Study on the Influence of Spectrum and Directivity on the Uncertainty of the Sound Power Determined Using the Substitution Method

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Introduction

A very common method for sound power determination is the substitution method. It works by replacing a sound source of unknown sound power by a sound source with known sound power output. By measuring both sources, the sound power of the unknown source can be determined. This procedure is described in ISO standard 3741, 3743-1 and 3747. These standards also detail uncertainties associated with the determined sound power level. Specifically, the term σ_{RO} describes the uncertainty that is due to the measurement procedure itself and includes attributions such as different sound emission patterns of the two sources. Upper bounds for σ_{RO} are given in the ISO standards for sources with a "relatively flat" spectrum. However, being a term that lumps together different uncertainty contributions, the influence of individual factors is not quantified. For this reason, a numerical study was conducted that focused on the two uncertainty contributors directivity of sound emission and spectral content of the two sources. The goal was to assess the uncertainty in the determined sound power of the unknown source that is due to differing characteristics of these two factors in the two measured sources. A study focusing on sources with different directivities has been performed previously by the German Federal Institute for Occupational Safety and Health [4]. However, this study focused on the comparison of sound power levels determined using different ISO standards.

The Substitution Method

As mentioned previously, the substitution method involves two measurements. First, the sound pressure levels ($L_{p,\mathrm{RSS}}$) of a source with known sound power output (reference sound source) are measured on an enveloping surface. Then, the reference sound source (RSS) is replaced by a source with unknown sound power output (device under test). The sound pressure levels ($L_{p,\mathrm{DUT}}$) of this device under test (DUT) are then measured on the same enveloping surface as the one used for the RSS (see Fig. 1). After averaging the measured sound pressure levels over the measurement surface, the sound power level of the DUT is calculated (Eq. 1) [3], where $L_{W,\mathrm{DUT}}$ denotes the sound power level of the device under test, $L_{W,\mathrm{RSS}}$ the known sound power levels of the reference sound source and $\overline{L_p}$ sound pressure levels averaged over the measurement surface.

$$L_{W,\mathrm{DUT}} = L_{W,\mathrm{RSS}} + \overline{L_{p,\mathrm{DUT}}} - \overline{L_{p,\mathrm{RSS}}} \qquad [\mathrm{dB}] \qquad (1)$$

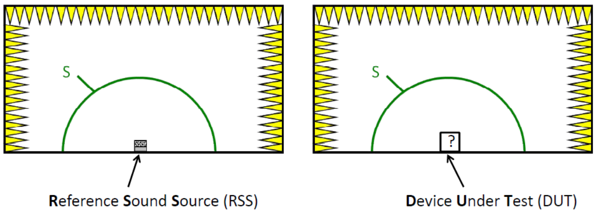

Figure 1: Schematic of the measurement for the application of the substitution method. Sound pressure levels of both sources are measured on an enveloping surface and the sound power of the DUT calculated (Eq. 1).

Reference Data

In order to apply the substitution method to the different numerically modeled devices under test reference data from a PTB reference sound source were needed. These data were obtained from a measurement on a RSS in the PTB hemi-anechoic room using a hemi-circular arc. This arc with 2 m radius had 24 microphones attached to it and was moved from the floor to an elevation angle of 84° in 1° steps. Measurements were performed in one-third octave bands, where the 1 kHz third octave band was used in the subsequent directivity analyses. Hence, the reference data from the RSS were comprised of the sound power level (L_W) as well as 84x24 sound pressure levels (L_p) of the RSS for every one-third octave band between 20 Hz and 20 kHz.

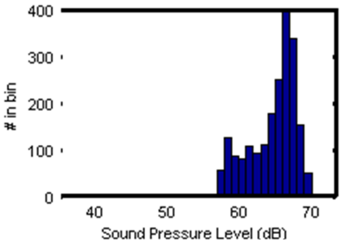

Figure 2: Histogram of the distribtuion of sound pressure levels for the reference sound source in the 1 kHz one-third octave band.

Directivity

The directivity of sound emission (Eq. 2) was used to develop models for devices under test that show sound emission patterns that are different from the one of the reference sound source.

$$\mathrm{DI} = \max(L_{p,i}) - \mathrm{mean}(L_{p,i}) \qquad [\mathrm{dB}] \qquad (2)$$

1000 models each were developed for directivity indexes of 0, 2, 4, 6, 8 and 10 dB. Each model consisted of 84x24 sound pressure level values to match the reference data (Fig. 3). Within each model these sound pressure levels corresponded in a first run to normally distributed sound pressures (p). In a second run p^2 values followed a uniform distribution.

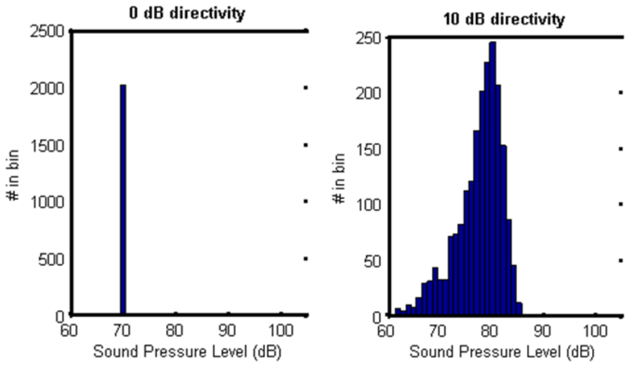

Figure 3: Samples of histograms for distribtuions of sound pressure levels for devices under test.

In a real life measurement setting obtaining more than 2000 sound pressure levels is of course not realistic. Hence, subsets with 2 to 200 points were chosen at random from each modeled DUT. This process was repeated 1000 times per subset size, corresponding to a Monte Carlo method with 10^6 variations per subset size and directivity index (Fig. 4).

Using the substitution method (Eq. 1) sound power levels ($L_{W,i}$) for the modeled devices under test were then calculated for each one of the 10^6 variations per subset. These values were compared to the sound power levels ($L_{W,\text{84x24points}}$) calculated using the complete 84x24 point model under the assumption that the sound power calculated with the complete model is the correct sound power level, i.e. that the sound field is sampled completely with those 2016 points. The standard deviation of the difference in sound power levels between the undersampled and completely sampled sound fields was used as indicator for the uncertainty contribution of the directivity of sound emission on sound power levels calculated using the substitution method (Eq. 3).

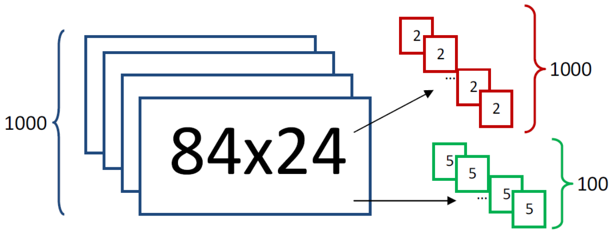

Figure 4: Schematic representation of the Monte Carlo method used, where each rectangle represents a matrix or vector of specified size. For each directivity index 1000 models were developed and from each model 1000 different combinations per subset size generated.

$$u = \text{std}(L_{W,i} - L_{W,\text{84x24 points}}) \qquad [\text{dB}] \qquad (3)$$

Results indicate that the uncertainty of the substitution method that is due to a difference in directivity between the two sources used decreases exponentially with an increase in measurement microphones. Furthermore, devices under test whose sound fields show small values in directivity indexes have a smaller uncertainty contribution from their directivity than DUTs whose sound emission show large directivities (Fig. 5). There was no significant difference in results for models that assumed normal distributions for sound pressures, p, and models that assumed uniform distributions for p^2.

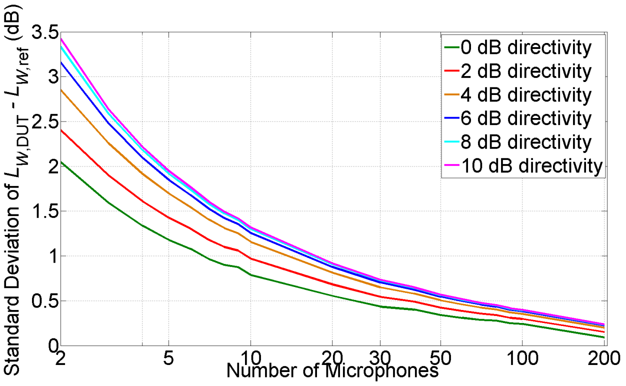

Figure 5: Results on the behavior of the uncertainty due to directivity differences in the sources used in the substitution method.

Spectral Content

A second numerical investigation was concerned with the question whether sound power levels that are calculated by applying an A-weighting (Eq. 4) after the use of the substitution method are different from sound power levels calculated by applying the substitution method to A-levels. Specifically, two approaches to calculate sound power levels for modeled DUTs were followed. One was to apply the substitution method to every one-third octave band and then post-calculate an A-weighted sound power level (Eq. 5). The thus calculated sound power level was regarded as reference sound power level ($L_{W,\text{ref}}$) as it describes the proper data analysis procedure. Secondly, a sound power level ($L_{W,\text{DUT}}$) was calculated by first calculating A-levels and then applying the substitution method to these A-levels (Eq. 6). The difference, ϵ, of the two calculated sound power levels gives a rule for the validity of $L_{W,\text{DUT}}$ (Eq. 7).

$$L_{W,A} = 10\lg\left[\sum_{i=1}^{n} 10^{\frac{L_{W,i}+A_i}{10\text{ dB}}}\right] \qquad [\text{dB}] \qquad (4)$$

$$\begin{aligned} L_{W,\text{ref}} &= (L_{W,i,\text{DUT}})_A \\ &= (L_{W,i,\text{RSS}} + L_{p,i,\text{DUT}} - L_{p,i,\text{RSS}})_A \qquad [\text{dB}] \quad (5) \end{aligned}$$

$$\begin{aligned} L_{W,\text{DUT}} &= L_{W,A,\text{DUT}} \\ &= L_{W,A,\text{RSS}} + L_{p,A,\text{DUT}} - L_{p,A,\text{RSS}} \qquad [\text{dB}] \quad (6) \end{aligned}$$

$$\epsilon = L_{W,\mathrm{DUT}} - L_{W,\mathrm{ref}} \qquad [\mathrm{dB}] \qquad (7)$$

The reference data used for this analysis were introduced before. However, those data sets only included values from PTB's hemi-anechoic room. To model acoustic environments corresponding to more diffuse rooms, the sound pressure levels of the reference sound source on a hemispherical surface with radius r=2 m were varied through the introduction of a room correction coefficient, K_2. For every one-third octave band, i, this correction coefficient was multiplied by a random number with mean 0 and standard deviation 1, denoted as $N_{[0,1]}$. Through the use of room correction coefficients K_2=0 dB, K_2=7 dB and K_2=12 dB three different acoustic environments were modeled (Eq. 8).

$$L_{p,\mathrm{RSS},K_2,i} = L_{W,\mathrm{RSS},i} - 10\lg(2\pi r^2) + K_2 N_{[0,1]} \qquad [\mathrm{dB}] \qquad (8)$$

Three different types of models were developed for devices under test.

1. Tolerance levels of 1 to 20 dB were defined around the one-third octave band sound pressure levels of the RSS. Within these tolerance levels modeled sound pressure levels followed a uniform distribution. 5x10^5 models per tolerance level were developed (Fig. 6).
2. A tonal DUT was modeled by defining a base sound pressure level of 70 dB which was modified for one randomly chosen third octave band so that this single one-third octave band was assigned a sound pressure level between 75 and 120 dB. 5x10^5 such models were developed (Fig. 7).
3. Broadband DUTs were modeled by prescribing the sound pressure level for the first third octave band and sequentially calculating all other third octave band sound pressure levels through addition/subtraction of a constant value in the range of ±5 dB (Fig. 8).

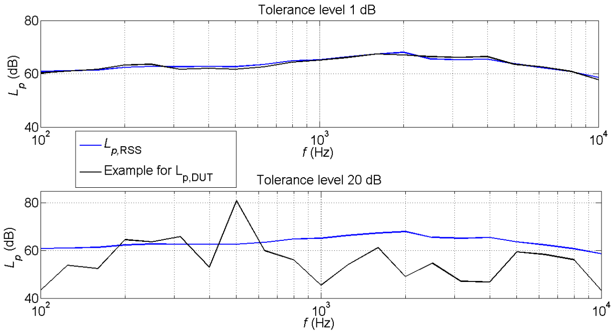

Figure 6: Examples of modeled DUTs where a tolerance level was defined around each one-third octave band sound pressure level of the RSS.

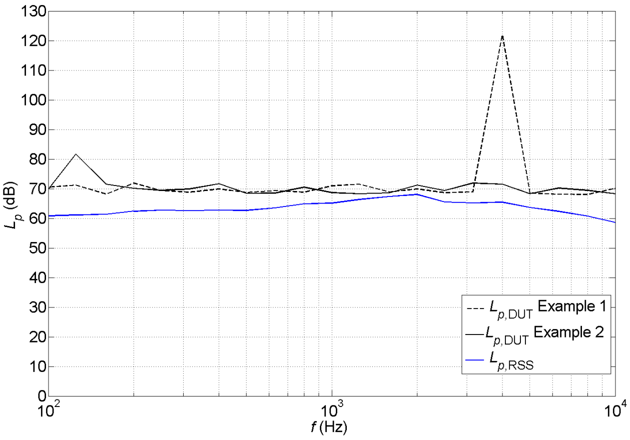

Figure 7: Examples of modeled DUTs where a single one-third octave band shows tonality.

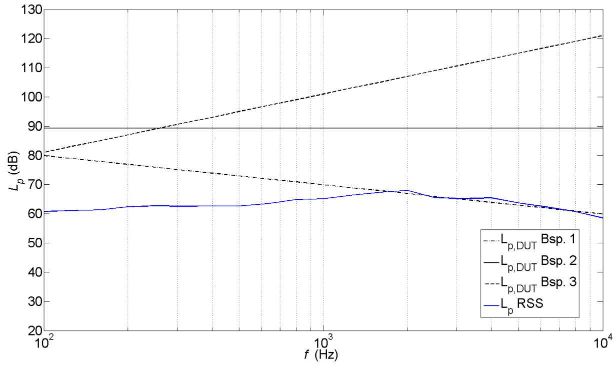

Figure 8: Examples of modeled broadband noise DUTs.

Results reported here show the value ϵ which describes the difference between the sound power levels calculated performing A-weightings prior or post to the use of the substitution method (Eq. 5-7). Furthermore, results were separated for each of the three types of modeled DUTs in free-field (K_2=0 dB) vs. non-free field (K_2=7 dB and K_2=12 dB) results.

The results agree for all types of modeled devices under test. Namely, in free field conditions the value of ϵ is not significant. For approximated free and diffuse fields (K_2=7 dB and K_2=12 dB), however, this does not hold. For these environments ϵ becomes significant (Figs. 9-11). This means that for these environments correct sound power levels are obtained only by applying the substitution method to each one-third octave band and then post-calculating an A-weighting (Eq. 5).

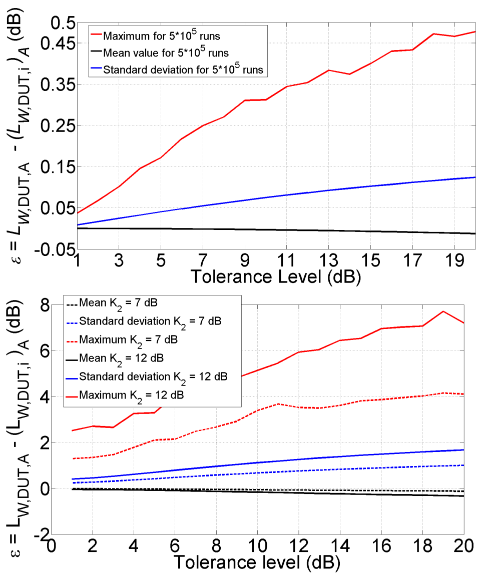

Figure 9: Results for DUTs modeled by defining tolerance levels. Top: $K_2 = 0$ dB

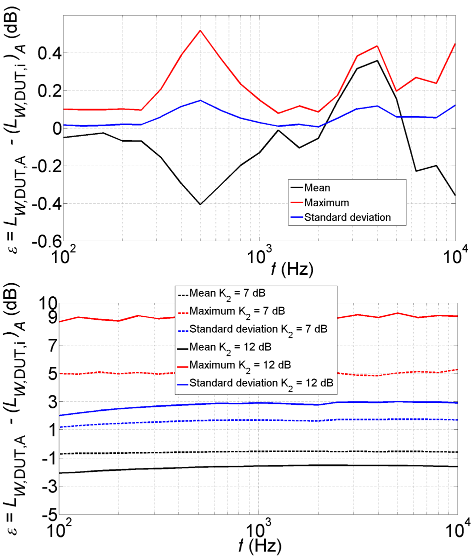

Figure 10: Results for tonal DUTs. Top: $K_2 = 0$ dB

Conclusion

This paper presents two investigations on uncertainties associated with the substitution method. The first dealt with numerical models describing devices under test that display directivity patterns which are different from the one that the reference sound source emits. Results show that the uncertainty of the calculated sound power decreases exponentially with an increase in measurement microphones used.

The second investigation showed that it is possible to calculate a correct sound power level for a device under test by applying the substitution method to A-levels. This does not hold for approximated free or diffuse fields, where sound powers have to be calculated by using the substitution method on each one-third octave band with subsequent A-weighting.

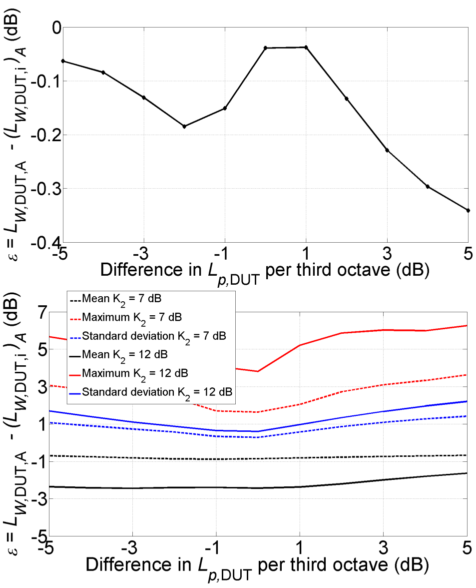

Figure 11: Results for broadband noise DUTs. Top: $K_2 = 0$ dB

References

[1] ISO 3741:2010 Acoustics - Determination of sound power levels and sound energy levels of noise sources using sound pressure - Precision methods for reverberation test rooms

[2] ISO 3743-1:2010 Acoustics - Determination of sound power levels and sound energy levels of noise sources using sound pressure - Engineering methods for small, movable sources in reverberant fields - Part 1: Comparison method for a hard-walled test room

[3] ISO/FDIS 3747:2010 Acoustics - Determination of sound power levels and sound energy levels of noise sources using sound pressure - Engineering/survey methods for use *in situ* in a reverberant environment

[4] Otremba, H.-O., Hoppe, G., Sehrndt, G.A.: Geräuschemission von Maschinen - Umgebungskorrekturen. Bundesanstalt für Arbeitsschutz und Arbeitsmedizin (Hg.), Dortmund/Berlin, 2001