}p\left(\theta,\phi\right)d\phi\right|^2\sin\theta d\theta}{\int_0^{\pi}\int_0^{2\pi}\left|p\left(\theta,\phi\right)\right|^2d\phi\sin\theta d\theta} \tag{4}$$

where it is assumed that an extremum of J is obtained when the source is at the center of a $\{p\left(\theta,\phi\right)\}_{\phi\in[0,2\pi)}$ contour, such that the area within is maximized. This is illustrated in Fig. 1

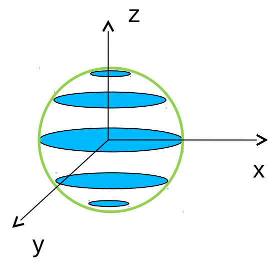

Figure 1: Pressure contours along ϕ in the x-y domain

The corresponding expression in the spherical harmonics domain is

$$J=-\frac{\sum_{n=0}^{N}\left|p_{n0}\right|^2}{\sum_{n=0}^{N}\sum_{m=-n}^{n}\left|p_{nm}\right|^2} \tag{5}$$

given here without proof, where the full derivation will be clarified in a journal paper. This objective function is calculated in the x-y plane, but it can easily adapted to the y-z and x-z planes using appropriate rotations of the original directivity pattern.

Experimental Results

In order to test the proposed acoustic-centering algorithm, trumpet recordings from a surrounding spherical array of 32 microphones were used. The measurements were performed in an anechoic chamber in the technical university of Berlin (TU-Berlin). [7–9] and the recording scheme can be seen in Fig. 2.

Figure 2: Spatial recording of a trumpet using a spherical array of 32 microphones in TU-Berlin

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Figure 3 shows the objective function values at 185 and 744 Hz, compared with the values of the objective function that was reported to achieve the best results in [5]. The results are displayed for each objective function in the x-y, y-z and x-z planes. At low frequencies both objective functions are convex. However, in the higher frequencies convexity is maintained only using the proposed algorithm.

Conclusion

An acoustic-centering algorithm was proposed such that the symmetry of the directivity pattern is maintained. It was shown that while state-of-the-art source-centering algorithms fail to converge at high frequencies, or, with high-order source directivity pattern, the proposed algorithm is convex under these conditions.

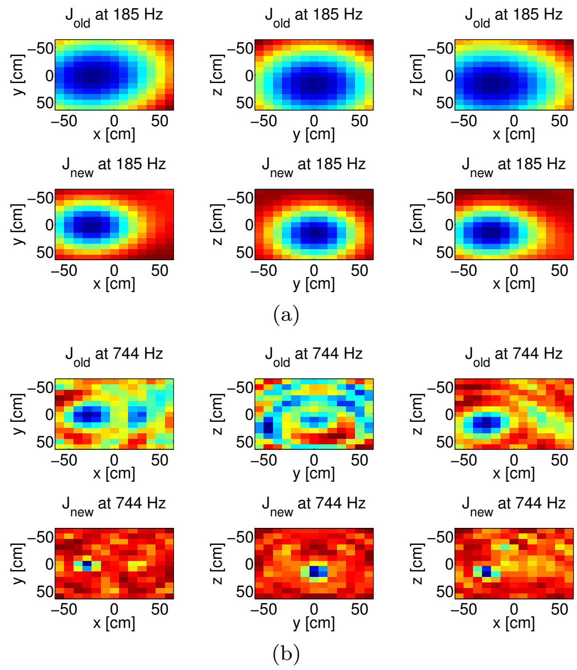

Figure 3: Objective functions at 185Hz (a) and 744Hz (b). State-of-the art objective function (top row) vs. proposed objective function (bottom row). Values represented in x-y, y-z and x-z planes in the left, middle and right columns, respectively.

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