A Simple Model of the Far-Field Directivity of an Open Circular Pipe with a Hot Flow

Drasko Masovic[1], Eugene Nijman[1], Jan Rejlek[1], Robert Höldrich[2]
[1] *Virtual Vehicle Research Center, 8010 Graz, E-Mails: drasko.masovic@v2c2.at, eugene.nijman@v2c2.at, jan.rejlek@v2c2.at*
[2] *IEM University of Music and Performing Arts Graz, 8010 Graz, E-Mail: robert.hoeldrich@kug.ac.at*

Introduction

Sound radiation from an open pipe belongs to classical acoustic problems. The first analytical solution for an open unflanged pipe (without a mean flow) was introduced by Levine and Schwinger in 1948 [1]. Increasing interest in aircraft noise and the development of aeroacoustics have resulted in more general solutions in the presence of a mean flow exhausted from the pipe. The most prominent one is probably derived by Munt [2], which is valid not only for hot interior and/or exterior flows, but also for higher acoustic modes inside the pipe. Since then, the solution has been many times experimentally confirmed [3, 4]. Nevertheless, it has several drawbacks. Firstly, the solution, although generally referred to as analytical, requires numerical calculation of several involved functions and complex integrals. Secondly, its complex mathematical apparatus, based on the Wiener-Hopf technique, remains most frequently incomprehensible and inapplicable for broader engineering community. Thirdly, very few physical interpretations of the results are given, with respect to the key acoustic phenomena and, thus, although remarkably accurate, Munt's model does not provide a clear physical explanation of the resulting sound field. Lastly, due to the highly idealized geometry and simple uniform mean flow considered, the accuracy of the solution is questionable in real situations, especially for capturing refraction in the mixing region of the jet [4].

With regard to that, the goal of this work is to present a simple, semi-analytical model of the far-field sound radiation from an open pipe with a hot flow. The model should keep the accuracy comparable to Munt's solution, within the limit of low Mach number of the mean flow and low frequency (Helmholtz number) of the incident sound. This makes the model more restrictive compared to Munt's solution (subsonic flows and broad frequency range). Still, many practical problems, such as intake, exhaust, and HVAC systems in vehicles and pass-by noise prediction, satisfy the given limitations.

The acoustically relevant phenomena will be treated according to the order in which they appear on the path between a source inside the pipe and the listener in the far field outside the pipe. These are, namely: incident sound inside the pipe, vortex-sound interaction at the pipe edge, and refraction in the mixing region of the jet. In the end, all three contributions will be combined and the obtained radiation pattern shapes will be compared to the ones obtained using Munt's solution and the experimental data.

Incident sound

The incident sound inside the pipe is assumed to originate from any sound source which is located deep inside the pipe, far from the opening. At sufficiently low Helmholtz number $ka < 1$, with the pipe radius a as the reference geometrical length scale, the sound wave inside the pipe is a one-dimensional plane wave. The low Mach number mean flow inside the pipe is considered to be uniform and the physical quantities can be split into their constant mean flow components (denoted here with index $_J$) and small, purely acoustical perturbations (denoted with the prime symbol). The linearized momentum equation then simplifies to:

$$\frac{\partial p'}{\partial x} \approx -\rho_J \frac{\partial v'}{\partial t}, \qquad [\mathrm{Pa/m}] \qquad (1)$$

where p denotes pressure, ρ is density, and v is velocity component parallel to the pipe axis, which is set along the x-coordinate axis. The incident sound wave, therefore, contributes to the exterior radiation by the injection of its momentum through the pipe's cross section. This radiation can be considered to be omnidirectional and represented by a monopole source located at the centre of the pipe opening. It should be noted that the mean flow and convection have no effect on the far-field directivity of this contribution. Convective amplification, as the property of the source inside a flow rather than a pure propagation phenomenon, takes place only at the location of the actual source. Therefore, no convection of incident sound can cause angularly dependent radiation outside the pipe. Instead, the pronounced far-field directivity at low frequencies is a consequence of vortex-induced sound, which will be discussed in the next section.

The far-field sound pressure at distant location $\mathbf{x}$ outside the pipe due to the monopole can now be estimated as:

$$p_m'(\mathbf{x},t) = -\frac{1}{4\pi|\mathbf{x}|}\iiint_V \frac{\partial}{\partial x}\left(\rho_J \frac{\partial v'}{\partial t}\right)dV \approx \frac{\rho_J \omega v'\left(a^2\pi\right)}{4\pi|\mathbf{x}|}, \qquad [\mathrm{Pa}] \qquad (2)$$

where ω denotes angular frequency of the sound and a is the pipe radius. In the case of a cold jet, this agrees with [5].

Vortex-sound interaction

When the incident sound wave reaches the thin trailing edge of the rigid pipe, it is diffracted in all directions and at all angles to the pipe axis. At low frequencies and without a mean flow, the resulting radiation is nearly omnidirectional [1], since the opening of the pipe acts as a compact source of the sound, as perceived by the listener outside the pipe. The contribution of the vortices shed from the pipe edge is of the second order and, therefore, negligible for small acoustic perturbations. However, this becomes substantially different when the pipe exhausts a jet, even under quasi-steady conditions at low Mach number velocities. High mean flow speed gradient established at the edge leads to the formation of an unstable shear-layer of vortical flow, which ensures the finiteness of the flow parameters at the sharp edge (Kutta condition). The resulting unsteady solenoidal perturbations can easily interact with the incoming sound waves.

The sound energy turned into the kinetic energy of the vortex implies that the vortex acts as an acoustic sink. This mechanism of sound attenuation dominates over the flow-induced sound generation at low frequencies. However, part of the acoustic energy is retrieved through the action of the vortex on the solid edge as a dipole radiation. Since the energy of the vortex originates from the incident sound wave, we cannot speak about a pure aeroacoustic source, although the entire process can be treated aeroacoustically. In order to quantify it, we start with the momentum equation in the form:

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \nabla p + \frac{1}{2}\rho \nabla v^2 + \rho \boldsymbol{\omega} \times \mathbf{v} = 0. \qquad [\mathrm{Pa/m}] \qquad (3)$$

At low Mach number values, the flow around the edge can be considered to be incompressible and the static pressure is assumed to be constant everywhere. After the linearization, we obtain:

$$\rho_0 \frac{\partial \mathbf{v'}}{\partial t} + \nabla p' \approx -\rho_J \boldsymbol{\omega} \times \mathbf{v}. \qquad [\mathrm{Pa/m}] \qquad (4)$$

Taking the uniform mean flow as the reference, we conclude that the dominating term which involves the interaction of the forming vortex with the incoming acoustic wave [5] has the form of the Coriolis force per unit volume. The reaction of the pipe wall on this force "regenerates" the sound. Thus, the resulting far-field pressure equals:

$$p_d'(\mathbf{x},t) = \frac{1}{4\pi|\mathbf{x}|} \iiint_V \nabla \cdot (\rho_J \boldsymbol{\omega} \times \mathbf{v}) dV. \qquad [\mathrm{Pa}] \qquad (5)$$

Next, we follow the approach of Hirschberg and Hoeijmakers [6], based on Vortex Sound Theory [5], and assume that the axial component of the Coriolis force generates the vortex ring at the pipe edge. The volume integral from the last expression can then be estimated as:

$$\frac{\partial}{\partial x}\left(\rho_J \frac{d(S_\omega \Gamma)}{dt} \right), \qquad [\mathrm{N/m}] \qquad (6)$$

where Γ and S_ω represent the circulation and the surface of the vortex ring, respectively. For very edge and shear-layer, the latter one approaches the area of the pipe opening. The amount of the circulation shed at the pipe edge gives in the first-order approximation:

$$\frac{d\Gamma}{dt} = \frac{d\Gamma}{dx}\frac{dx}{dt} \approx \frac{(v_J + v')^2}{2} \approx v_J v'. \qquad [\mathrm{m^2/s^2}] \qquad (7)$$

Here, dx/dt represents the convection velocity of vorticity, which can be roughly approximated as $(v_J + v')/2$, that is, the average velocity between the (uniform) jet and still surrounding. Inserting this in the volume integral, the far-field sound pressure can be obtained as:

$$p_d'(\mathbf{x},t) \approx \frac{\rho_J \omega v' (a^2 \pi)}{4\pi|\mathbf{x}|} M_J \frac{c_J}{c_\infty} \cos(\theta) \qquad [\mathrm{Pa}] \qquad (8)$$
$$= p_m' M_{ac} \cos(\theta),$$

where $M_{ac} = v_J/c_\infty = M_J c_J/c_\infty$ is the so-called acoustic Mach number.

Unlike the monopole source, which is not convected, the dipole source is located just outside the pipe and inside the flow, which leads to the convective amplification. If the flow around the source is assumed to be nearly uniform, the convective amplification has an analytical solution and it is proportional to M_{ac}^2, which is very small in low Mach number flows.

Refraction

Refraction is a pure sound propagation phenomenon inside an inhomogeneous medium with varying effective sound speed, due to the non-uniformity of either temperature or mean flow velocity. In the case of an open pipe with a flow, the non-uniformity is limited to the mixing region of the jet, where the values of the mean flow parameters spatially decay to the values in the still medium far from the opening.

Being a spatially distributed phenomenon, refraction is more difficult to approximate than the compact monopole and dipole sources treated so far, apart from the trivial case when the entire mixing region is acoustically compact. The high-frequency approximation of geometrical acoustics is unsuitable for low-frequencies considered here, as it leads to the overestimation of the zone of silence. Fortunately, the effect of the refraction is significant only at small angles to the pipe axis, within a so-called zone of (relative) silence, where the thickness of the mixing region (which is at the axis typically $\sim O(10a)$ [7]) is comparable to the sound wavelength, that is for approximately $ka > 0.2$. As the thickness of the mixing region decreases with the angle to the pipe axis (becoming as small as the thickness of the shear-layer at the pipe edge at angle to the axis $\theta = 90°$), refraction effects become negligible. Therefore, the zone of relative silence is very narrow in round jets and most of the applications in practice do not require accurate estimation of the refraction. We will, therefore, take a very crude approach, by estimating refraction as the distributed artificial source in a similar fashion as in Lighthill's aeroacoustic analogy.

Assuming small unsteady perturbations of the low Mach number mean flow and constant static pressure, the momentum equation in the mixing region reads:

$$\frac{\partial \mathbf{v}'}{\partial t} + \frac{\nabla p'}{\rho_0} + \mathbf{v} \cdot \nabla \mathbf{v} = 0. \qquad [\mathrm{m/s^2}] \qquad (9)$$

The reference flow is now the cold quiescent flow at infinity and the relevant "sound sources" are described by the terms:

$$-\frac{\nabla p'}{\rho_0} \frac{T_0 - T_\infty}{T_0} - \mathbf{v} \cdot \nabla \mathbf{v}, \qquad [\mathrm{m/s^2}] \qquad (10)$$

where also the ideal gas equation with constant pressure ($p_0 = \rho_0 R T_0 = const$) was used. The first term is then attributed to the refraction due to the temperature gradient and the second one due to the variations of the mean flow velocity. The contribution of the refraction to the far-field sound pressure is thus:

$$\begin{aligned} p_r'(\mathbf{x},t) = &-\frac{1}{4\pi|\mathbf{x}|} \iiint_V \rho_0 \nabla \cdot \left(\frac{\nabla p'}{\rho_0} \frac{T_0 - T_\infty}{T_0} \right) dV \\ &-\frac{1}{4\pi|\mathbf{x}|} \iiint_V \rho_0 \nabla \cdot (\mathbf{v} \cdot \nabla \mathbf{v}) dV, \end{aligned} \qquad [\mathrm{Pa}] \qquad (11)$$

The second integral can be expanded due to the fact that $\nabla \cdot (\mathbf{v} \cdot \nabla \mathbf{v}) = \nabla \cdot (\boldsymbol{\omega} \times \mathbf{v}) + \nabla (\mathbf{v} \cdot \mathbf{v})/2$. The influence of vortices on the sound propagation is further neglected and in the first-order approximation: $\mathbf{v}\cdot\mathbf{v}/2 \sim v_0 v'$. We scale $p' \sim \rho_0 c_0 v'$ in the first integral and all spatial derivatives as $\sim 1/\lambda$. Assuming that the mixing region has only one relevant dimension (its thickness), which is a function of the angle to the axis, $L(\theta)$, we estimate the volume integral as $a^2 \pi L(\theta)$ and for the mean flow variables we take values of the uniform jet, ρ_J, v_J, and T_J. It should be noted that $L(\theta)$, seen as the path length of the sound, has already been used in the literature for characterisation of the zone of relative silence [8]. The very rough simplification introduced here should serve as an estimation of the effect of the refraction in an integral sense. Putting everything together gives the following estimation of the last contribution:

$$\begin{aligned} p_r'(\mathbf{x},t) &\approx -\frac{(a^2\pi) L(\theta) \rho_J v'}{4\pi|\mathbf{x}|\lambda^2} \left(c_J \frac{T_J - T_\infty}{T_J} + v_J \right) \\ &= -p_m' \left(\frac{T_J - T_\infty}{T_J} + M_J \right) \frac{L(\theta)}{(2\pi)^2 a} ka. \end{aligned} \qquad [\mathrm{Pa}] \qquad (12)$$

The far-field pressure is negative and refraction behaves as an "acoustic sink" distributed throughout the mixing region.

The introduced thickness of the mixing region has to be evaluated empirically. It should decay fast with the angle to the axis θ and nearly equal the pipe wall thickness at $\theta = 90$º. In the following, we will use the exponential function $L(\theta) = 100a\exp(-4.5\theta)$, which at the pipe axis takes the value of 100 pipe radiuses. This length roughly corresponds to the decay of the maximum jet velocity to 10% of its value [7]. Since $L(\theta{=}90°) \approx 1.7$mm, the estimated thickness of the mixing region at the pipe edge approaches closely the thickness of the pipe, which is taken as $a = 2$mm. The range of validity of the proposed refraction model is also expected to be limited by the upper value of Helmholtz number. This limit is a consequence of the fact that already at moderately high Helmholtz number values, such as $ka > 0.3$, the mixing region thickness becomes much larger than the sound wavelength and the approximation of the spatial derivatives, as done here, becomes inappropriate. The sound propagation in such a case fits better to the assumptions of geometrical acoustics.

Results and discussion

Summarizing the results above, the far-field radiation pattern shape can be estimated as:

$$RP(\theta) = 20\log_{10}\left(\frac{p_m' + p_d' + p_r'}{p_m'} \right). \qquad [\mathrm{dB}] \qquad (13)$$

Figure 1 shows the shapes of the radiation patterns, calculated using the last expression for maximum Mach number of the jet 0.25 and temperature 300°C and two different Helmholtz number values, 0.1 and 0.3. The patterns are normalized according to the total radiated power and compared to the corresponding measured values [4] and Munt's solution [2]. As shown to provide a better match, the maximum value of the acoustic Mach number was used for the estimation of the radiation pattern, while the cross-section average value was used for Munt's solution, as in [4]. This difference might be due to the slight underestimation of the convection velocity of the vortices by the average velocity between the jet and the motionless surrounding medium, which then also leads to the underestimation of the dipole radiation. The introduced model of the jet's mixing region generally fits well with the measurement results of the far-field directivity pattern.

The derived model can be further used for the estimation of the free-space directivity of a vehicle's tail-pipe as a source of low-frequency sound at the lowest orders of an internal combustion engine. With the supposed radius of the pipe of 2cm, the estimated radiation pattern shapes for the second order of a four-stroke, four-cylinder engine are shown in Figure 2. The results are given for a full throttle constant acceleration from 1500 rpm to 5500 rpm in the steps of 1000 rpm, whereas, the mean flow properties are estimated from the measurement results reported in [9].

Helmholtz number values are well below 0.1, so the refraction effects are negligible. Nevertheless, the directivity deviates strongly from the omnidirectional radiation as the engine speed increases, reaching up to around 7dB of the difference between the sound pressure level downstream and upstream from the tail-pipe opening. This pronounced directivity should, therefore, be taken into account when estimating pass-by noise, or transmission of the exhaust system noise into the vehicle's interior. While the car body is expected to affect the directivity at higher Helmholtz numbers, it is the mean flow of the exhausted gases which can be a cause of a dipole-like contribution even at the lowest frequencies and engine orders. At the same time, the reflections from the ground should not have a strong effect on the directivity at very low frequencies, when the distance of the pipe opening from the ground is much smaller than the sound wavelength.

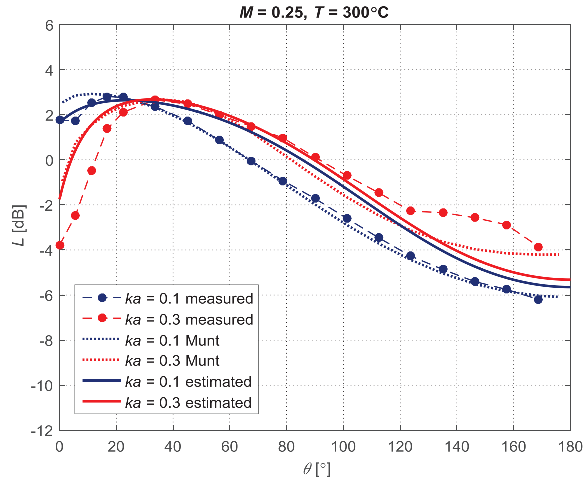

Figure 1: Comparison of the simple model with Munt's solution and measurements for a hot jet; normalization based on the total radiated energy.

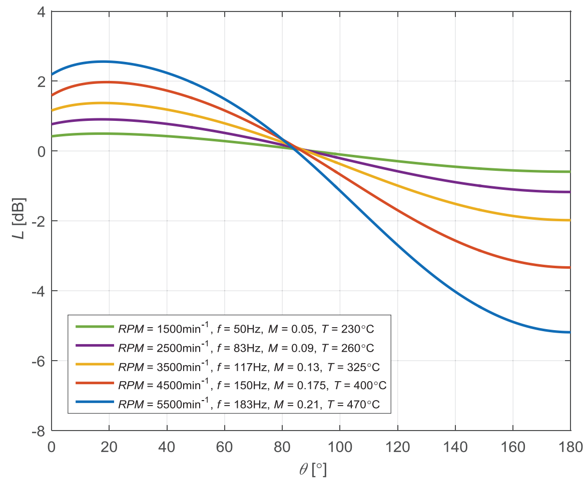

Figure 2: Estimated shapes of the radiation patterns of the second engine order, during a full throttle constant acceleration, based on the derived sound radiation model.

The derived model can also be used for the approximation of the edge condition, within a solution of convective wave equation [10]. The treatment of the vortex-sound interaction as discussed here can be used for the inclusion of vortex effects, which are normally not covered by a single-unknown wave equation.

Conclusion

The proposed, simple model of the exterior sound radiation from an open circular pipe at low frequencies and low Mach number velocities gives a satisfactory match with the measurement results and more involved Munt's solution. It can be used for the estimation of far-field directivity in engineering applications. Although less accurate than Munt's solution, it gives a clear insight into the governing phenomena and the flow-acoustic interaction. Moreover, the effect of the refraction within the zone of relative silence at the pipe axis is captured in more details, with the aid of an empirical model of the jet.

Literature

[1] Levine, H., Schwinger, J.: On the radiation of sound from an unflanged circular pipe. Physical Review 73 (1948), 383-406

[2] Munt, R.M.: The interaction of sound with a subsonic jet issuing from a semi-infinite cylindrical pipe. Journal of Fluid Mechanics 83 (1977), 609-640

[3] Pinker, R. A., Bryce, W. D.: The radiation of plane-wave duct noise from a jet exhaust, statically and in flight. 3rd AIAA Aero-Acoustics Conference, Palo Alto, California (1976), AIAA Paper No. 76-581

[4] Masovic, D., Zotter, F., Nijman, E., Rejlek, J., Höldrich, R.: Directivity measurements of low frequency sound field radiated from an open cylindrical pipe with a hot mean flow. 9th ISNVH Congress, Graz (2016), SAE Technical Paper 2016-01-1822

[5] Howe, M. S. Acoustics of Fluid-Structure Interactions. Cambridge University Press, 1998

[6] Hirschberg, A., Hoeijmakers, M.: Comments on the low frequency radiation impedance of a duct exhausting a hot gas. Journal of the Acoustical Society of America – Express Letters 136 (2014) EL84-EL89

[7] Witze, P. O.: Centerline velocity decay of compressible free jets. AIAA Journal 12 (1974) 417-418

[8] Lush, P. A.: Measurements of subsonic jet noise and comparison with theory. Journal of Fluid Mechanics 46 (1971) 477-500

[9] Van Der Walt, D. C.: Measurement technique to assess the acoustic properties of a silencer component for transient engine conditions. Journal of Sound and Vibration 243 (2001) 797-821

[10] Masovic, D., Nijman, E., Rejlek J., Höldrich, R.: Towards a boundary condition for convective wave equation and sound diffraction at a trailing edge inside a flow. DAGA 2017 Kiel (2017)

Acknowledgments

The research work of Drasko Masovic has been funded by the European Commission within the ITN Marie Curie Action project BATWOMAN under the 7th Framework Programme (EC grant agreement no. 605867).

The authors acknowledge the financial support of the COMET K2 - Competence Centres for Excellent Technologies Programme of the Austrian Federal Ministry for Transport, Innovation and Technology (BMVIT), the Austrian Federal Ministry of Science, Research and Economy (BMWFW), the Austrian Research Promotion Agency (FFG), the Province of Styria and the Styrian Business Promotion Agency (SFG).