Statistical Uncertainty Analysis of an Acoustic System

Samira Mohamady, Michael Vorländer
Institute of Technical Acoustics, RWTH Aachen University,
smo@akustik.rwth-aachen.de

Abstract

An analytical uncertainty evaluation of a simple acoustic system was proposed by the authors in a previous publication (ISMA 2014-236). In this follow-up study first a detailed statistical analysis of the acoustic system in the Schroeder region is presented. The statistical analysis is applied to the system with uncertain input parameters for tonal excitation, which are changes in sensor positioning and temperature. The analysis is continued with calculation of extrema and bandwidth of the absolute sound pressure variation. Consequently, a comparison of both analyses is performed to derive a final statement of uncertainty evaluation of acoustic transfer functions in the cross-over region between the modal range and the statistical range.

Introduction

Measurements of vibro-acoustic systems are subject to uncertainties mainly due to the measurement procedure. In this research, we look at this issue with at first introducing a simple acoustic system with input uncertainty parameters. Then it is defined the confidence interval due to this uncertainty. Accordingly the statistical uncertainty behavior of transfer function (TF) in Schroeder region is investigated.

Probability uncertainty analysis

Figure 1 shows the block diagram of an uncertainty modelling. Accordingly, the first block from the left side is the excitation input signal which could be either broad band or tonal excitation.

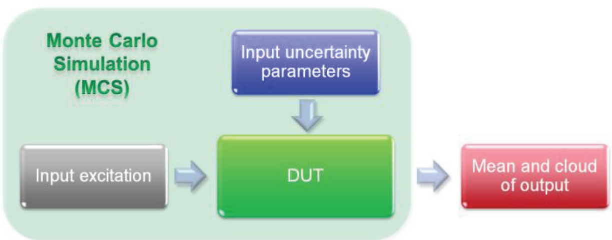

Figure 1: Parametric uncertainty analysis block diagram

The second input block is the input uncertainty parameters and is divided into interior and exterior parameters. Any variation in geometry of the structure, of material properties or rigidity of joints is a part of interior uncertainty parameters, however, temperature changes or displacement in excitation or the receiver point is a part of exterior uncertainty parameters (Figure 2). It is clear that input uncertainty parameters are not limited to the following categories and could be extended in more details. In the following it has been assumed that the interior parameters are fixed and the remaining variations are due to exterior parameters only.

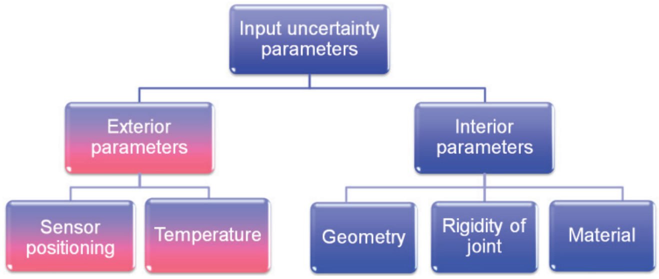

Figure 2: Parametric uncertainty analysis

A third block in Figure1 is a device under test (DUT). Two simple acoustical and structural systems as DUT were defined. The first system is a rectangular enclosure with hard boundary conditions as an example for an acoustic system, and a second is a thin aluminum plate representing a 2D structural system. In both cases Monte Carlo simulations were used to simulate these three blocks. In order to develop a Monte Carlo simulation for sensor positioning, first in a rectangular enclosure a sound point source is placed inside the enclosure and in the top left of the enclosure an analyzing surface is determined with dimension of $10 \times 10\ cm$ (the middle point of the surface is located at $x_s = 0.68\ m,\ y_s = 0.12m,\ z_s = 0.2\ m$). In each side of the surface 88 analyzing point is embedded which together gives 7744 calculating points, the simulation setup can be seen in Figure 3.

Meanwhile, the probabilistic uncertainty analysis is considered. Therefore the middle of the analyzing surface is considered as a center of distribution and 400 samples are taken from the analyzing surface. This distribution can be observed in Figure 4 with green color.

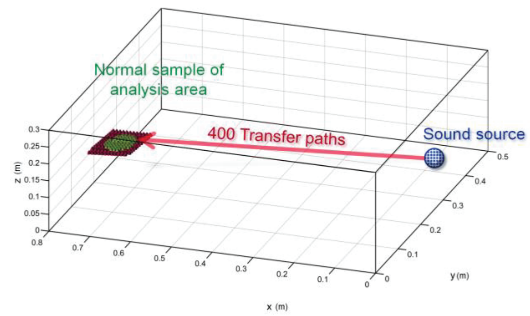

Figure 3: MCS of acoustic model

The second exterior input uncertainty parameter is temperature. A detailed analysis of the temperature variation in performed and is given in Mohamady et al. [1].

Results of uncertainty analysis

Figure 4 shows the histogram of the 400 samples of sensor positions.

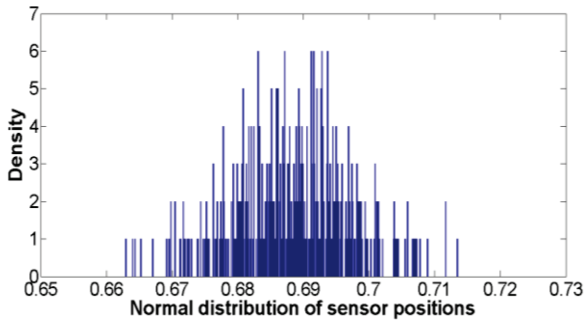

Figure 4: Histogram of 400 sensor positions

The frequency response of the 400 transfer function (TFs) is plotted in Figure 5.

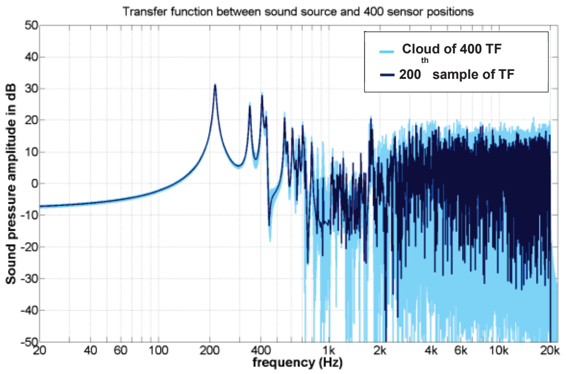

Figure 5: Transfer functions between sound source and 400 receivers

In this plot light blue shows the cloud of 400 TFs and dark blue is the sample number 200th as an example. The relative standard deviation of sound pressure amplitude due to position uncertainty can be calculated using:

$$\Delta L = 20\log(1 + \frac{\sigma_p}{\bar{p}}) \qquad [1] \qquad (1)$$

where σ_p and $\bar{p}$ are standard deviation (STD) and mean value of sound pressure amplitude, respectively. The result of this analysis is depicted in Figure 6(a). It is clear that with increasing the frequency, the wavelength decreases, and therefore any changes in sensor positioning leads to the larger errors. However, above the Schroeder frequency error level seems to stay constant. To further examine this observation, the added power of error is calculated in each 1/3 octave band and plotted in Figure 6(b). Saturation of added power of error at 8 dB can be observation in higher frequencies. In the next section we look precisely at the higher frequencies (statistical region) to evaluate the error saturation at 8dB.

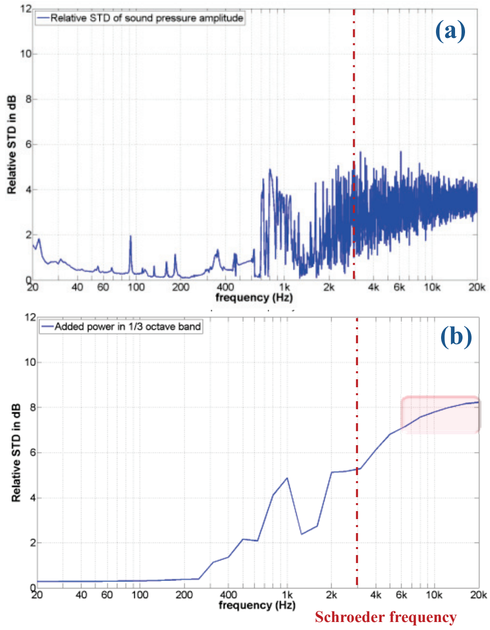

Figure 6: a) Relative standard deviation of sound pressure amplitude, b) power added of error analysis in 1/3 octave band

Statistical uncertainty analysis

An uncertainty analysis of the modal overlap region shows combination of two random processes, namely random deviations of sound pressure amplitude belong to each transfer function and amplitude deviation due to sensor positioning. We have obtained the histogram of sound pressure amplitude in overlap region of each transfer function. Figure 7 shows the histogram of the sound pressure amplitude above the Schroeder frequency for sample number 200th as an example. Finally there would be 400 histograms to be considered. According to Schroeder, the sound pressure amplitude in overlap region follows a Rayleigh distribution, so it is fitted a Rayleigh distribution to each histogram. Then it is calculated the mean and variance of all 400 transfer functions. Figure 8 shows the histogram of 400 Rayleigh fits and variances.

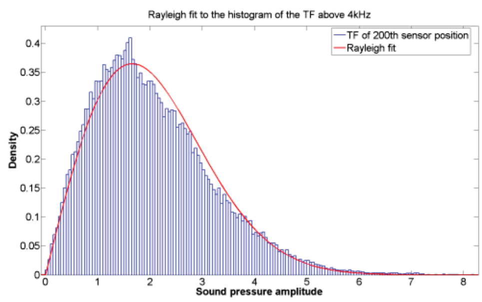

Figure 7: Rayleigh fit for distribution of 200th samples

The results are shown in Figure 8.

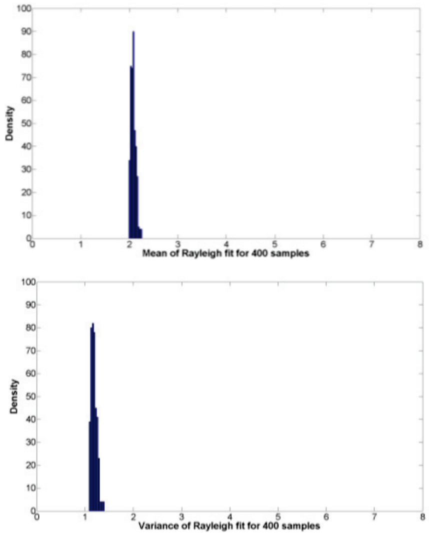

Figure 8: Histogram of mean and variances of all 400 Rayleigh fit distributions.

The mean and variation analysis illustrates a very narrow deviation which proves the saturation observed in the error analysis. In order to verify the analytic result the experimental study is presented in the next section.

Experimental setup

The experiment is developed with the same specification of the analytical system. The walls are made of MDF wood, and a small window is integrated on one side of the enclosure to monitor the experiment during measurement.

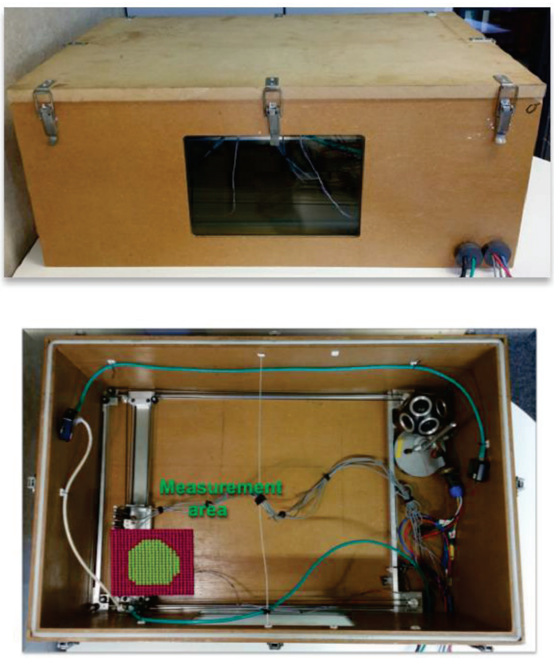

Figure 9: (a) Experimental set up, (b) Interior view of the box

An omni-directional loudspeaker is used to excite the system and four Sennheiser KE4 electret microphones are used to measure the sound pressure over the analysis surface. Temperature and humidity are monitored during the measurement using three temperature sensors. According to the monitoring results, temperature and humidity are kept constant during measurement which is depicted in Figure 10. The maximum deviation of both temperature and humidity are less than 1% which can be neglected.

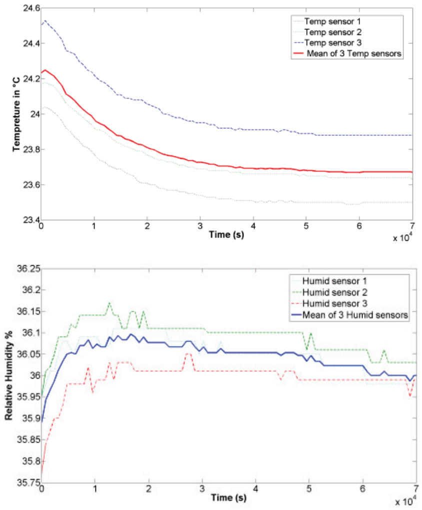

Figure 10: Temperature and humidity monitored during sensor positioning measurement

A deviation analysis (Equation 1) of the measured transfer paths is performed and plotted in Figure 11.

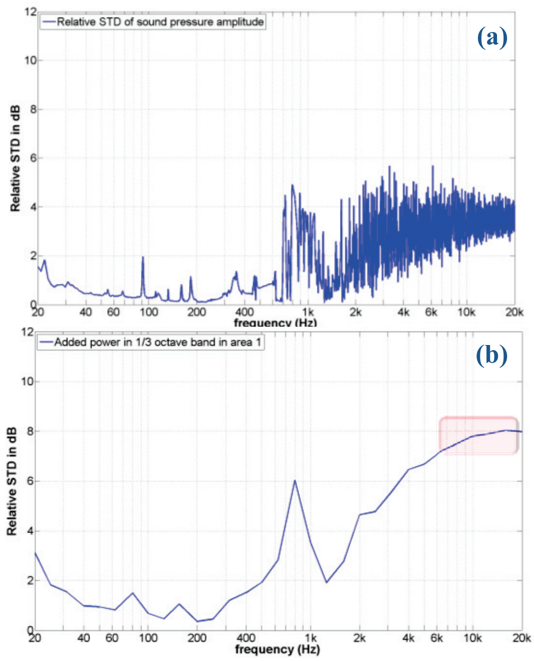

Figure 11: a) Relative standard deviation of sound pressure amplitude, b) power added in 1/3 octave band measured experimentally

The same deviation interpretation of the analytical work can be seen here as well. The verification of the analytical analysis of the enclosure is given in the next section.

Verification of uncertainty analysis

The probability uncertainty analysis is verified using the experimental data. The result of verification is depicted in Figure 12.

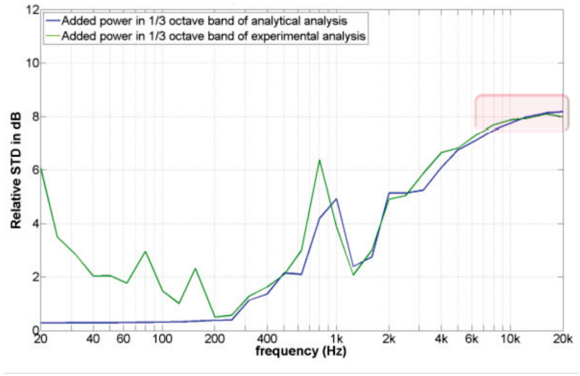

Figure 12: Verification of uncertainty analysis

Both analyses show the same behavior. However, the experimental analysis shows larger errors at lower frequencies due to the noise during the measurement. The saturation at 8 dB can be seen in verification analysis as well. To check if the saturation is position dependent or not we have selected four measurement regions inside the box and performed the same analysis. These regions are shown in Figure 13.

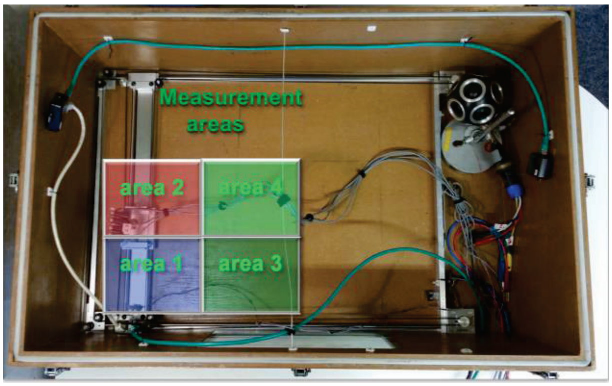

Figure 13: Measurement areas with middle normal sampling

The results of error analysis in 1/3 octave is shown in Figure 14.

Conclusion

In this study it is studied analytically the influence of the sensor positioning uncertainty in a simple acoustic system. Then it is verified the analysis using experimental set-up. Saturation of 8 dB in modal overlap region was obtained and the statistical analyses of this region were performed to determine the saturation in this area. Finally four analysis areas were measured to prove position independency of the saturation in Schroeder region.

Future work

In this work the DUT was a purely acoustic system. In the next study a vibro-acoustic system is considered with adding a plate on top side of the enclosure. The aluminum plate itself is also studied and the uncertainty due to the wave propagation in aluminum plate will be considered as well.

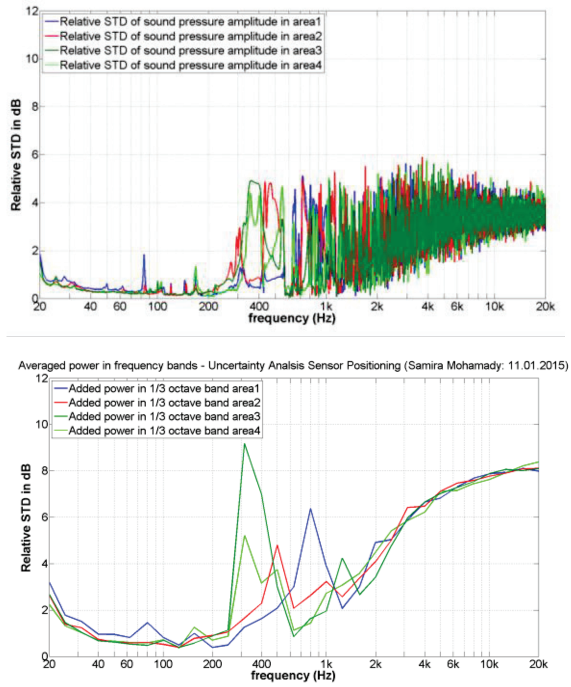

Figure 14: Error analysis in 1/3 octave band

Literature

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