Sensitivity of nonlinear distortion in loudspeakers to the change of parameters

Andrzej Dobrucki, Maurycy Kin, Piotr Otręba

Wrocław University of Science and Technology, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland
{andrzej.dobrucki, maurycy.kin, piotr.otreba}@pwr.edu.pl

Introduction

Algorithms of reduction of nonlinear distortion in loudspeakers with DSP require the identification of the parameters. These parameters can be either linear, as the mass of membrane and voice-coil, stiffness of the suspension, mechanical and electrical resistances, inductance of the voice-coil and *Bl*-factor; or nonlinear which describe nonlinearities of the compliance of suspensions, *Bl*-factor and voice-coil inductance. If the loudspeaker is placed in an enclosure, the additional parameters which characterize enclosure should be taken into account. These parameters are e.g. acoustical compliance of the enclosure, acoustical mass and resistance of the canal. The identification of the parameters is always characterized by an uncertainty. When the parameters are identified incorrectly, the algorithm of reduction will give incorrect results and instead of reduction even the increase of distortion can occur. Then, the determination of sensitivity of nonlinear distortion to change of parameters is important. As the measure of the nonlinear distortion the coefficient of harmonic has been defined. The sensitivity to the Thiele-Small parameters and normalized nonlinear parameters has been determined. The nonlinearities have been described with second-orders curves of displacement.

Equations of loudspeaker with nonlinearities

The equivalent electrical circuit of a loudspeaker is presented in Fig. 1.

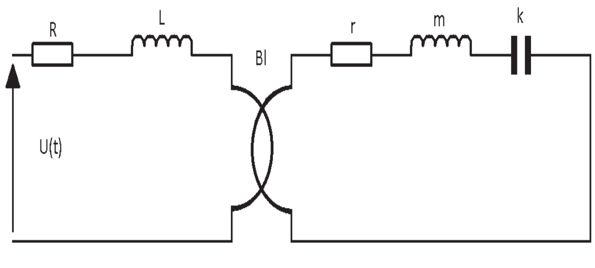


Figure 1. Equivalent circuit of an electrodynamic loudspeaker

The equation system, resulting from the eqiuvalent electrical circuit of loudspeaker has the following form in the linear case:

$$\begin{aligned} \frac{dv}{dt} &= \frac{Bl}{m} i - \frac{r}{m} v - \frac{k}{m} x \\ \frac{dx}{dt} &= v \\ \frac{di}{dt} &= \frac{E_m}{L} \cos \omega t - \frac{R}{L} - \frac{Bl}{L} v \end{aligned} \qquad (1)$$

For nonlinear case the system has the following form:

$$\begin{aligned} \frac{dv}{dt} &= \frac{Bl(x)}{m} i + \frac{1}{2m} \frac{dL(x)}{dx} i^2 - \frac{r}{m} v - \frac{k(x)}{m} x \\ \frac{dx}{dt} &= v \\ \frac{di}{dt} &= \frac{E_m}{L(x)} \cos \omega t - \frac{R}{L(x)} i - \frac{Bl(x)}{L(x)} v - \frac{1}{L(x)} \frac{dL(x)}{dx} iv \end{aligned} \qquad (2)$$

In this system the dependence of force factor, electrical inductance and mechanical stiffness on displacement are taken into account. In comparison with the linear case two additional terms appear in the system (2). In the first equation this is a reluctance force and in the third one the term connected with the dependence of the magnetic flux in the air gap on the displacement. In both terms the derivative $dL(x)/dx$ occurs. The normalization and normalized values are introduced to the equation systems (1) and (2):

$x_0 = \dfrac{E_m Bl}{kR}$ - normalizing displacement,

$v_0 = \omega_s x$ - normalizing velocity,

$I_0 = \dfrac{E_m}{R}$ - normalizing current,

$\omega_s = \sqrt{\dfrac{k}{m}}$ - resonant angular frequency,

$\xi = \dfrac{x}{x_0}$ - normalized displacement,

$\eta = \dfrac{v}{v_0}$ - normalized velocity,

$\iota = \dfrac{i}{I_0}$ - normalized current,

$\nu = \dfrac{\omega}{\omega_s}$ - normalized exciting frequency,

$\tau = \omega_s t$ - normalized time,

$Q_{MS} = \dfrac{\omega_s m}{r}$ - mechanical quality factor,

$Q_{ES} = \dfrac{\omega_s m}{Bl^2 / R}$ - electrical quality factor,

$Q_L = \dfrac{\omega_s L}{R}$ - quality factor of the inductance.

The parameters ω_s , Q_{MS} and Q_{ES} are known as the Thiele-Small parameters of linear loudspeaker in the low-and middle frequency range. They are a useful tool for analysis and design of loudspeaker. Other normalized parameters which occur in this paper are introduced by the authors. These parameters characterize nonlinear loudspeaker. Introducing the normalized parameters to the Equation (1), this linear equation system becomes the form:

$$\begin{aligned} \frac{d\eta}{d\tau} &= \iota - \frac{1}{Q_{MS}}\eta - \xi \\ \frac{d\xi}{d\tau} &= \eta \\ \frac{d\iota}{d\tau} &= \frac{1}{Q_L}\cos\nu\tau - \frac{1}{Q_L}\iota - \frac{1}{Q_L Q_{ES}}\eta \end{aligned} \qquad (3)$$

We assume that the nonlinearities of k, Bl and L are the described with polynomials of the 2nd order:

$$\begin{aligned} k(x) &= k(x) = k_0\left(1 + k_1 x + k_2 x^2\right); \\ Bl(x) &= Bl_0\left(1 + b_1 x + b_2 x^2\right); \\ L(x) &= L_0\left(1 + l_1 x + l_2 x^2\right) \end{aligned} \qquad (4)$$

These nonlinearities are transferred into dimensionless, normalized form:

$$\begin{aligned} F(\xi) &= 1 + b_1 x_0 \xi + b_2 x_0^2 \xi^2; \\ G(\xi) &= 1 + k_1 x_0 \xi + k_2 x_0^2 \xi^2; \\ H(\xi) &= 1 + l_1 x_0 \xi + l_2 x_0^2 \xi^2 \\ H'(\xi) &= l_1 x_0 + 2 l_2 x_0^2 \xi \end{aligned} \qquad (5)$$

Introducing (5) into the nonlinear equation system (2) it has the form:

$$\begin{aligned} \frac{d\eta}{d\tau} &= F(\xi)\iota - \frac{1}{Q_{MS}}\eta - G(\xi)\xi + \frac{1}{2}Q_L Q_{ES} H'(\xi)\iota^2; \\ \frac{d\xi}{d\tau} &= \eta; \\ \frac{d\iota}{d\tau} &= \frac{1}{Q_L H(\xi)}\cos\nu\tau - \frac{1}{Q_L H(\xi)}\iota - \frac{1}{Q_L Q_{ES}}\frac{F(\xi)}{H(\xi)}\eta - \frac{H'(\xi)}{H(\xi)}\iota\eta \end{aligned} \qquad (6)$$

Equation system (6) has been solved using the Runge-Kutta procedure with application of the MathCAD software. In the last step the acoustic pressure at the distance d from the loudspeaker center is computed: It is given by the following equation:

$$\begin{aligned} p &= \frac{\rho_0 S}{2\pi d}\frac{dv}{dt} = \frac{\rho_0 S}{2\pi d}\omega_s^2 x_0 \frac{d\eta}{d\tau} = \\ &= \frac{\rho_0 S}{2\pi d}\omega_s^2 x_0 \left(F(\xi)\iota - \frac{1}{Q_{MS}}\eta - G(\xi)\xi + \frac{1}{2}Q_L Q_{ES} H'(\xi)\iota^2 \right) \end{aligned} \qquad (7)$$

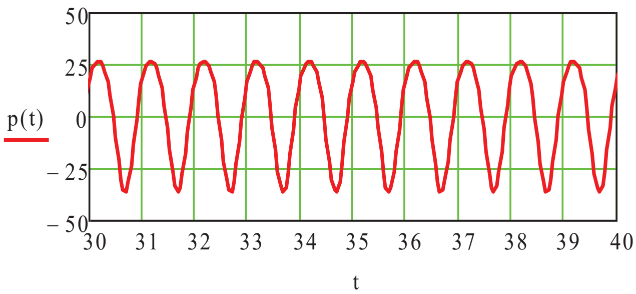
Computing

The modeling of nonlinear distortions has been conducted for a set of initial parameters. Some of these parameters have been varied and the sensitivity was calculated. The maximum values of nonlinear parameters l_1, l_2, b_1, b_2, k_1, k_2 have been chosen as follows: l_1 = -200 H/m, l_2 = 20000 H/m^2, b_1 = 200 T, b_2 = -20000 T/m, k_1 = 200 N/m^2, k_2 = 20000 N/m^3, The values are similar for evaluate the ranking of nonlinear parameters, the signs reflect typical character of nonlinear cirves. For testng of one of these parameters ist value has been changed from 0 to maximum amount, while the other parameters have been equal to 0. Other parameters have following values: normalizing displacement x_0=0.02 m, Q_{ES}= 0.7, Q_{MS} = 7, Q_L=0.4. There are typical values, the normalizing displacement depends on the excitation voltage and ist value is rather high. The time dependence of acoustical pressure and ist fourier transform (spectrum) have been computed. Typical plots of acoustical pressure and ist spectrum are presented in Figure 2. The calculations have been done for resonant frequency (ν=1), bacause for this frequency the value of displacement is highest. The nonlinear distortion have been characterized by the total harmonic distortion coefficient h:

$$h = \frac{\sqrt{p_2^2 + p_3^2 + \ldots}}{\sqrt{p_1^2 + p_2^2 + p_3^2 + \ldots}} \qquad (8)$$

where p_i is i-th spectral component of acoustic pressure. The sensitivity of the nonlinear distortion for the change of parameter u is defined as:

$$S_u^h = \frac{d(\log h)}{d(\log u)} \qquad (9)$$



a)

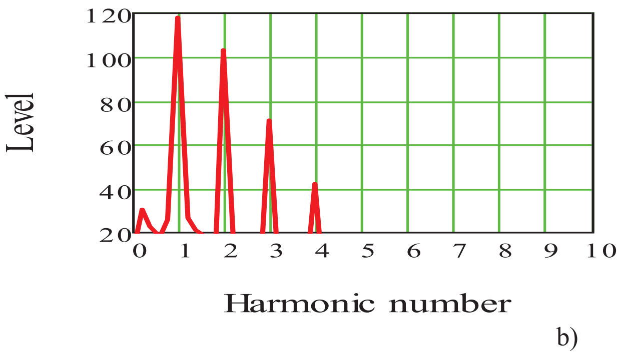


Figure 2. Acoustic pressure (a) and its spectrum obtained from calculations (b).

The higher value of senswitivity means that the harmonic distortion is more sensitive for change of the given parameter. For example, the dependence of log(*h*) on log(*l*1) is presented in Figure 3.

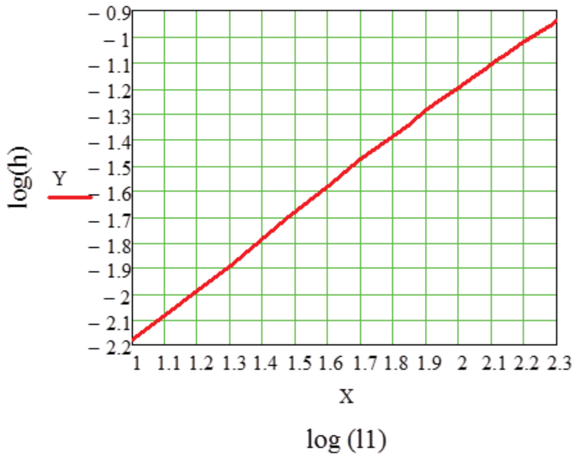


Figure 3. Dependence of the logarithm of harmonic distortion coefficient on nonlinear parameter *l*1, characterizing nonlinear inductance.

The dependence presented in Fig. 3 is almost linear. The slope of the straight line is the sensitivity. Its value is 0.952. It does not depend on other nonlinear parameters. The dependence of harmonic distortion *h* on nonlinear parameter *l*2, which also characterizes nonlinearity of electrical inductance is more complicated. It is nonlinear and it strongly depends on parameter *l*1. The dependence of log(*h*) on log(*l*2) for three values of *l*1 is presented in Figure 4.

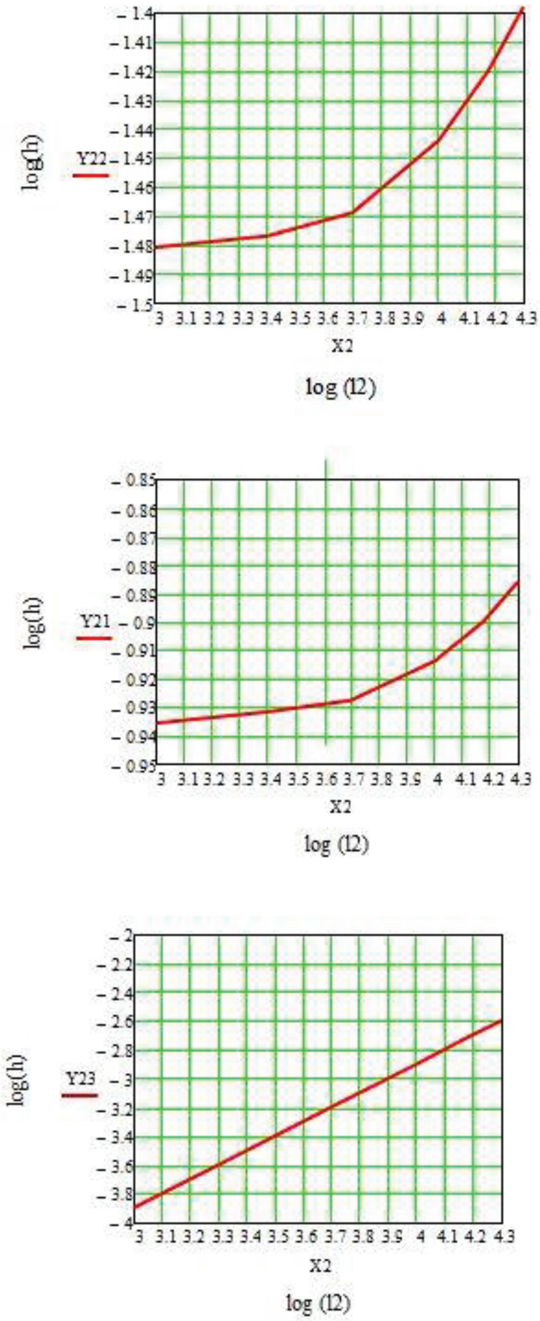


Figure 4. Dependence the logarithm of harmonic distortion coefficient *h* on nonlinear parameter *l*2, for three values of *l*1: a) *l*1=200, b) *l*1=50, c) *l*1=0.

The dependence is linear only for *l*1=0. For this value only odd harmonics appear. The sensitivity for this case does not depend on *l*2 and it is equal to 0.999. For *l*1=50 and for *l*1=200, the sensitivity depends on *l*2 and also on *l*1. For *l*1=50 minimal value of sensitivity is equal to 0.017 for low values of *l*2 (near 0) and the maximal value is equal to 0.098 for high values of *l*2 (near 20000). For *l*1=200 minimal value of sensitivity is equal to 0.011 for low values of *l*2 (near 0) and the maximal value is equal to 0.062 for high values of *l*2 (near 20000). Pay attention that the sensitivity for *l*2 is much smaller than for *l*1, and it decreases when *l*1 increases.

Similar dependencies occur for nonlinear parameters of *Bl* (*b*1 and *b*2) and mechanical stifness (*k*1 and *k*2). The sensitivities of THD-factor for nonlinear patameters *l*1, *b*1 and *k*1 are presented in Table 1. All nonlinear parameters, except the tested one are equal to zero.

Table 1. Sensitivities of THD for nonlinear parameters *l*1, *b*1 and *k*1

Sensitivity of THD for	*l*1	*b*1	*k*1
Sensitivity value	0.952	0.898	1.003

Next parameter, which influences the nonlinear distortion is an amplitude of excitation. In normalized notation it is described by the parameter *x*0. The sensitivity of THD for *x*0 depends on nonlinear parameters *l*1, *b*1 ,*k*1, *l*2, *b*2 and *k*2. For maximal values of these parameters i.e. l_1 = -200 H/m, l_2 = 20000 H/m^2, b_1 = 200 T, b_2 = -20000 T/m, k_1 = 200 N/m^2, k_2 = 20000 N/m^3, the sensitivity $S_{x0}^{h} = 1.022$. It is seems to be interesting to know partial sensitivities of *x*0 for nonlinear parameters *l*1 – *k*2 when only one of them is different than 0. The results are presented in Tables 2 and 3.

Table 2. Sensitivities of THD for *x*0, for nonlinear parameters *l*1, *b*1, *k*1, *l*2, *b*2, *k*2 equal to 0 except one listed in the table

Nonlinear parameter	*l*1= -200	*b*1= 200	*k*1= 200
Sensitivity value for *x*0	0.944	0.99	0.988

Table 3. Sensitivities of THD for *x*0, for nonlinear parameters *l*1, *b*1, *k*1, *l*2, *b*2, *k*2 equal to 0 except one listed in the table

Nonlinear parameter	*l*2= 20000	*b*2= -20000	*k*2= 20000
Sensitivity value for *x*0	1.965	1.765	0.16

Concluding remarks

The analysis of sensitivity of properties of electronic devices for the changes of parameters allows for correct design of devices [1]. The parameters of loudspeakes eqiuvalent circuit influence their properies. When they are different than used for design, they cause that loudspeaker system has other properties than assumed. Some parameters influence more for total characteristics, and for other the properties of loudspeakers are less sensitive. Usually, as the tested characteristics the frequency response is chosen. However, loudspeakers have relatively high level of nonlinear distortion. The ranking of nonlinearities is presented in [2] and [3]. It is an interesting thing to know the sensitivity of nonlinear distortion for change of loudspeaker parameters. This paper is a trial to determine these sensitivities for simple case of single loudspeaker without any enclosure. The generalized Thiele-Small parameters have been chosen for description of loudspeaker properties. When the sensitivity of distortion for these normalized parameters is known, it can be easily determined for physical parameters. Interpretation of the relative sensitivity is such: when its value is lower than unity, the nonlinear distortion is less sensitive for changes of this parameter and when it is greater than unity, the distortion is more sensitive for changes of this parameter. Many simulations have been done. The results of these simulations show that the sensitivity of nonlinear distortion for nonlinear parameters of the first order is close to unity, what means that the distortion increases approximately linearly with change of these parameters. The sensitivity for the second order nonlinear parameters is very low and it cannot be taken into account. The nonlinear distortion is also sensitive for the excitation level. The sensitivity depends on nonlinear parameters – for parameters of the first order it is close to unity, but for the second order it is close to two, except of nonlinearity of the stiffness of suspension. Results confirmed previous consideration that the nonlinear distortion is sensitive for nonlineality of electrical inductance and force factor but it is less sensitive for nonlinearity of stiffness.

Acknowledgment

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