Optimum Thickness of a Viscoelastic Constrained Layer for damping Beam Vibrations

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Introduction

In electronic industry, data storage devices have to exchange date in increasing faster ways. The mechanics of these devices are often controlled by high bandwidth servo controllers. Compactness, high precision and high stiffness of the mechanical components are primary requirements for proper functioning. The most common examples are hard disks and CD–DVD readout mechanisms.

Particularly, this paper treats a vibration problem of a radial motor of a CD–mechanism. The motor is displayed in figure 1. The swing arm of the mechanism is powered by a coil which is moulded in the arm. The magnetic circuit of the motor consists of two steel plates. The upper plate carries the magnets. The lower plate conducts the magnetic field. In the lower plate, vibrations are induced magnetically by the coil current. At the plate resonances, the radial motor servo can become unstable, causing an annoying high frequency noise radiated from this plate.

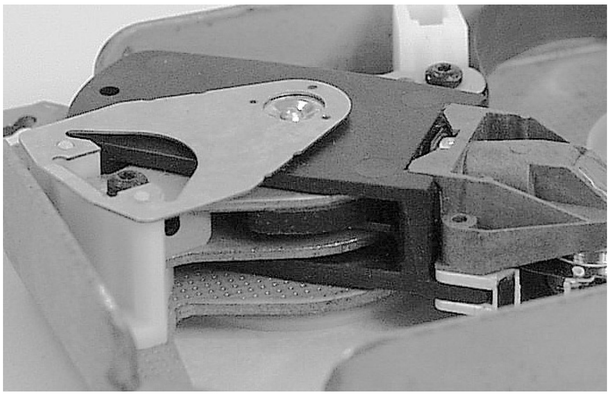

Figure 1: Photograph of the radial motor of a swing-arm CD-mechanism. The lower steel plate of the motor resonates at 5.9 kHz and must be damped.

The transfer function between displacement at the motor plate edge and the current through the coil is measured to localize the vibration. This measurement is displayed in figure 2. At 5.86 kHz, the plate resonates with a damping of 3%. To solve the servo stability problem, this resonance should be approximately 20 dB damped.

The analysis of this problem will be carried out as follows. First, the undamped plate will be analysed as an Euler-Lagrange beam [1], clamped in the center. The motor plate will then be damped by adding a thin viscoelastic layer covered by a thin steel foil. The analysis of this three layer structure will then be performed [1, 2, 3]. After reconstruction of the displacements and the forces or moments, the elastic energy and the dissipated power during one vibration cycle will be determined. These energies will be evaluated numerically and the damping will be determined in terms of the damping layer thickness. An optimum damping layer thickness appears and will be applied to the motor plate, which efficiently damps the resonances of the plate.

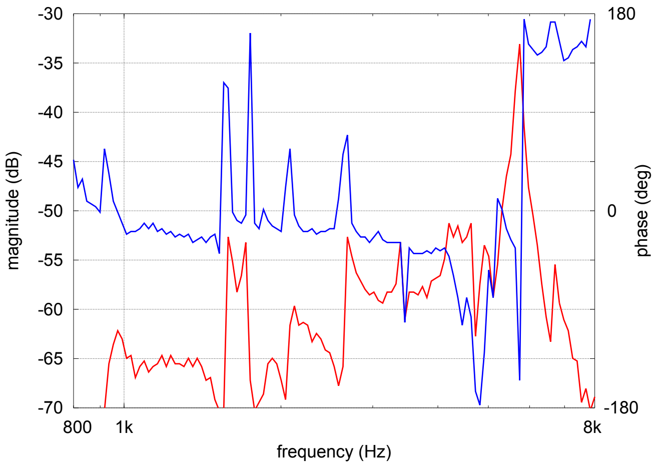

Figure 2: Transfer function measurement between plate end displacement and coil current (red line: magnitude, blue line: phase).

Analysis of the undamped stator plate.

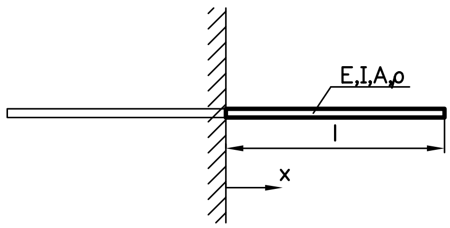

Figure 3: Model of the lower steel plate of the radial motor.

The stator plate is symmetric and can be considered as a beam clamped at its symmetry axis, at presented in figure 3. The differential equation describing the motion of each point of the beam in free vibration equals:

$$\frac{E_1\, I_1}{\rho\, A_1}\, \frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} = 0 \tag{1}$$

wherein E_1 denotes the modulus of elasticity of the beam's material, I_1 the bending moment of inertia, ρ the specific mass of the beam, A_1 the cross section of the beam, y the beam's displacement at any position x along the beam and t the time. The boundary conditions for a single edge clamped beam are:

$y(0) = 0$: no displacement at the fixation point;
$\frac{\partial y}{\partial x}(0) = 0$: no rotation at the fixation point;
$\frac{\partial^2 y}{\partial x^2}(l) = 0$: no bending moment at the free edge;
$\frac{\partial^3 y}{\partial x^3}(l) = 0$: no shear force at the free edge.

Assuming the beam carries out a sinusoidal motion, the solution of this differential equation is obtained by Laplace transformation:

$$y = \frac{M_0}{2\,\gamma^2}\,(\cosh\gamma x - \cos\gamma x) + \frac{D_0}{2\,\gamma^3}\,(\sinh\gamma x - \sin\gamma x) \quad (2)$$

wherein

$$\gamma = \sqrt[4]{\frac{\rho\, A_1\, \omega^2}{E_1\, I_1}} \quad (3)$$

and M_0 and D_0 are related to the fixation point bending moment and shear force respectively. They are connected by the relation

$$-\frac{M_0\,\gamma}{D_0} = \frac{\sinh\gamma l + \sin\gamma l}{\cosh\gamma l + \cos\gamma l} = \frac{\cosh\gamma l + \cos\gamma l}{\sinh\gamma l - \sin\gamma l} \quad (4)$$

which is only satisfied by the values where the equation

$$\cosh\gamma l\,\cos\gamma l + 1 = 0 \quad (5)$$

is valid. Numerically, the values of γl are 1.875, 4.6941, 7.8547 etc.... These values determines the eigenfrequencies of the beam.

Particularly, for this beam with a length of 13 mm, a width of 27 mm and a thickness of 1.2 mm, the eigenfrequency of the first bending mode equals 5.93 kHz, which is approximately equal to the measured value of 5.86 kHz. The ratio $\frac{M_0\,\gamma}{D_0}$ amounts -105.88.

The three layer structure.

To damp the resonance of the plate, a visco-elastic layer constrained with a metal foil is applied. A three layer structure arises, where only the middle layer has damping properties and is presented in figure 4. The bending stiffness of the cover foil is negligible compared to the beam. Consequently, only traction and compression forces can appear by the cover foil. The energy loss in the middle layer, in one cycle of the vibration, is a measure for the damping coefficient.

To determine the damping, the shear angle of the damping layer must be found. The shear angle is related to the compression force present in the cover foil. De deformation $\delta(x)$ of the cover foil, caused by the compression force passed through the damping layer equals

$$\delta(x) = H\,\frac{\partial y}{\partial x} - \xi \quad (6)$$

wherein H the distance between beam neutral grain and cover foil, ξ the cover foil displacement along the beam

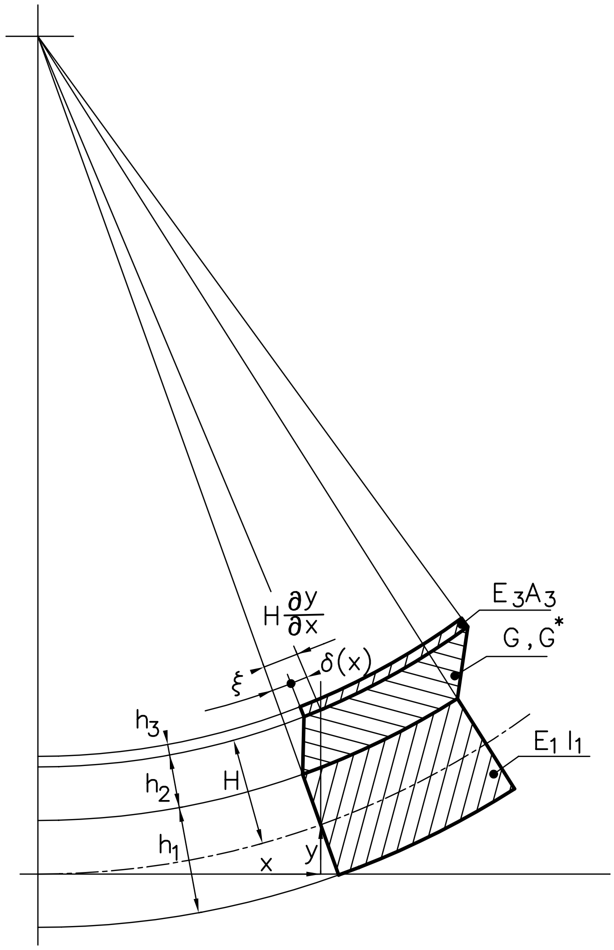

Figure 4: Three layer beam in bending.

axis. Consequently, the compression force in the cover foil amounts

$$T = E_3\,A_3\,\frac{\partial\,\delta(x)}{\partial x} = E_3\,A_3\,(H\,\frac{\partial^2 y}{\partial x^2} - \frac{\partial \xi}{\partial x}) \quad (7)$$

wherein E_3 the elastic modulus and A_3 the crossection of the cover foil. The shear force in the visco-elastic layer becomes

$$\frac{\partial T}{\partial x} = b\,G\,\frac{\xi}{h_2} = E_3\,A_3\,(H\,\frac{\partial^3 y}{\partial x^3} - \frac{\partial^2 \xi}{\partial x^2})\ . \quad (8)$$

wherein b the width of the beam, G the elastic shear modulus and h_2 the thickness of the visco-elastic layer. The differential equation for the displacement of the cover foil is obtained from equation (8).

$$\frac{\partial^2 \xi}{\partial x^2} + \mu^2\,\xi = H\,\frac{\partial^3 y}{\partial x^3}\ . \quad (9)$$

wherein

$$\mu = \sqrt{\frac{b\,G}{h_2\,E_3\,A_3}} \quad (10)$$

When the stiffness increase of the beam by the damping layer is negligible, the displacement y of the undamped beam can be used at the right hand side of equation (2).

$$\frac{\partial^2 \xi}{\partial x^2} + \mu^2\,\xi = \frac{H}{2}\,[M_0\,\gamma\,(\sinh\gamma x - \sin\gamma x) + D_0\,(\cosh\gamma x + \cos\gamma x)]\ . \quad (11)$$

The boundary conditions for the displacement ξ of the cover foil are:

$\xi(0) = 0$: no displacement of the cover foil at the fixation point;

$\frac{\partial \xi}{\partial x}(0) = M_0\, H$: the alteration of the foil displacement is a consequence of the compression force present at the fixation point, obtained from the fixation bending moment.

Taking into account these boundary conditions, the solution of the differential equation is obtained by application of the Laplace transform.

$$\begin{aligned}\xi = & \frac{H}{2\,(\gamma^4-\mu^4)}\,(M_0[(\gamma^3-\mu^2\,\gamma)\sinh\gamma x \\ & +(\gamma^3+\mu^2\,\gamma)\sin\gamma x - 2\,\mu^3\,\sin\mu x] \\ & + D_0[(\gamma^2-\mu^2)\cosh\gamma x - \\ & (\gamma^2+\mu^2)\cos\gamma x + 2\,\mu^2\,\cos\mu x])\end{aligned} \tag{12}$$

This expresses the displacement of the cover foil along the beam axis. This displacement is a measure of the energy loss in the viscoelastic layer. Some physical insight in what will happen when the thickness of the visco-elastic layer alters, can be obtained by evaluating the limits of equation (12) when μ goes to zero and infinity respectively. When μ becomes zero, this is, when the shear modulus of the viscoelastic layer becomes zero, no force can be transported through the viscoelastic layer and the length of the cover foil remains the same length as the beam neutral grain. This is confirmed by evaluating the limit

$$\lim_{\mu\to 0}\xi = H\,\frac{\partial\, y}{\partial\, x} \tag{13}$$

from the expression (12). If μ goes to infinity, this is, when the thickness of the damping layer becomes zero, no shear between foil and beam can occur and the relative displacement between foil and beam is zero. This is also confirmed by evaluating the limit

$$\lim_{\mu\to\infty}\xi = 0 \tag{14}$$

from the expression (12).

Knowing the present forces and displacements, the elastic and dissipative energy in the three layer structure can be determined. The damping coefficient will be obtained from their ratio. The basic expression to calculate elastic energy is for traction

$$d\,W_t = \frac{\sigma^2}{E}\,d\,V \tag{15}$$

and for shear

$$d\,W_a = \frac{\tau^2}{G}\,d\,V\ . \tag{16}$$

The total elastic energy W_e consists of the bending energy of the beam, the elastic shear energy in the visco-elastic layer end the traction energy in the cover foil.

$$\begin{aligned}W_e = & \int_0^l \frac{M(x)^2}{2\,E_1\,I_1}\,d\,x \\ & + \int_0^l \frac{G\,b}{h_2}\,\xi(x)^2\,d\,x + \int_0^l \frac{T(x)^2}{2\,E_3\,A_3}\,d\,x\ .\end{aligned} \tag{17}$$

The dissipative energy will be determined over one cycle of vibration. The starting point is likewise

$$d\,W_d = \frac{\tau^2}{G^*}\,d\,V \tag{18}$$

wherein G^* is the viscous part of the damping layer shear modulus, τ is shear tension and V is volume.

When the cover foil carries out a sinusoidal motion

$$\xi_t = \xi\,\cos\omega t \tag{19}$$

then the velocity becomes

$$\dot{\xi}_t = -\xi\,\omega\,\sin\omega t\ . \tag{20}$$

The shear tension in the visco-elastic layer is

$$\tau = \frac{G^*}{\omega\,h_2}\,\dot{\xi}_t = -\frac{G^*}{h_2}\,\xi\,\sin\omega t\ . \tag{21}$$

With

$$d\,V = b\,h_2\,d\,x \tag{22}$$

the energy, dissipated over one cycle of the vibration per unit length becomes

$$\frac{d\,W_d}{d\,x} = \int_0^{2\pi}\frac{b\,G^*}{h_2}\,\xi^2\sin\omega t\,d\omega t\ . \tag{23}$$

resulting in

$$\frac{d\,W_d}{d\,x} = \pi\,\frac{b\,G^*}{h_2}\,\xi^2\ . \tag{24}$$

The amount of dissipated energy becomes over the total length

$$W_d = \int_0^l \pi\,\frac{b\,G^*}{h_2}\,\xi^2\,d\,x. \tag{25}$$

Both integrals (17) and (25) are numerically evaluated and the damping coefficient ζ will be calculated as the ratio between dissipative and elastic energy

$$\zeta = \frac{1}{2\,\pi}\,\frac{W_d}{W_e}\ . \tag{26}$$

Optimal damping layer thickness

The calculations are carried out for the motor plate using industrial glues as visco-elastic layer and cover foil in stainless steel. A nice combination is presented in figure 5, which is a combination of a visco-elastic layer with 20 MPa elastic modulus and 10% damping with a cover foil in stainless steel with 35 μm thickness. In the figure, the damping coefficient is plotted against the damping

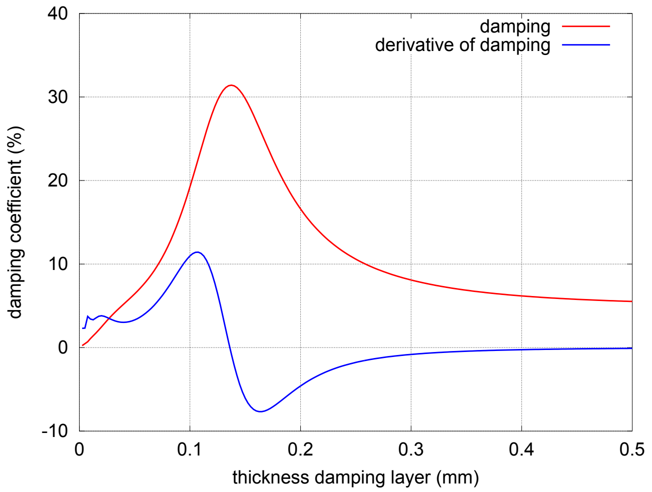

Figure 5: Damping coefficient as function of damping layer thickness for a damping layer with 20 MPa shear modulus, 10% damping and constrained by a steel cover foil of 35 μm thickness.

layer thickness. The damping starts at zero when the damping layer thickness is zero, increases until it reaches a maximum and decreases again. The damping goes theoretically to zero at infinite thickness, but these calculations becomes inaccurate for thick damping layers. The optimum thickness of the damping layer is located where the derivative of the damping curve equals zero.

The maximum damping is proportional with the imaginary part of the shear modulus of the damping layer. It's value is independent of the real part of the shear modulus and also of the cover foil stiffness.

The optimum thickness of the damping layer is not dependent of imaginary part of the shear modulus, it is dependent of the real part of the shear modulus of the damping layer and the cover foil stiffness. The optimum damping layer becomes thinner when the real part of the shear modulus decreases or when the cover foil stiffness increases. The non-dependency of the optimum damping layer thickness of viscous properties is advantageous in practice, because these properties are seldom well known.

The selection of the elastic modulus of the damping material is the most important parameter. Once a good modulus is selected, the layer thickness can be tuned by adjusting the cover foil stiffness.

It appears also that the method " trial and error" leads seldom to good results. The optimum is often far away and a damping of only a few percents is obtained.

In the case of the motor plate, a damping layer of 0.12 mm thickness with an elastic shear modulus of 20 MPa with a cover foil thickness in 35 μm stainless steel is the most optimal choice. The total thickness increase of the plate amounts only 0.15 mm, which can be easily build in. Higher moduli lead to very thin layers, which are difficult to reproduce in a production process. The effect of the damping is demonstrated in figure 6. In comparison with figure 2, the damping is increased by 18 dB, which solves the resonance problem. Also the other resonances between 1.5 and 2 kHz are efficiently damped.

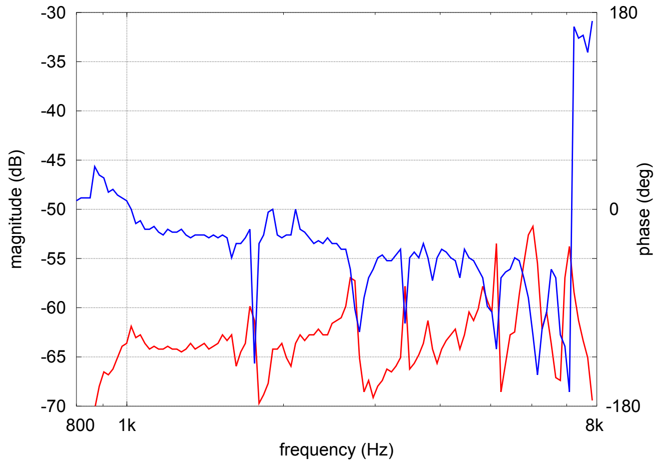

Figure 6: Effect of the optimized damping layer on the motor plate. The resonance at 5.9 kHz is damped 18 dB compared to figure 2. (red line: magnitude, blue line: phase).

Conclusion.

A method for optimizing damping in constrained viscoelastic damping layers is presented. Relatively high damping values can be reached in a small space, without affecting the structure stiffness. The maximum value of the damping is proportional with the imaginary part of the damping layer's shear modulus and independent of the elastic properties of the damping layer and the cover foil. The optimum layer thickness is only dependent of elastic properties of the materials.

References

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