Uncertainties of Airborne Source Characterization using Matrix Inversion

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Introduction

Matrix inversion methods are widely used in structure-borne **T**ransfer **P**ath **A**nalysis (TPA) for indirect force determination based on measurements of acceleration signals and inertance matrices at the interface between the source and the receiving structure [1]. Instead, for airborne TPA, the sound shares are often synthesized using microphone measurements at each side near the source in combination only with airborne sound sensitivities measured with a small loudspeaker. This may be sufficient to estimate the transmission of airborne sound energy from a source to a receiver. However, for a detailed analysis of the contribution of particular structural units of a complex source, a correct definition of source characteristics is one of the most important steps. In the case of airborne sound, matrix inversion methods also allow evaluating the volume velocity $Q(\vec{r},t)$ of a complex source as a superposition of assumed monopoles composing this source. The determination of the volume velocity Q of the complex source results in a vector of Q-values for each of the components. Calculations are based on measured sound pressure signals in the near field around the source and measured transfer functions from the locations of assumed monopoles to the measurement points, using an external volume velocity source (loudspeaker with tube). Although the logic of the method is not new, there are no exact rules concerning the realization of the measurements, the calculation procedure and the limits of applicability of the method. For example, the choices of the number and locations of the measurement points, as well as the assumed number and distribution of monopoles, influence the results. Different authors, who propose the matrix inversion method for the airborne case, usually present only general ideas about the approach not mentioning the chosen parameters [2], or specify the criteria not mentioning the reasons of the choice [3]. The following research aims to find the uncertainties in the results caused by different chosen parameters or by specific parameters of the source itself through the example of a simple model consisting of several loudspeakers. The findings will help to proceed to the exact specification of the method.

Description of the matrix inversion method

The method allows representing the complex sound source as a superposition of K monopoles. The desired volume velocity value of the source is represented as a vector Q_{op} of the monopoles' Q-values (eq. 1)

$$Q_{\mathrm{op}} = \begin{pmatrix} Q_{\mathrm{op},1} \\ \vdots \\ Q_{\mathrm{op},K} \end{pmatrix} \qquad \left[\frac{\mathrm{m}^3}{\mathrm{s}}\right] \qquad (1)$$

where $Q_{\mathrm{op,k}}$ represents a monopole $Q(\vec{r}_{\mathrm{k}},t)$ at the position $\vec{r}_{\mathrm{k}}$.

To find the vector Q_{op} the following data are needed:

1. Sound pressure values generated by the source under operational conditions are measured in L points around the source in the near-field area: vector p_{op}.

$$p_{\mathrm{op}} = \begin{pmatrix} p_{\mathrm{op},1} \\ \vdots \\ p_{\mathrm{op,\,L}} \end{pmatrix} \qquad [\mathrm{Pa}] \qquad (2)$$

The relation between the source volume velocity and generated sound pressure is expressed by a matrix of transfer functions H between all of the assumed monopole positions and each of the measurement points.

$$p_{\mathrm{op}} = H \cdot Q_{\mathrm{op}} \qquad [\mathrm{Pa}] \qquad (3)$$

Using eq. (3) the vector of Q-values can be calculated using matrix inversion (eq. (4))

$$Q_{\mathrm{op}} = H^{-1} \cdot p_{\mathrm{op}} \qquad \left[\frac{\mathrm{m}^3}{\mathrm{s}}\right] \qquad (4)$$

or by means of pseudo-inversion for the case of $K \neq L$ of the transfer function matrix H

$$Q_{\mathrm{op}} = \mathrm{pinv}(H) \cdot p_{\mathrm{op}} \qquad \left[\frac{\mathrm{m}^3}{\mathrm{s}}\right] \qquad (5)$$

2. Transfer functions are determined by placing an external volume velocity source Q_{k} one by one close to each position of the K assumed monopoles while measuring sound pressure values produced by the Q-source in L points:

$$H = \begin{pmatrix} H_{1,1} & \cdots & H_{1,\mathrm{K}} \\ \vdots & \ddots & \vdots \\ H_{\mathrm{L},1} & \cdots & H_{\mathrm{L,K}} \end{pmatrix} \qquad \left[\frac{\mathrm{Pa}}{\mathrm{m}^3/\mathrm{s}}\right] \qquad (6)$$

where

$$H_{\mathrm{l,k}} = p_{\mathrm{l}}/Q_{\mathrm{k}} \qquad \left[\frac{\mathrm{Pa}}{\mathrm{m}^3/\mathrm{s}}\right] \qquad (7)$$

Knowing the operational sound pressure values and transfer function matrix, the pseudo-inversion can be applied to find the desired Q-vector according to eq. (5).

As mentioned in the introduction, there are no exact rules concerning

- the number and positioning of the assumed monopoles,
- the number and the positioning of the measurement points,
- the measurement of the transfer functions.

This paper presents how some of the mentioned parameters of the airborne source affect the prediction of volume velocity.

Measurement setup

As a model of an airborne source an arrangement of several loudspeakers, considered as monopoles, is used in order to study the influence of the assumptions about the number and positions of the unknown monopoles.

In this paper, the influence of the number and positioning of microphones is not considered. Sound pressure values around the source were measured by means of a stationary microphone array consisting of 19 ¼ inch microphones uniformly distributed on the hemispherical measurement surface (blue hemisphere in Figure 1).

All measurements were performed in a full anechoic chamber.

In real applications, such as the source characterization of a combustion engine, engineers usually use a measurement surface having the same shape as the engine itself (microphones at a constant distance to the surface of the engine). In the present case, it is assumed that the source consists of simple components on a hemispherical shape (red dashed hemisphere in Figure 1), i.e., the diaphragms of the loudspeakers were positioned on the surface of the assumed hemisphere.

The chosen simple model also allows for avoiding uncertainties related to the transfer function measurements. The external volume velocity source can be placed directly inside the assumed hemisphere (replacing the loudspeakers by the external Q-source), whereas in the case of a real source like an engine the external Q-source can only be placed close to the positions of the assumed monopoles.

With this simple model, the uncertainties related to the estimation of the volume velocity caused by the choice of the number and the positioning of the assumed monopoles are studied.

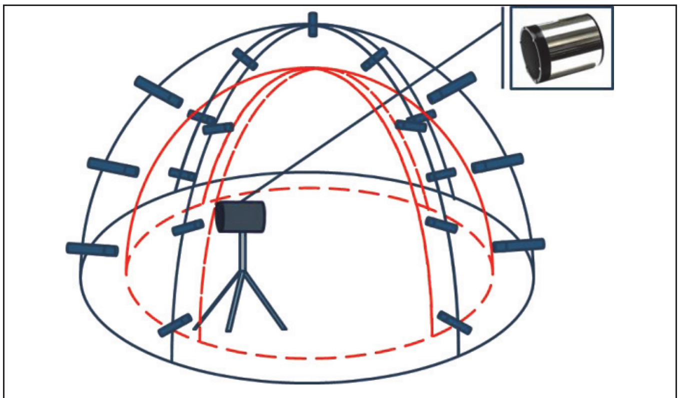

Figure 1: Experimental setup: loudspeaker fixed on a tripod, blue frame: hemispherical setup with 19 ¼ inch microphones, red dashed frame: the assumed shape of the studied source.

Experiments

Case of one component, consideration of the positions of assumed monopoles

First, it was studied how accurate the results are if the simplest model consisting of only one component is used. For this purpose the external volume velocity source was used as the source itself.

It was assumed that the source of interest is not just a component, but a hemisphere (red dashed line in Figure 1) consisting of an unknown number of components. For the representation of the unknown source, 12 monopoles distributed on the hemisphere were assumed to be positioned in front of 12 (out of 19) microphones.

The first operational measurement was carried out with a Q-source placed in one of the positions of the 12 assumed monopoles using a broadband signal (frequency range: 100-2000 Hz). The comparison of the predicted Q-value at the assumed hemisphere and the measured volume velocity produced by the Q-source is shown in Figure 2.

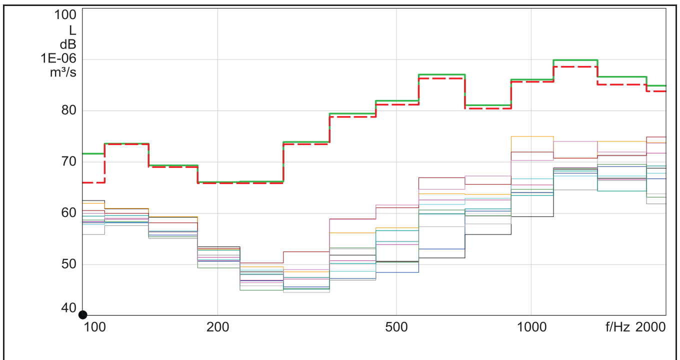

Figure 2: Comparison of the 3rd octave spectra of predicted and measured volume velocity values. Green solid line: measured Q, red dashed line: predicted Q of the assumed monopole at the matching position of the real Q-source, other lines: predicted Q of the assumed monopoles at the other positions.

An almost perfect match between the predicted and measured Q-values can be seen in Figure 2. The predicted Q-values at the other positions are almost negligible for the considered frequency range (ca. 20 dB below the Q-value of the real source). The results obtained for the dominant source will not change if only one monopole at the position of the real source is considered for the prediction.

To see how the results change if a wrong assumption is made about the position of the assumed monopoles, measurements were carried out moving the Q-source along the hemisphere systematically at angles $\alpha \leq 30°$ (Figure 3).

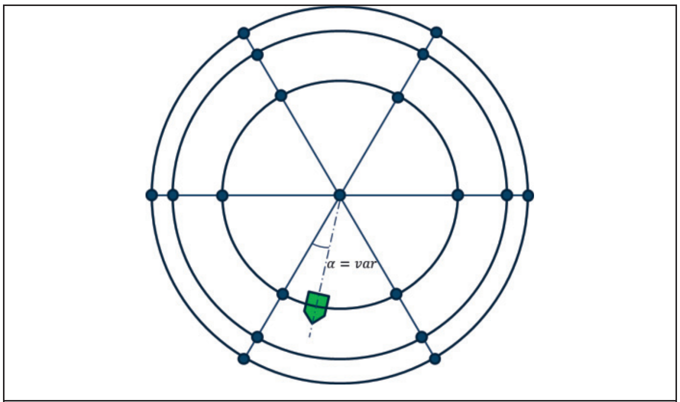

Figure 3: Changing the volume velocity position: top view of the measurement setup. Blue dots: positions of the microphones, green pentagon: position of the volume velocity source.

It was found that within an angle of 5° the obtained Q-values do not differ significantly, but for larger angles, the results cannot be interpreted in the correct way. For $\alpha = 15°$ the one source component cannot be recognized well at frequencies between 800 and 1000 Hz. The results for $\alpha = 30°$ do not show at all a dominant component in the entire frequency range of interest (Figure 4).

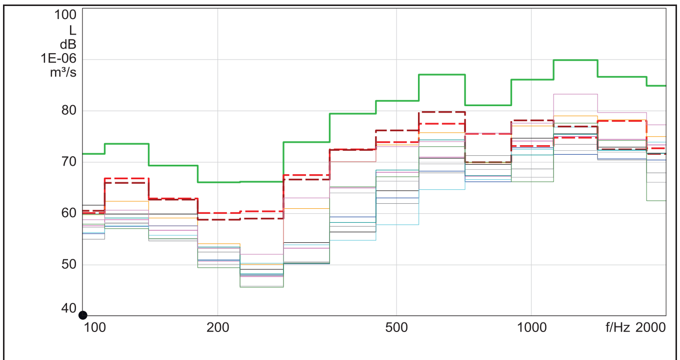

Figure 4: Comparison of the 3rd octave spectra of predicted and measured volume velocity values for the case $\boldsymbol{\alpha = 30°}$. Green bold solid line: measured Q, red bold dashed lines: predicted Q for the two positions around the real source position.

It can be seen that the one simple component is split into 12 assumed monopoles: this does not reflect the real situation. Thus, the large number of assumed monopoles does not provide better results in terms of precision. If only two monopoles close to the real position (blue dots left and right to the green pentagon in Figure 3) of the Q-source are assumed, the simple component splits into two assumed monopoles. Figure 5 shows 3rd octave spectra of the synthesized sound pressure signals generated by the two derived monopoles (at the microphone positions close to the positions of these monopoles, taking into account the transfer functions between each source position and the two microphone positions) compared to the 3rd octave spectra of the sound pressure signals measured under operational conditions. The deviations between the 3rd octave spectra of predicted and measured sound pressure signals are below 3 dB for the frequency range 125–1000 Hz (only two source components), although the exact source could not be found.

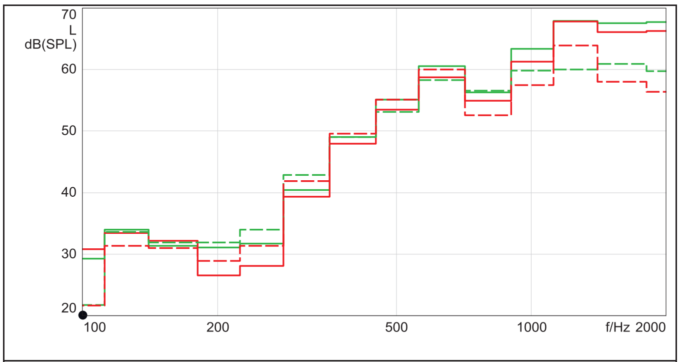

Figure 5: Comparison of the 3rd octave spectra of predicted sound pressure signals obtained from the synthesis of only two found monopoles (dashed lines) and sound pressure signals measured using the volume velocity source shown in Figure 3 (solid lines). Red and green colors indicate two different positions left and right to the real source position.

Case of two components, consideration of the positions of assumed monopoles and consideration of crosstalk

For the next experiment, an arrangement of two loudspeakers (as shown in Figure 1) was used (Figure 6). Different excitation signals for loudspeaker 1 (always at position 1) were used in order to study the influence of crosstalk due to the two loudspeakers: a.) low correlation with signal 2 and b.) high correlation with signal 2.

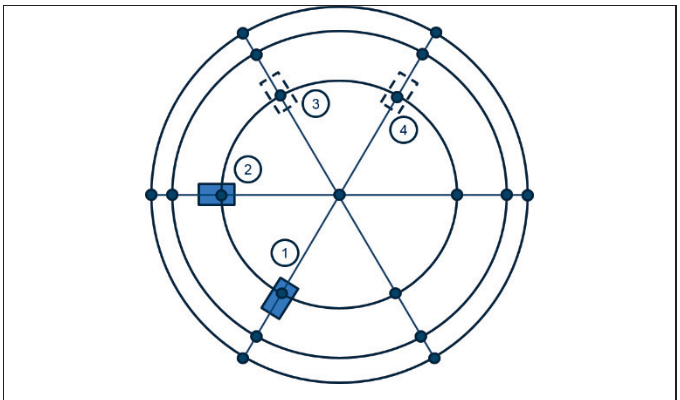

Figure 6: Experimental setup: two loudspeakers fixed on tripods: loudspeaker 1 always positioned at position 1, loudspeaker 2 at positions 2, 3, and 4, successively.

The position of loudspeaker 2 was changed from position 2 to the positions 3 and 4 (successively) always using the same excitation signal (Figures 8-10).

The prediction of the volume velocity Q for loudspeaker 2 shows only a minor influence of the correlation between the excitation signals 1 and 2. However, a stronger influence of the position of the loudspeaker can be observed. For better comparison, the data from Figures 8-10 are shown together for the three different positions in one graph for the case of low correlation (Figure 11) and for the case of high correlation (Figure 12).

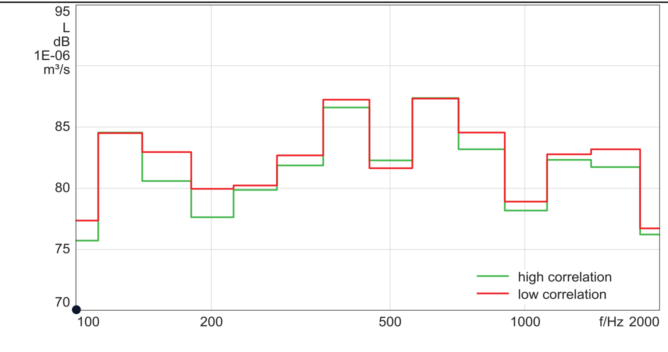

Figure 8: Predicted *Q* for loudspeaker 2 at position 2 for high and low correlation between signal 1 and signal 2.

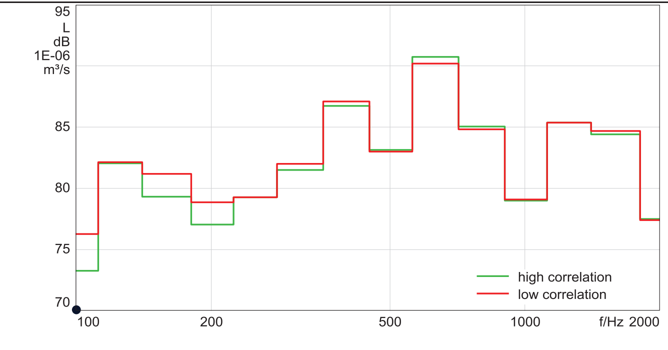

Figure 9: Predicted *Q* for loudspeaker 2 at position 3 for high and low correlation between signal 1 and signal 2.

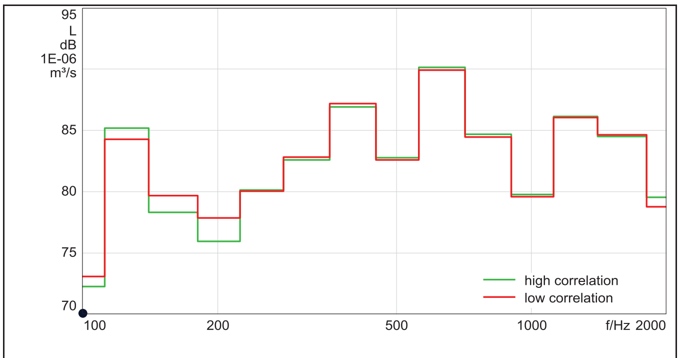

Figure 10: Predicted *Q* for loudspeaker 2 at position 4 for high and low correlation between signal 1 and signal 2.

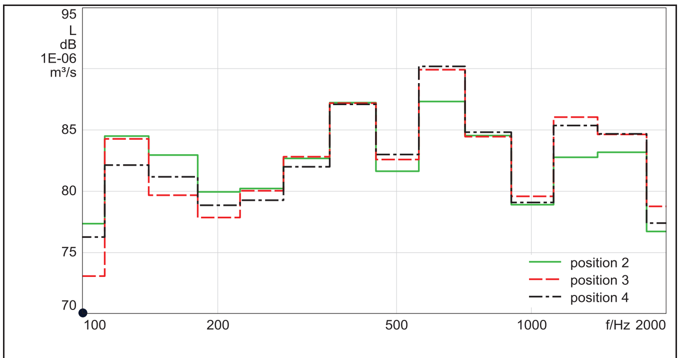

Figure 11: Predicted *Q* for loudspeaker 2 at position 2-4 for low correlation between signal 1 and signal 2.

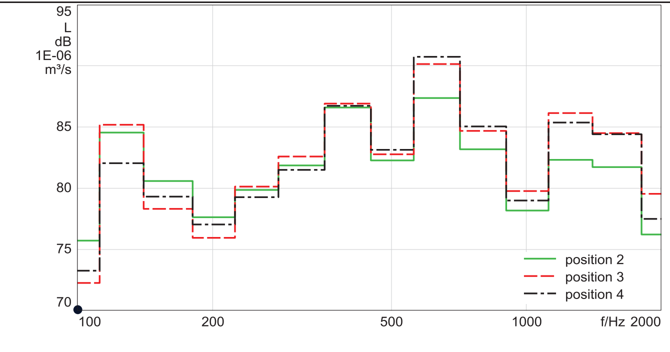

Figure 12: Predicted *Q* for loudspeaker 2 at position 2-4 for high correlation between signal 1 and signal 2.

The predicted volume velocities *Q* for loudspeaker 2 are similar for position 3 and 4. The predicted *Q* differs especially for positon 2 compared to predictions for the other positions. At position 2, loudspeaker 2 is radiating more laterally whereas at positions 3 and 4 it is radiating more frontally. A possible reason for the different predictions could be a nonuniform radiation of the loudspeaker. Further measurements must be performed to determine the angle-dependent source characteristic of each loudspeaker.

Conclusion

The prediction quality of volume velocity for simplified airborne sources, consisting of an arrangement of loudspeakers considered as monopoles, was studied using matrix inversion with respect to the number, position, and the correlation of the assumed sources.

The assumption about the positions of the monopoles has a high impact on the prediction result, even in the simplest case of a single monopole. For an improvement of the prediction quality more transfer functions to assumed source positions are required. An increase of the number of transfer functions will lead to a better prediction only if a denser grid of source points around the real source positions is chosen.

The correlation of the excitations signals used for the two loudspeaker arrangement shows only a minor effect on the results.

The positioning of the source has a stronger effect on the predicted volume velocity. The reason for this finding must be studied in a next step. A possible reason could be that the loudspeakers do not represent ideal monopoles.

References

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