A Comparison of Intonation Estimators for Reed Woodwinds

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Abstract

The low frequency sound transmission behavior resulting from the bore and tone hole geometry determines the intonation properties of woodwind instrument to a large extent. The prediction of sounding frequencies from the air column modes however is difficult at the required precision level, because of the nonlinear reed - resonator interaction. Due to this coupling significant pitch deviations from the fundamental air column frequency occur.
To investigate this for the bassoon, a numerical study is presented employing a minimal model of the reed-resonator interaction. Some aspects of the model are explored here using previously obtained experimental data from artificial mouth experiments. The computed sounding frequencies from the model are compared to empirical estimates relying only on the impedance curve of the resonator, and to pitch estimated from a perception experiment with a musician.
Independent of the pitch estimator, the results point to the importance of higher modes for the sounding frequency of the bassoon, especially for notes in the low tonal range.

Introduction

The task to predict sounding frequencies from geometry information is of practical relevance in woodwind design. While calculation and measurement of acoustic resonator properties in terms of input impedance is well understood, the interpretation of impedance curves in terms of intonation and playing properties of the instrument is still difficult.
Several recipes have been suggested for the practical purpose to determine favorable playing frequencies of wind instruments from input impedance curves.
The simplest, intuitive approach is to assume, that the sounding frequency will be near the largest input impedance peak Z_{max} in the magnitude spectrum. The corresponding frequency estimator is the peak frequency $f(|Z|_{max})$.
To account for the possibility, that the pitch may not only determined by a single air column resonance, but also by higher overtones, Wogram [1] suggested a "sum function"

$$SF(f_0) = \sum_{n_h=1}^{N_h} \mathfrak{Re}\{Z(n_h\, f_0)\}, \qquad (1)$$

where $n_h = 1..N_h$ are the ordinal numbers of the partials taking into account[1]. The corresponding sounding frequency estimator is the frequency, where SF has a maximum, thus $f(SF_{max})$.
Another approach, largely related to the idea of a sum function is the "weighted intonation average"

$$D_{w.av}(f_0) = \frac{1}{\sum_{n_m} |Z_{n_m}|} \sum_{n_m=1}^{N_m} D_{n_m} |Z_{n_m}|, \qquad (2)$$

where $n_m = 1..N_m$ are the ordinal numbers of the most harmonic modes with respect to f_0, and D_{n_m} is the modal detuning, given by the ratio of the modal frequency f_{n_m} to the harmonic frequency $n_m f_0$ in Cent[2]. The corresponding frequency estimator is the frequency of a local minimum in the averaged detuning[3] $f(D_{w.av,min})$.
In the approaches outlined above, the reed is taken into account as a passive, linear acoustic element which prolongs the air column at its input end. Measured impedance curves can, after this simple correction, be directly used to obtain the pitch estimators.
In this paper we want to compare this type of sounding frequency estimator to a minimal physical model of reed-resonator interaction, as suggested by Kergomard [4]. Here, the reed is included as a non-linear generator element with two control parameters representing mouth pressure and embouchure tightness. The coupled model is able to perform stable, self-sustained oscillations. By use of measured control parameters for all notes on the bassoon [2], we determined frequencies at which the system settles and compare these to the purely passive estimators.
The paper is organized as follows: Firstly, a description of the coupled model is given. Then a brief outline on the experiment to determine the control parameters is presented. This is followed by some exemplary simulation results. Finally we compare the sounding frequencies estimated with the various approaches and discuss their practical use.

Material and Methods

The implementation of the model which is used here, was suggested by Doc *et al.* [5].

Description of the coupled model

The resonator is modeled as a superposition of n independent acoustical modes on the left hand side of the following differential equation system relating pressure p and volume-flow rate u as

$$\ddot{p}_i + \frac{\omega_i}{Q_i}\dot{p}_i + \omega_i^2 p_i = C_i \omega_i \dot{u} \qquad (3)$$

where $i = 1..n$ is the ordinal number of the modes, and $(C, \omega, Q)_i$ are parameters triples of the corresponding single

[1] N_h depends on the frequency range of interest $N_h \leq f_{max}/f_0$

[2] If a frequency f is a semitone higher than a reference frequency f_{ref}, the corresponding detuning will be $1200\, log_2(f/f_{ref})$ Cent = 100 Cent.

[3] This estimator was originally proposed in [2], but it was brought to our attention recently that a similar estimator was suggested earlier in unpublished work by Krüger [3].

DOF transfer functions (see Eq. (6)) .
The generator is represented by the excitation function u on the right hand side of the equation system. This function depends non-linearly on p and describes the pressure-flow characteristics of the reed valve

$$u = \begin{cases} \zeta(1-(\gamma-p))\sqrt{|\gamma-p|}sgn(\gamma-p) & \text{for } \gamma-p \leq 1 \\ 0 & \text{for } \gamma-p \geq 1 \end{cases} \quad (4)$$

where γ, ζ are the control parameters related to the mouth pressure and the embouchure tightness, respectively.
All quantities introduced above are non-dimensionalized as

$$\begin{aligned} p =& \frac{\tilde{p}}{p_M} \\ u =& \frac{Z_c\,\tilde{u}}{p_M} \\ \gamma =& \frac{p_m}{p_M} \\ \zeta =& Z_c S_{in}\sqrt{\frac{2}{\rho\, p_M}} \end{aligned} \quad (5)$$

where $\tilde{p}, \tilde{u}$ are the fluctuating reed pressure and volume-flow-rate in Pa and m^3/s, respectively; p_m, p_M are the constant mouth pressure and, the constant pressure needed to close the reed, respectively; S_{in} is the constant reeds inlet area; and ρ, Z_c are the density of air and the characteristic impedance at the resonator input cross section. Note, that $\gamma - p$ in Eq.(4) is the non-dimensional pressure difference across the reed and the reed pressure p in Eq.(3) is calculated as $p = \sum_i p_i$.

Resonator parameters

The resonator parameters are $(C, \omega, Q)_i$, i.e. the modal amplitude, the modal angular frequency and the modal damping of the i^{th} impedance peak. Not all impedance peaks have been taken into account here, but the number of modes n has been set to eight. Further a pre-selection has been done: For each of the harmonics $i f_0$ only one impedance peak has been used [6]. The selection was based on the minimum detuning criterion. The $3\,n$ resonator parameters where determined by fitting a frequency domain model

$$Z\omega = Z_c\, j\omega \sum_i \frac{C_i\,\omega_i}{(j\,\omega)^2 + j\omega\frac{\omega_i}{Q_i} + \omega_i^2} \quad (6)$$

to measured input impedance data, that was corrected for a reed equivalent volume $V_{eq} = 1.9$ cm^3. The upper frequency limit was set to 3 kHz.

Generator parameters

The two embouchure parameters γ, ζ have been determined from blowing experiments with an artificial mouth [2], which had a sensors to measure the pressures $\tilde{p}, p_m$ and the mean flow $\int \tilde{u}dt$. The reed's intake cross section S_{in} was measured optically through transparent walls of the housing. A calibration measurement of the force exerted to the reed by the artificial lip under quasi-stationary conditions allows to determine the embouchure parameters during a blowing experiment[7].

Implementation and Solution

We implemented the model in MATLAB and use the built-in ODE solver *ode23s* and an initial pressure disturbance $p_1(t = 0) = 0.01$ as suggested by Doc *et al.* [5]. Largely depending on the number of modes n in Eq.(3), the computational time is about 5-15 times the physical time on an intel CORE i5 notebook.

Results

In this section some exemplary calculation results of the model are shown. To demonstrate the influence of the number of radiator modes we present a simulation case study based on experimental data from a German bassoon *Sonora* (Fa. Oscar Adler, Markneukirchen, Germany), played with a bocal *N6* (Fa. Guntram Wolf Holzblasinstrumente, Kronach, Germany), and a plastic double reed *270 M* (Conn-Selmer Inc., Elkhart, Indiana, USA). The standard fingering of the note E2 ($f_0 = 83$ Hz) was applied, and the embouchure parameters were measured as $\gamma = 0.3, \zeta = 2.6$.

Influence of higher modes

For the same initial pressure disturbance a set of simulations was run, in which the number n of modes was varied from one to eight. For $n = 1$ and $n = 2$ no stable periodic regime evolved. However, for $n \geq 3$ we found the system settling in limit cycle regimes. The corresponding waveforms are shown in Figure 1.

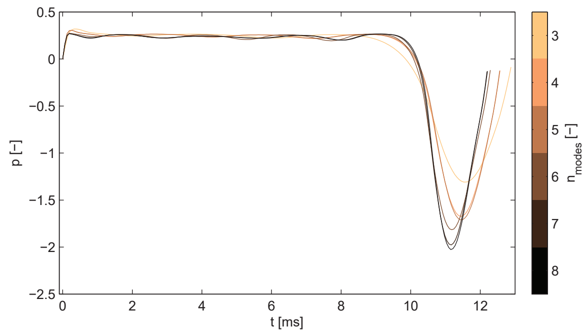


Figure 1: Simulated pressure waveforms for the bassoon note E2 ($f_{0,nom} = 83$ Hz), number of modes n varies from three to eight

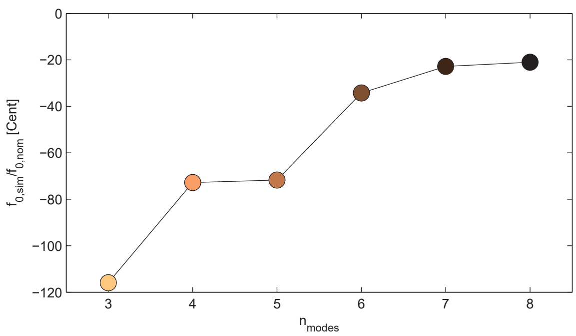


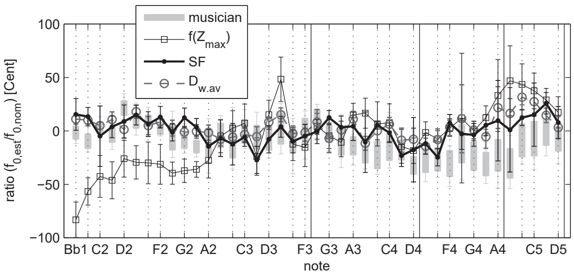
Figure 2: Influence of the number of modes on the pitch. Deviation in Cent between the nominal sounding frequency $f_{0,nom} = 83$ Hz and the limit cycle oscillation frequency $f_{0,sim}$ of the model

As a criterion for the settling of the system, the instantaneous frequency was used. Once its change was smaller than 0.1 Cent the calculation was interrupted. Interestingly, the sounding frequency varies with the number of modes: For the configuration under study, we found the pitch deviation to decrease from -120 Cent to -20 Cent when increasing the number of modes from three to eight (Figure 2).

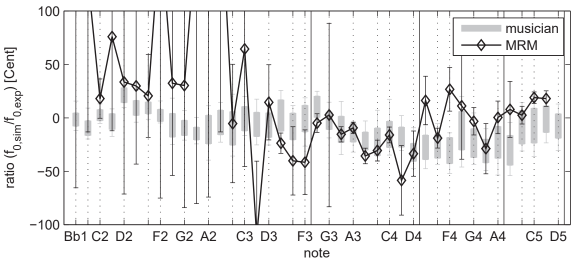
Comparison of intonation estimators

For this deliberately chosen amount of modes $n = 8$, calculations were carried out for a variety of tupels (γ, ζ) at which a note could be played in tune with the artificial mouth. The experimental data set used for the calculation encompasses experimental data from the full tonal and dynamical range on 5 different bassoon-bocal combinations.

As a matter of course, the data were not normally distributed, but are still given here as mean value $\pm$ standard deviation for graphical reasons. For simplicity these data are represented by mean value and standard deviation of the detuning $f_{0,sim}/f_{0,exp}$ (Figure 3(b)). The gray boxplots in Figures 3 a) and b) are results obtained from an experiment with a musician, who was asked to play without embouchure corrections [8].



(a) f_0-estimators from an impedance measurement



(b) f_0-results from the simulation

Figure 3: Comparison of frequency estimators: first impedance peak ($f(Z_{max})$), sum function (SF), amplitude averaged pitch deviation ($D_{w.av}$), and minimal non-linear reed model (MRM). The results are given in form of a tuning deviation $f_{0,est.}/f_{0,nom.}$. The gray boxplot represents the subjective tuning as perceived by a musician [8].

Discussion

Intonation estimators taking more than one acoustical mode into account yield results that resemble intonation curves of a musician. It is most important, that the reed's effective volume is chosen properly. For the case of the bassoon studied here, the deviations are still up to a half semitone. Especially in the higher registers, the estimators predicted higher pitches as compared to the musician.

In contrast to the empirically suggested pitch estimators, a physical model of reed-resonator can demonstrate effect of higher modes on the pitch generation. For a wide range of measured control parameters, we find a periodic solutions of general resemblance to the measured oscillation regime. Moreover, the non-linear model features a good quantitative agreement in oscillation amplitude and frequency.

From the viewpoint of non-linear dynamical modeling, the observed pitch deviations within a few percent seem to be a reasonably good result from a minimal model. However, in view of the very tight tuning tolerances allowed in music, more research is needed to identify a precise enough tuning estimator.

Acknowledgment

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