Modelling the Sound Diffraction at Modified Noise Barriers

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Introduction

Sound diffraction at noise barriers leads to a decrease of wanted noise abatement. This effect is more pronounced in the low frequency range. One possibility to alleviate this is by modifying the top of the barriers. To calculate the insertion loss the Boundary Element Method (BEM) is often used but in most cases only in two dimensions. This leads to two difficulties. First the source is modeled as a coherent line source instead of an incoherent point source. This is far away from reality for the situation of moving vehicles radiating sound while passing by the barrier. Secondly the sound arriving to the barrier from larger angles in reference to the perpendicular case is not modeled correctly. This may lead to an overestimation of the noise abatement of the barrier. Here a model will be presented, consisting of solving the Fresnel integrals of the Fresnel diffraction theory for an acoustically hard barrier edge.

ISO 9613-2 and RLS-90

An simplified calculation of the diffraction of an edge is part of the **ISO 9613–2** [1] and given in equation (1).

$$D_z = 10 \cdot \lg \left[3 + \frac{C_2}{\lambda} \cdot C_3 \cdot z \cdot K_{met} \right] \tag{1}$$

with

z — indirection in m

λ — wave length in m

$C_2 = 20$ — with ground effect

$C_3 = 1$ — only one diffraction

$K_{met} \approx 1$ — no meteo–correction

This calculation is simplified in the **RLS–90** [2].

$$D_z = 10 \cdot \lg \left[3 + 80 \cdot z \cdot K_{met} \right] \tag{2}$$

In the following the calculations of the Fresnel diffraction will be compared with this approximation of the RSL–90.

Road Traffic Noise Spectrum

Diffraction is depending on the frequency of the acoustic wave (see section *ISO 9613-2 and RLS–90*). Considering this in the calculation of the diffraction with the Fresnel formulation (see section *Fresnel*) in principle the road traffic noise spectrum **Road Traffic Noise Spectrum** [3] is used. For simplification an approximation is used for the acoustic weight aw:

$$aw\,(f) = -\frac{4}{\lg\,(2)} \left| \lg\,(f) - 3 \right| \tag{3}$$

whereby

$f = \frac{c}{\lambda}$ — frequency in Hz

The velocity of sound propagation c is assumed to be 340 m/s.

Fresnel Diffraction

The Fresnel diffraction occures when a wave emitted from a point source is reaching a screen with an opening like an hole, a slit or an infinite edge. The distance of the receiver point is also finite.

In [4] a complete deduction of the mathematical representation is given. The results are shown in the next chapter.

Theory

The sound pressure derived from Fresnel diffraction is given by:

$$u_p = A \cdot (C + iS) \tag{4}$$

Here A is the amplitude:

$$A = -\frac{ik}{2\pi} \cdot \cos\delta \cdot \frac{e^{ik(R+R_0)}}{RR_0} \tag{5}$$

with

$\cos\delta$ — cosine of the angle of sight, i.e. the angle between the line from source to receiver and horizontal line

R — distance from source to barrier (not to the edge!) in direct line to the receiver in m

R_0 — distance from the barrier (not from the edge!) to the receiver in m

The integrals C and S are derives from the complex Fresnel integrals:

$$C = a \cdot \left[re\,(U) \cdot re\,(V) - im\,(U) \cdot im\,(V) \right] \tag{6}$$

and

$$S = a \cdot [im\,(U) \cdot re\,(V) + re\,(U) \cdot im\,(V)] \tag{7}$$

with:

$$a = \frac{\lambda}{2 \cdot \left(\frac{1}{R} + \frac{1}{R_0}\right) \cdot \cos\delta} \tag{8}$$

The Fresnel integals U and V are given by:

$$U\,(w) = \int_0^w \cos\left(\frac{\pi}{2} \cdot u^2\right) du \tag{9}$$

and

$$V\,(w) = \int_0^w \sin\left(\frac{\pi}{2} \cdot u^2\right) du \tag{10}$$

Here w is an auxilary variable:

$$w = x \cdot \cos\delta \cdot \sqrt{\frac{2}{\lambda} \cdot \left(\frac{1}{R} + \frac{1}{R_0}\right)} \tag{11}$$

with

x — vertical distance of the line from the source to the receiver below the edge in m

For each receiver point a different co-ordinate system is used. This co-ordinate system must be rotated if source and receiver are not perpendicular to the edge.

The intensity of the wave (coherently) diffracted at the edge is:

$$I = |u_p|^2 = \frac{I_0}{2} \cdot \left\{\left[U\,(w) + \frac{1}{2}\right]^2 + \left[V\,(w) + \frac{1}{2}\right]^2\right\} \tag{12}$$

The reference intensity for the direct wave without edge is given by:

$$I_0 = 4 \cdot |A|^2 \cdot a^2 = \frac{1}{(R + R_0)^2} \tag{13}$$

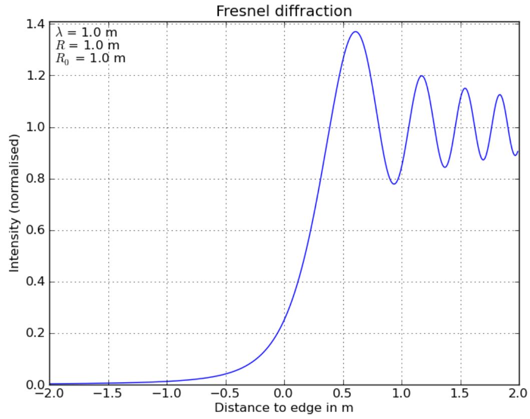

Figure 1: Relative intensity of diffraction at an infinite edge

Examples

In Figure 1 the diffraction at an infinite edge is shown. Here the distances R and R_0 are 1 m. The wavelenght λ is also set to 1 m.

At the edge the relative intensity is 0.25. The relative intensity is increasing to a maximum value of nearly 1.4 which occures at about 0.6 m above the edge. The wavelenght of flucuation around an relative intensity of 1.0 starts with approximately 1 m and is getting shorter while the amplitude of the oscillation is decreasing from about 0.4 exponentally towards 0.

In Figure 2 the normalised level of intensity of diffraction at an infinit edge is shown.

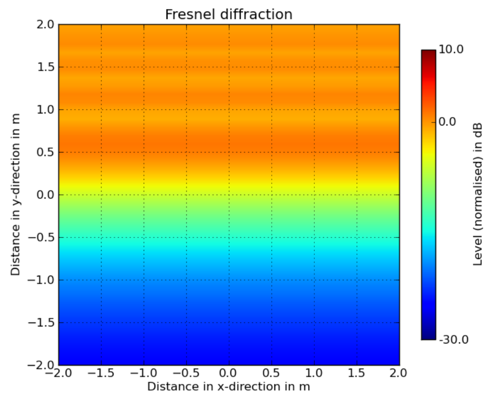

Figure 2: Normalised level of intensity of diffraction at an infinit edge

The x–direction is along the edge while the y–direction is perpendicular in the plan of the screen.

Cutting the Edge

Due to the principle of superposition it is possible to cut the edge into semi-infinite slits and combine them later. If the total width of the slits $\sum b_i$ is much longer than the distance from source to receiver $R + R_0$ the result remains the same when adding the intensities from all slits.

There are two reasons for cutting the edge into slits. The first reason is to be able to calculate partly coherent propagation (see *Partly Incoherent Diffraction*). The second reason is to be able to change the geometry of the edge (see section *Changing the Edge*). It is also possible to combine both calculations (see *Both Effects*). For better comparison of the different calculations the normal diffraction is also conducted with the cutted edge (see *Coherent Diffraction*).

Coherent Diffraction

The geometry remains here and for the following calculations the same. The point source is at the horizontal distance of 7.5 m to the screen with a height of 5.625 m. The source point can be seen above the view point of 15 m height, when the horizontal distance of the receiver point to the screen is 12.5 m.

The normal diffraction of an edge for a road traffic spectrum is calulated by energetic summation of the 24th octave band results from 250 Hz to 4 kHz.

The total coherent intensity of the partial sound pressures $u_{p,f,i}$ as a sum over all frequency f and all semi-infinite slits i is given by:

$$I_{coherent} = \sum_f 10^{0.1 \cdot aw(f)} \cdot \left| \sum_i u_{p,f,i} \right|^2 \qquad (14)$$

with

$aw\,(f)$ — acoustic weight of the Road Traffic Noise Spectrum (see *Road Traffic Noise Spectrum*).

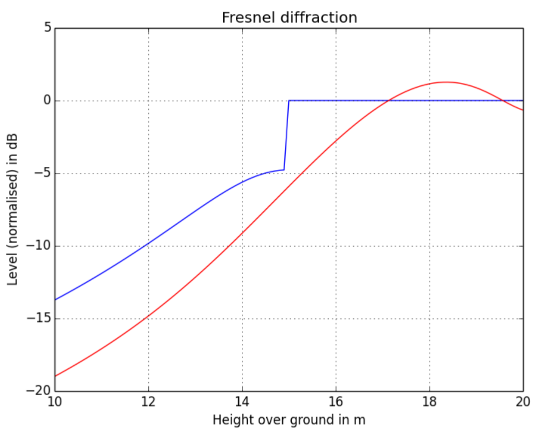

Figure 3: Coherent Fresnel diffraction and diffraction of RLS–90

In Figure 3 the coherent Fresnel diffraction and the diffraction of RLS–90 is shown.

Due to the geometry the edge is at a location of 15 m. Here the equation (2) is discontitious with a step of about 5 dB. The curve calculated with Fresnel diffraction is below the curve of RLS–90 for all receiver positions below the edge.

Partly Incoherent Diffraction

The total partly incoherent intensity of the partial sound pressures $u_{p,f,i}$ as a sum over all frequency f and all semi-infinite slits i is given by:

$$I_{incoherent} = \sum_f 10^{0.1aw(f)} \left| \sum_i \left[(1+\kappa)\,|u_{p,f,i}|^2 + \kappa u_{p,f,i}^2 \right] \right|^2 \qquad (15)$$

with

$aw\,(f)$ — acoustic weight of the Road Traffic Noise Spectrum (see *Road Traffic Noise Spectrum*).

κ — coherence coefficient; 1 if coherent and 0 if incoherent.

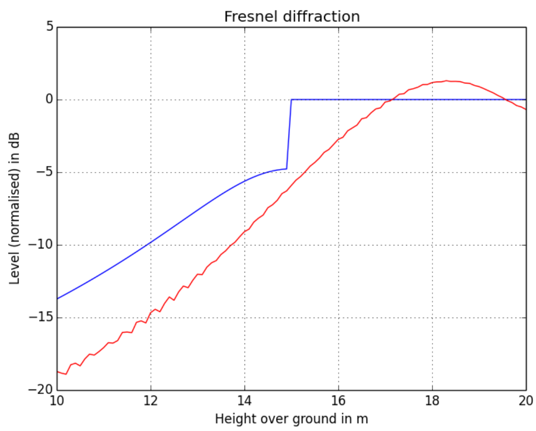

Figure 4: Partly incoherent Fresnel diffraction and diffraction of RLS–90

In Figure 4 the partly incoherent Fresnel diffraction and the diffraction of RLS–90 is shown. The coherence coefficient is 0.2. This leads to some noise on the Fresnel diffraction curve.

Changing the Edge

The edge is not modified by moving the semi-infinite slits up and down alternately (see Figure 5).

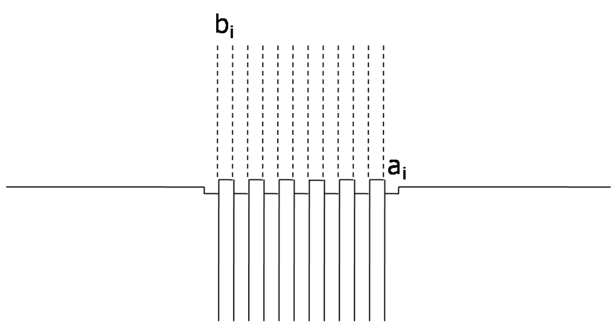

Figure 5: Modified edge

The width b_i and the the amplitude of modification the height a_i of the semi-infinite slits is choosen as λ and $\frac{\lambda}{2}$.

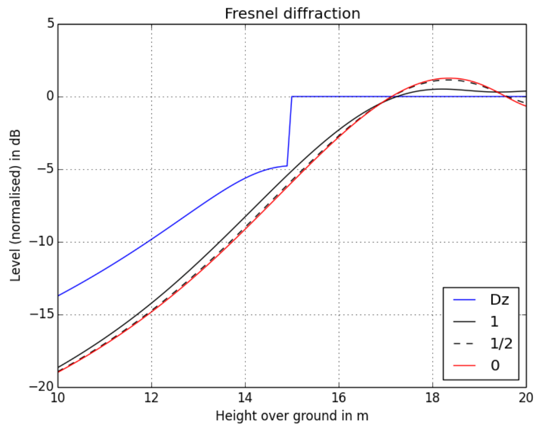

Figure 6: Fresnel diffraction of changed edge and diffraction of RLS–90

In Figure 6 the result of the modifications are shown for $a_i = b_i = \lambda$ (1) and for $a_i = b_i = \frac{\lambda}{2}$ $(\frac{1}{2})$ compared to the unchange case (0) and the diffraction of RLS–90.

In the shadow zone below the edge the diffraction of the changed edge with $a_i = b_i = \lambda$ (1) is less efficient while the increase above the edge is decreased significantly. For $a_i = b_i = \frac{\lambda}{2}$ $(\frac{1}{2})$ there is no change visible.

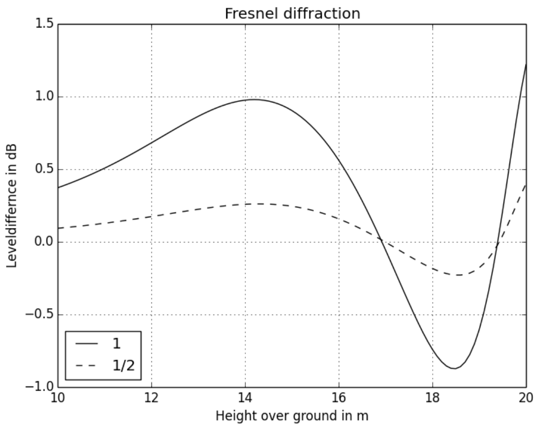

Figure 7: Difference in Fresnel diffraction of changed edge compared to unchanged edge

In Figure 7 the difference in Fresnel diffraction of changed edge compared to unchanged edge is shown.

The decrease of the level above the edge in the case of $a_i = b_i = \lambda$ (1) is about 1 dB.

Both Effects

The combined effect of partly incoherence and changed edge is shown in Figure 8.

The noise is much more pronounced for the changed edge than for the unchanged edge. The cohernence coefficient is again 0.2. Fortunaty the noise above the edge is less than below the edge.

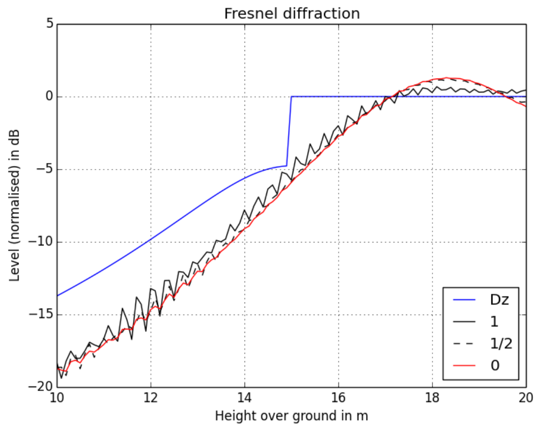

Figure 8: Partly coherent Fresnel diffraction of changed edge and diffraction of RLS–90

Outlook

The insertion loss for a complete pass–by of a vehicle, modeled as a coherent or partly incoherent point source, can be calculated with some effort. The problem in calculation of a whole pass–by is to rotate the co-ordinate system of the Fresnel diffraction model.

The diffraction can be calculated also with Finite Difference in Time Domain, **FDTD**. Two and three dimensional models have been developed and recently tested sucessfully (see Figure 9).

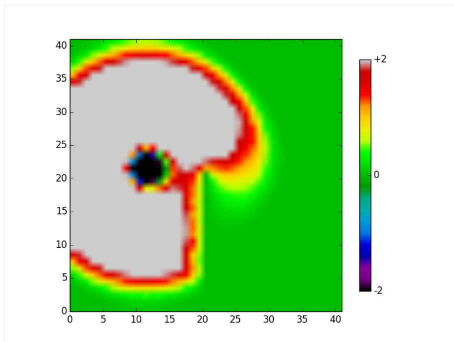

Figure 9: Sound pressure at an absorning screen from 3D-FDTD calculations

The parameter of this calculation where: $f = 56$ Hz, $dx = dy = dz = 0.1$ m, $dt \approx 1/3$ ms, $T \approx 10$ ms.

References

[1] **ISO 9613–2**: Acoustics — Attenuation of sound during propagation outdoors — Part 2: General method of calculation

[2] Richtlininen für den Lärmschutz an Straßen, **RLS–90**, Ausgabe 1991

[3] **EN 1793–3**: Road traffic noise reducing devices — Test method for determining the acoustic performance — Part 3: Normalised traffic noise spectrum

[4] Born, M.: Optik — Ein Lehrbuch der elektromechanischen Lichttheorie, Springer 1972.