Virtual Analog Modeling of Guitar Amplifiers with Wiener-Hammerstein Models

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Abstract
Virtual analog modeling of guitar amplifiers is an ongoing research topic and its aim is the recreation of an analog system as exactly as possible. A mathematical model is created which can be used reproducibly and independently of aging hardware and temperature dependence. In this work, the popular sound of vintage tube amplifiers can be combined with new digital recording techniques. In this work, the reference system is characterized by measurements. A Wiener–Hammerstein model is used to emulate the analog reference system. They consist of a series connection of an input filter (LTI), a nonlinear transfer-function and an output filter (LTI). Linear and nonlinear blocks of the model are adapted separately and the parameters of the nonlinear block are optimized with the Levenberg–Marquardt method. The proposed optimization routine yields convincing results if the parameters and input signals are chosen carefully. The results are verified by objective error measurements as well as a listening test.

1. Introduction
Virtual analog modeling of guitar amplifiers is a relevant topic for researchers and commercial products. It describes the process of building a digital model out of an analog system. There exist two main approaches when modeling an analog system. The first approach uses all available information about the reference device and is called white box modeling. The schematic of the reference device is analyzed, the characteristics of the nonlinear circuit elements (like diodes, vacuum tubes, transistors, etc.) are measured and a digital model is constructed based on this information [1, 2]. This approach yields convincing results, since the behavior of the circuit itself is calculated but the resulting model is computationally very demanding. A complex circuit with multiple nonlinear elements can not be calculated in real-time without simplifications.
Due to this limitation and the technical expertise needed to construct the digital models for white box approaches audio companies often use a gray box modeling approach, where a generic model is adapted to input/output measurements. Several commercial products make use of filters and nonlinear mapping functions to create distortion in digital products [3, 4]. These functions relate the amplitude of their input signal to their output signal according to a nonlinear function like polynomials or a hyperbolic tangent. In [4] the creation of a virtual analog guitar amplifier is described and it becomes obvious that it is a quite tedious process. In [3] an automated procedure is used to identify any guitar amplifier by input/output measurements. This approach works well if the output of the guitar amplifier is recorded with a microphone in front of the speaker cabinet.
In this work an automated procedure is presented which is based on input/output measurements to adapt a block oriented Wiener–Hammerstein model, consisting of a series connection of input filter, nonlinear block and output filter. The filters are measured with two exponentially swept sine waves of different amplitudes and the parameters of the nonlinear block are optimized with the Levenberg–Marquardt method [5, 6].

2. Digital Model
The Wiener–Hammerstein model was chosen because it represents the fundamental principle of any musical distortion circuit. It is referred to as ‘the fundamental paradigm of electrical guitar tone’ by [4]. The model consists of a series connection of a filter, a nonlinear stage and another filter, as shown by Fig. 1. The first filter \( H_1(z) \) determines which frequencies are going to be distorted. In most guitar amplifiers it is a simple first-order high-pass or band-pass filter. The distortion is introduced in the nonlinear block, adding harmonic overtones to each fundamental frequency. The second filter \( H_2(z) \) determines how the distortion sounds by shaping the overtones created by the nonlinear block. This model seems very simple, especially since a real guitar amplifier usually features a pre-amplifier and a power-amplifier which both introduce distortion and have a filter between them. Previous research indicates however, that a more accurate model of a guitar amplifier is not inherently better when using system identification approaches [8].
The nonlinear block is the same as presented in [8, 9]. It is based on a nonlinear mapping function and has been extended with an optional dry/wet mixing stage, a signal-dependent bias-point shifting stage as well as pre- and post-gains.

3. System Identification
This section gives a detailed explanation of the necessary steps to adapt the digital model to the measurements. First, the measurement setup is explained. Afterwards the used input signals and optimization methods for the identification of linear and nonlinear subsystems are explicated.
3.1 Hardware Measurements

All guitar amplifiers were measured using a USB audio interface (RME Fireface UC). The output of the audio interface was calibrated to directly reproduce a digital amplitude, meaning that a sine wave with a digital amplitude of ±1 corresponds to an analog sine wave with an amplitude of ±1 V.

The first output of the interface is connected to the guitar amplifier and the output of the amplifier is connected to a power attenuator or ‘dummy load’, which is an equivalent impedance filter to a guitar cabinet whose electronic components are dimensioned for the high voltages and currents of a guitar amplifier output. The signal from the dummy load is picked up and fed to the first input of the audio interface as depicted by Fig. 2. Additionally a loop-back connection from another output to another input is made by simply connecting the output to the input. All signals passing through the amplifier are simultaneously send via the loop-back connection to measure and eliminate the influence of the audio interface. The loop-back signals are also used as the input signals for the system identification. Another advantage of using the loop-back connection is that all signals are automatically time-aligned.

The dummy load was constructed according to [10]. Its circuit diagram is shown in Fig. 3. The circuit electrically behaves like a real-world speaker cabinet but produces no sound which is helpful when measuring guitar amplifiers with a saturating power amplifier because a real cabinet would produce considerable sound pressure levels.

3.2 Linear Subsystems

All linear subsystems were measured with exponentially swept sine waves according to [11]. The input and output filters are identified with two measurements. The first measurement is taken with a very low amplitude which has to be chosen carefully because no distortion must occur. Depending on the gain of the reference amplifier the amplitudes ranged from 0.1 mV to 10 mV to have a reasonable SNR. The resulting frequency response \( H_{\text{total}}(z) \) contains the influence of all filters of the reference device. Afterwards the same measurement is repeated with a high amplitude of 1 V. The resulting frequency response only contains the influence of the filters after the distortion. This is depicted in Fig. 4: a signal is filtered by \( H_1(z) \) can then be calculated by dividing both measured frequency responses. Afterwards all filters of the digital model have been measured and can directly be used.

In view of a possible real-time implementation of the digital model the measured frequency responses were approximated with peak filters, a high frequency shelving and a low frequency shelving filter, designed according to [12]. This reduces the number of multiplications and additions needed compared to a finite impulse response filter with a reasonable frequency resolution.

3.3 Nonlinear Block

A signal flow graph of the nonlinear block is shown in Fig. 5. The distortion is introduced by the mapping function \( m(x) \) which is based on a hyperbolic tangent and was already used in [7–9]. Additional enhancements of the nonlinear block are a side chain envelope detector directly in front of the mapping function which simulates a signal dependent bias point shift which is occurring in tube amplifiers and an optional blend stage mixing the clean and the distorted signal.

After the linear subsystems have been identified, the parameters of the nonlinear block are optimized. Since the Levenberg–Marquardt method is gradient-based, it is necessary to have a good initial parameter set. Otherwise, the optimization process might converge into a local minimum and the identification will be unsuccessful. For this reason a grid search is carried out for the parameters \( g_{\text{pre}} \) and \( g_{\text{post}} \) because they have the most influence on the model output.
The final step is the parameter optimization with a real-world guitar input signal. The cost-function is based on psycho-acoustical observations. The signal flow graph for calculating the cost-function is shown in Fig. 6. The input signals for this method are the reference signal and the output of the optimized digital model (which should sound the same). They are both transformed into the time-frequency domain by short-time Fourier transform (STFT). Then redundant information from the Fourier transform is removed and the magnitude spectrogram is calculated by computing the absolute value of the complex matrix. Afterwards, the spectrogram of the residual (RES) is calculated by subtracting the model spectrogram from the reference spectrogram. Before calculating the final score, the frequency bins of the Fourier transform are pooled by calculating the mean value of the bins corresponding to a certain frequency region according to a semi-tone spectrum starting at 27.5 Hz which is the lowest note that can be played on a standard tuning 5-string bass guitar. The final score is calculated by summing all error values.

4. Results

Evaluating how well the model performs is no trivial task. Objective scores like the root-mean-square error do not necessarily represent the human perception of the error because they are sensitive to small phase shifts between the optimized digital model and the analog reference system which are not detectable for a human listener. For this reason an objective similarity score was used, based on the cost-function which was used to adapt the models. Additionally a listening test was performed because the performance of the model should also be rated by human test subjects.

The similarity score is nearly identical to the cost-function. Except that the psycho-acoustical frequency bin pooling is performed for the reference spectrogram as well and the final score is calculated by dividing the sum of the residual by the sum of the reference to achieve a relative error.

Several amplifiers were modeled in different settings. The tone section of each amplifier was set to a neutral value (all knobs in the 12 o’clock position) and the ‘Gain’ and ‘Volume’ knobs were altered from a low value (9 o’clock position) to medium value (12 o’clock position) to high value (3 o’clock position). A high gain value results in pre-amplifier distortion and a high volume value results in power-amplifier distortion. The settings are marked with acronyms, e.g. ‘HGMV’ meaning ‘high gain medium volume’. Additionally to the similarity score the error to signal ratio (ESR) is given, which relates the energy of the time-domain error to the energy of the reference signal. The scores are shown in Tab. 1. The error to signal ratio is not very well suited for evaluation because the results do not represent the human perception of differences between model and reference. If the ESR has a low value, however, the results are good.

A listening test was conducted to see how well the adapted models perform for a human test subject. The listening test aimed at rating the similarity of the adapted model in relation to the analog reference device. The test subjects were presented with a reference item and two test items. The items should be rated according to how similar they sound to the reference, where 100 represents no detectable difference between the item and the reference and 0 represents a very annoying difference. The used similarity scale is shown in Tab. 2.

One of the test items was a hidden reference, which was the same audio-file as the reference item, the other item was the output of the digital model. The test had 35 participants from which only 19 were used for the final evaluation. The other participants were not able to detect the hidden reference or always rated the hidden reference with scores below 80 and were therefore excluded from the evaluation.

The results of the listening test are visualized in Figs. 7 – 9. The bar displays the 50% quantile (median) for each item. The lower and upper bounds of the box represent the 25% quantile or the 75% quantile respectively. Outliers are depicted as +.

The general trend for each amplifier is that the results get worse if the nonlinearity of the reference system increases (listening test and objective scores) but no ampli-

![Table 1: Objective scores for evaluation of the optimized digital model.](image)

<table>
<thead>
<tr>
<th>Amp (Setting)</th>
<th>Pick-Up</th>
<th>Similarity</th>
<th>ESR</th>
</tr>
</thead>
<tbody>
<tr>
<td>01 Bassman 100 (Clean)</td>
<td>HM3</td>
<td>0.0358</td>
<td>0.0100</td>
</tr>
<tr>
<td>02 Bassman 100 (Clean)</td>
<td>SC</td>
<td>0.0058</td>
<td>0.0077</td>
</tr>
<tr>
<td>03 Roost SR22 (HGMV)</td>
<td>HM2</td>
<td>0.1355</td>
<td>0.0296</td>
</tr>
<tr>
<td>04 Roost SR22 (HGMV)</td>
<td>HM3</td>
<td>0.1838</td>
<td>0.0555</td>
</tr>
<tr>
<td>05 JCM 900 (LGMV)</td>
<td>SC</td>
<td>0.0423</td>
<td>0.0540</td>
</tr>
<tr>
<td>06 JCM 900 (LHGV)</td>
<td>HM3</td>
<td>0.3172</td>
<td>1.7936</td>
</tr>
<tr>
<td>07 JCM 900 (MGHV)</td>
<td>BM</td>
<td>0.2131</td>
<td>0.2846</td>
</tr>
<tr>
<td>08 JCM 900 (MGMV)</td>
<td>HM3</td>
<td>0.2902</td>
<td>0.5919</td>
</tr>
<tr>
<td>09 JCM 900 (HGMV)</td>
<td>HM3</td>
<td>0.3942</td>
<td>0.9386</td>
</tr>
<tr>
<td>10 JCM 900 (HGHV)</td>
<td>HM2</td>
<td>0.4175</td>
<td>0.7413</td>
</tr>
</tbody>
</table>

![Table 2: Listening test rating score with corresponding identifier.](image)

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imperceptible</td>
<td>80 – 100</td>
</tr>
<tr>
<td>Minor Differences</td>
<td>60 – 80</td>
</tr>
<tr>
<td>Differences</td>
<td>40 – 60</td>
</tr>
<tr>
<td>Major Differences</td>
<td>20 – 40</td>
</tr>
<tr>
<td>Annoying Differences</td>
<td>0 – 20</td>
</tr>
</tbody>
</table>
The similarity score corresponds quite well to the results of the listening test, for example when comparing the scores for the Madamp A15Mk2 (Fig. 8) to the results of the listening test. If the similarity score is below 0.1, differences between model and reference signal are nearly imperceptible, as is shown by the results of the listening test. Each item which has a similarity score below 0.1 is rated as ‘imperceptible’ in the listening test. The listening test was performed without an anchor, which is an intentionally bad test item usually used to scale the subjective range for each test subject. This can explain the relatively large spread of the results. In future work an anchor should be used for the listening test.

5. Conclusion

A Wiener–Hammerstein model has been used to emulate analog guitar amplifiers with system identification methods. A psycho-acoustically motivated cost-function was used to calculate the error between reference device and digital model and a similarity score, based on the cost-function, was used to objectively rate the quality of the proposed method. The results are very convincing and although the model does not recreate the reference system perfectly, it comes very close. A listening test was performed and on average no model was rated worse than ‘minor differences’.

References