Directivity Filter for Sequential Loudspeaker Array Measurements

Johannes Klein, Marco Berzborn, Michael Vorländer
Institute of Technical Acoustics, RWTH Aachen University, 52056 Aachen, Germany,
Email: johannes.klein@akustik.rwth-aachen.de

Introduction

Room impulse response measurements including directivity are becoming a vital component of room acoustics. The results of these measurements allow for the room acoustical analysis regarding specific source receiver combinations and enable the realistic simulation of acoustical scenarios in virtual environments. Besides the widely applied microphone arrays, electro-acoustic sources with a steerable directivity are necessary to retain all degrees of freedom during the measurements. Sequential measurements using a rotating source help to enlarge the virtual number of transducers, resulting in a high spatial resolution [1]. To allow for the measurement with an arbitrary source directivity, the original source directivity has to be compensated for with control filters. Due to a trade-off between excessive noise gain and accuracy this process introduces an error. Therefore the goal of the filter design is the minimization of this error.

Filter Application

The directivity filter matrix $\hat{\mathbf{D}}^{+}$ is the generalized inverse of the source directivity matrix $\hat{\mathbf{D}}$. The source directivity matrix contains the measured spherical harmonic coefficient vectors of all loudspeaker array transducers. The goal of the filtering process is to impose a specific target directivity $\hat{\mathbf{t}}_{\mathrm{org}}$ on the source. The multiplication of the filter matrix and the target directivity vector yields a weight vector containing one coefficient for each transducer. The applications of the weight vector to the original source directivity then results in a reconstructed directivity

$$\hat{\mathbf{t}}_{\mathrm{rec}} = \hat{\mathbf{D}}\hat{\mathbf{D}}^{+}\hat{\mathbf{t}}_{\mathrm{org}}. \tag{1}$$

Filter Generation

The goal of the filter generation is the exact directivity reconstruction according to Eq. 1 within the valid frequency operating range of the array or complete MIMO system (see [2, 3]) while preserving a reasonable noise gain limit. Since the filter matrix is the inverse of the source directivity matrix, the problem of the filter generation is reduced to finding a suitable inversion method for the generally not square matrix $\hat{\mathbf{D}}$. The following methods are commonly applied for this task.

Moore-Penrose Pseudo-Inverse

The Moore-Penrose pseudo-inverse computes the least squares solution to a general system of linear equations. It does not take into account any acoustical properties and their representation in the spherical harmonic domain. The Moore-Penrose pseudo-inverse of the source directivity matrix $\hat{\mathbf{D}}$ is defined as

$$\hat{\mathbf{D}}_{\mathrm{p}}^{+} = \left(\hat{\mathbf{D}}^{H}\hat{\mathbf{D}}\right)^{-1}\hat{\mathbf{D}}^{H}. \tag{2}$$

Duraiswami Tikhonov Inverse

The inversion of the source directivity matrix $\hat{\mathbf{D}}$ is an ill-posed problem resulting in a not unique solution. Furthermore, $\hat{\mathbf{D}}$ shows low-pass behavior in the spherical harmonic domain causing its inverse to potentially amplify the noise in high orders, which contain little energy. The Tikhonov regularization solution

$$\hat{\mathbf{D}}_{\mathrm{r}}^{+} = \left(\hat{\mathbf{D}}^{H}\hat{\mathbf{D}} + \varepsilon\mathbf{V}\right)\hat{\mathbf{D}}^{H} \text{ with } \mathbf{V} = \mathbf{I} \tag{3}$$

as an $\mathbf{L}_2$-regularization gives preference to solutions with smaller norms, minimizing the noise amplification. Setting

$$\mathbf{V} = \left(1 + \mathbf{n}(\mathbf{n}+1)\right)\mathbf{I} \tag{4}$$

takes into account the spherical harmonic properties of $\hat{\mathbf{D}}$ and applies an additional order low-pass [4]. The result is a spatially smoothed directivity.

Moore-Penrose with Soft Limiting

In order to retain an error-minimized solution to the inverse problem while introducing an advantageous limit to the noise amplification (see [5]), Eq. 2 can be extended to

$$\hat{\mathbf{D}}_{\mathrm{s}}^{+} = \hat{\mathbf{D}}_{\mathrm{p}}^{+} \circ \arctan\left(\frac{\pi}{2\alpha} \circ \left|\hat{\mathbf{D}}_{\mathrm{p}}^{+}\right|\right) \circ \frac{2\alpha}{\pi \circ \left|\hat{\mathbf{D}}_{\mathrm{p}}^{+}\right|}. \tag{5}$$

Eq. 5 uses the arctangent function to provide a soft-knee limit. This approach does not take into account the spatial properties of the directivity.

Filter Comparison

The three filter generation methods are assessed regarding their resulting filter properties using the measured source directivity matrix of the sequential array seen in Fig. 1. The loudspeaker directivity data has been measured with a maximum spherical harmonic order of $\mathbf{n} = \mathbf{82}$. However, for the filter computation the order has been cut to $\mathbf{n} = \mathbf{20}$. All computations have exemplary been executed for one of the midrange speakers. The regularization coefficient for Eq. 3 has been set to $\varepsilon = 10^{-5}$. The soft limiting coefficient has been set to

Figure 1: Sequential Array.

$\alpha = 10^{\frac{30}{20}}$, resulting in a limitation at an absolute value of 30dB. The magnitude and phase of the filters over frequency are shown in Fig. 2 to 4 for the spherical harmonic orders of $n = 0$, $n = 10$ and $n = 20$.

It can be seen that the filter computed with Eq. 5 follows the Moore-Penrose inverse filter of Eq. 2 in all frequency ranges where the amplification stays well below 30dB. With increasing order and decreasing frequency the limitation has a larger effect and prevents the excessive noise amplification. This will also have an effect on the spatial precision of the reconstructed target directivity in Eq. 1. However, since the directivity is not very pronounced in low frequencies this effect is tolerable within limits.

Due to its spatial smoothing property the filter computed according to Eq. 3 and 4 shows a very uneven behavior in the frequency domain, including very high amplification factors.

The analysis of Fig. 6 shows that the filter matrix with soft limit is by far the best conditioned in the low frequency range, meaning that this filter will provide the lowest noise amplification.

Resulting Matching Error

The matching simulation [2, 3] regards the error introduced by noise due to high gain values of the filter. In this example the measurement signal-to-noise ratio is set to 40dB. The total noise is the dominant error for low frequencies and determines the range of validity. Due to the high dependence on the filter condition number the source matching error is the lowest for the filter generated according to Eq. 5, while retaining a tolerable accuracy in the spatial domain.

Conclusion

The Moore-Penrose pseudo-inverse aims at a high reconstruction accuracy, regardless of the noise gain. The noise

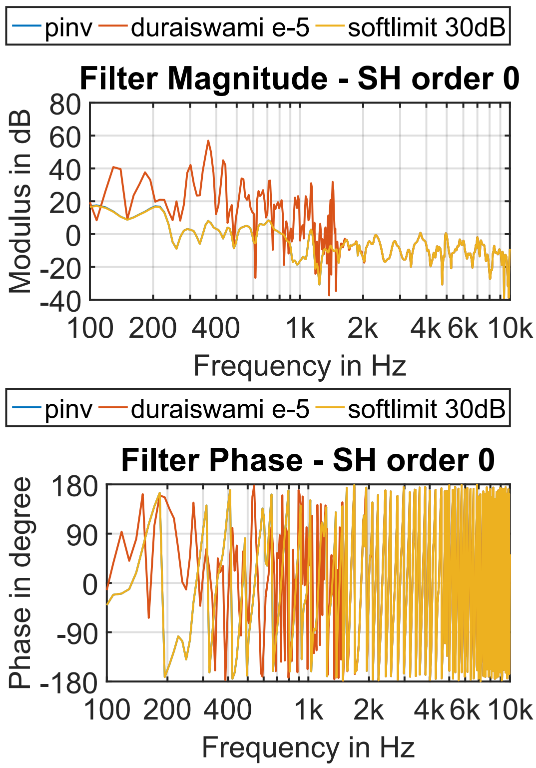

Figure 2: Filter magnitude (top) and phase (bottom). Spherical harmonic order 0.

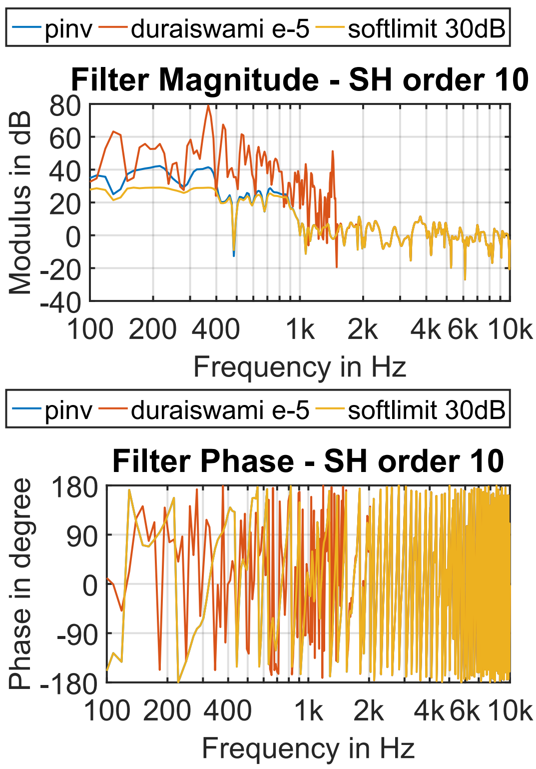

Figure 3: Filter magnitude (top) and phase (bottom). Spherical harmonic order 10.

gain is especially high for low frequencies and high orders.

The Duraiswami Tikhonov regularized inverse aims at a smooth reconstruction in the spatial domain. While this behavior can be desirable for applications such as HRTF

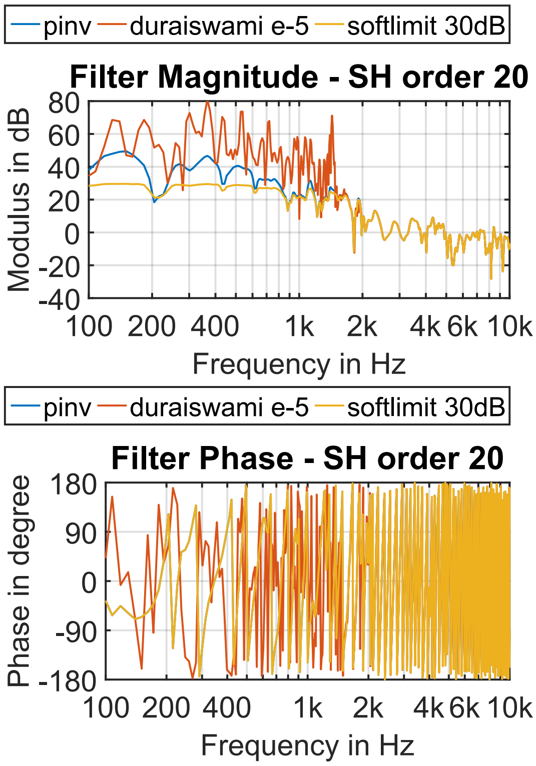

Figure 4: Filter magnitude (top) and phase (bottom). Spherical harmonic order 20.

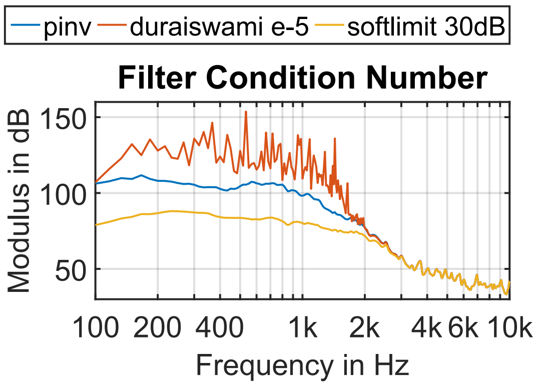

Figure 5: Filter condition number over frequency.

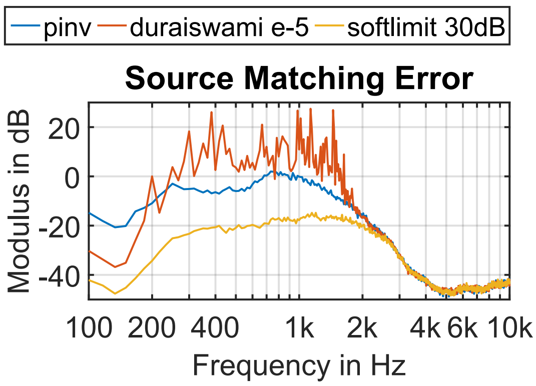

Figure 6: Matching error over frequency.

reconstructions it also leads to excessive noise gain for low frequencies in all orders.

The Moore-Penrose pseudo-inverse with soft limiting shows the same properties as the classic Moore-Penrose pseudo-inverse. Given the knowledge about the signal-to-noise-ratio during the source directivity measurement it offers a good possibility to limit the gain to a specific value. Since the limiting is done for every speaker individually, this measure will decrease the accuracy in the ranges affected by the limiting. However, it largely widens the range of validity. In conclusion, the Moore-Penrose pseudo-inverse with soft limiting offers a good - but computationally heavier - trade-off between spatial accuracy and low noise.

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