Authentic Modeling of Guitar Amplifiers and Effect Boxes

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Introduction

Amp modeling, the emulation of nonlinear guitar amplifiers or effect pedals by using digital signal processing algorithms, is a topic of continuous interest. The common approaches can be roughly classified into two categories. On the one hand there is the black box modeling, in which only the quantities that can be measured at the system inputs and outputs are available and used [1]. On the other hand, we speak of white box modeling if knowledge of the interior (usually the schematics) is required for development. The most common methods are wave digital filters [2] and state-space representation [3, 4].

This paper discusses the nonlinear state-space method, shows examples of simplification, and how nonlinear components such as discrete semiconductors or vacuum tubes are integrated.

Nonlinear State-Space Models

The state-space representation is an accepted tool to describe arbitrary physical systems. By defining suitable internal states, a system can be modeled as a set of input, output and state variables that are related by first-order differential equations. For electric circuits the number of required state variables is equal to the number of independent energy storage elements, i. e. capacitors or inductors.

Continuous-Time Description

A general, nonlinear state-space description is the basis of our model. As proposed in [3], we define a set of first-order differential equations,

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{C}\boldsymbol{i}(\boldsymbol{v}) \quad (1)$$
$$\boldsymbol{y} = \boldsymbol{d}\boldsymbol{x} + \boldsymbol{e}\boldsymbol{u} + \boldsymbol{f}\boldsymbol{i}(\boldsymbol{v}) \quad (2)$$
$$\boldsymbol{v} = \boldsymbol{G}\boldsymbol{x} + \boldsymbol{H}\boldsymbol{u} + \boldsymbol{K}\boldsymbol{i}(\boldsymbol{v}) \quad (3)$$

with state variables $\boldsymbol{x}$ and $\dot{\boldsymbol{x}}$, inputs $\boldsymbol{u}$ and outputs $\boldsymbol{y}$. $\boldsymbol{A}$ is called the state-, $\boldsymbol{B}$ the input-, $\boldsymbol{d}$ the output- and $\boldsymbol{e}$ the feedthrough-matrix. The matrices $\boldsymbol{C}$, $\boldsymbol{f}$ and $\boldsymbol{K}$ enhance the representation and link the voltages $\boldsymbol{v}$ and currents $\boldsymbol{i}$ of all nonlinear circuit elements.

For typical circuits $\boldsymbol{y}$ will be scalar (one output) while $\boldsymbol{u}$ will be a vector with entries for the input signal and all supply voltages. The states $\boldsymbol{x}$ commonly are the voltages across the capacitors and currents through the inductors, respectively. Please note that equation (3) is implicit.

Discrete-Time Description

For the implemetation as a DSP algorithm a discretization of the system description has to be done. The equivalent, canonicalized discrete-time system is given by

$$\boldsymbol{x}(n) = \bar{\boldsymbol{A}}\boldsymbol{x}(n-1) + \bar{\boldsymbol{B}}u(n) + \bar{\boldsymbol{C}}\boldsymbol{i}(\boldsymbol{v}(n)) \quad (4)$$
$$y(n) = \bar{\boldsymbol{d}}\boldsymbol{x}(n-1) + \bar{\boldsymbol{e}}u(n) + \bar{\boldsymbol{f}}\boldsymbol{i}(\boldsymbol{v}(n)) \quad (5)$$
$$\boldsymbol{v}(n) = \bar{\boldsymbol{G}}\boldsymbol{x}(n-1) + \bar{\boldsymbol{H}}u(n) + \bar{\boldsymbol{K}}\boldsymbol{i}(\boldsymbol{v}(n)) \quad (6)$$

with discrete state vectors $\boldsymbol{x}(n)$, $\boldsymbol{x}(n-1)$, inputs $\boldsymbol{u}(n)$ and nonlinear currents $\boldsymbol{i}(\boldsymbol{v}(n))$. The transfer of the system matrices is done by means of conversion equations. For example, $\bar{\boldsymbol{A}} = \left(\frac{2}{T}\boldsymbol{I} + \boldsymbol{A}\right)\left(\frac{2}{T}\boldsymbol{I} - \boldsymbol{A}\right)^{-1}$ includes a trapezoidal rule discretization and a canonicalization. Similar conversion equations are available for all matrices. A detailed derivation and further discretization techniques are discussed in [3].

Nonlinear Circuit Elements

Active circuit elements like diodes, transistors, and (still relevant for music electronics) vacuum tubes have a nonlinear relationship between voltages and currents. In our system the integration of such components is made possible by equations (3) and (6) and the attached terms in the state-space equations.

The term $\boldsymbol{i}(\boldsymbol{v})$ represents the nonlinear current-voltage relationship, which mathematically describes the currents of all the nonlinear circuit elements as a function of their terminal voltages. The description is memoryless and therefore static.

Equation (6) can be divided into a part $\boldsymbol{p}(n)$, which depends only on the states and inputs, and a part $\bar{\boldsymbol{K}}\boldsymbol{i}(\boldsymbol{v}(n))$, which expresses the reaction of the currents on $\boldsymbol{v}(n)$ and has no dependency on $\boldsymbol{x}(n)$ and $\boldsymbol{u}(n)$, following

$$\boldsymbol{v}(n) = \underbrace{\bar{\boldsymbol{G}}\boldsymbol{x}_c(n-1) + \bar{\boldsymbol{H}}\boldsymbol{u}(n)}_{\boldsymbol{p}(n)} + \bar{\boldsymbol{K}}\boldsymbol{i}(\boldsymbol{v}(n)), \quad (7)$$
$$0 = \boldsymbol{p}(n) + \bar{\boldsymbol{K}}\boldsymbol{i}(\boldsymbol{v}(n)) - \boldsymbol{v}(n). \quad (8)$$

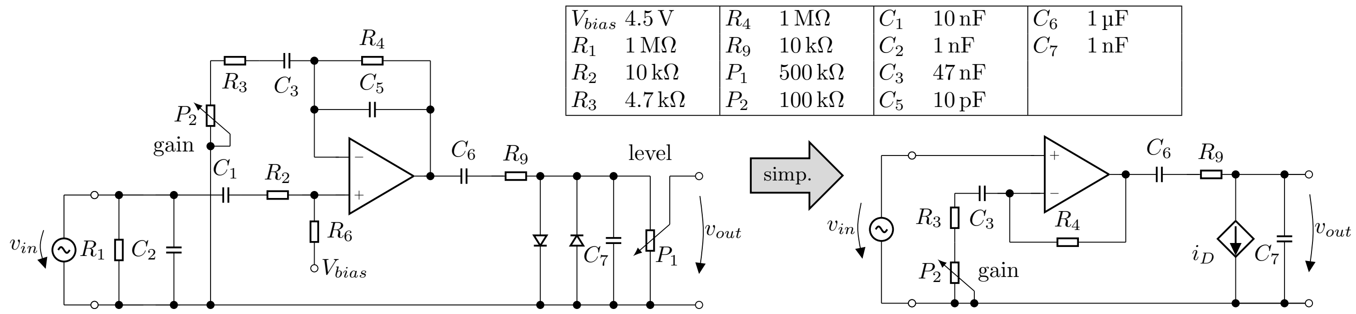
This root-finding problem must be solved for each sample, which can be done either during runtime or in advance in form of look-up tables.

Applications

In the following, the approach will be applied to two example circuits, one with semiconductors and one with vacuum tubes.

Nonlinear Model of a Distortion Pedal

The analyzed circuit (schematics given in Figure 1) only needs a few components to create a singing guitar sound, rich in harmonic distortions. The circuit shown here was



V_{bias}	4.5 V	R_4	1 MΩ	C_1	10 nF	C_6	1 µF
R_1	1 MΩ	R_9	10 kΩ	C_2	1 nF	C_7	1 nF
R_2	10 kΩ	P_1	500 kΩ	C_3	47 nF		
R_3	4.7 kΩ	P_2	100 kΩ	C_5	10 pF		

Figure 1: Full schematic of the distortion circuit (left), simplified circuit (right) and component values.

essentially copied from the famous *Distortion +* effect box (built by `MXR` since the mid-1970s), but there are many other successful guitar pedals with a very similar structure.

For our experiments, this circuit is particularly interesting because beneficial simplifications are possible. However, there are also some pitfalls and limitations that make a discussion worthwhile.

Circuit Analysis

Core of the circuit is an opamp of type 741 (a property to be discussed later) wired as a non-inverting amplifier. The gain is determined by R_4, P_2 and R_3 and is adjustable with P_2 (important: rev. log. taper) in the range of approx. 3 ... 46 dB. The antiparallel diodes strongly limit the output amplitude and are mainly responsible for the distortions. Germanium types have been used primarily, in particular type 1N270 or 1N34.

Simplifications

The circuit shown here can be further simplified:

1. Assuming the output is not considerably loaded, potentimeter P_1 can be replaced by a scaling factor.
2. R_1, C_2, C_5 have negligible influence and can be dropped.
3. Same for R_1 and C_5, assuming an ideal opamp.
4. No need for V_{bias} as an input, drop R_6 and V_{bias}.
5. Combine both diodes to one nonlinear element i_D.

The resulting simplified circuit is given in Figure 1 on the right.

Discussion: The assumption of an ideal operational amplifier has greatly simplified modeling. However, it should be noted that this is not generally valid without further ado and, in particular, is not permitted for all operating conditions with this circuit. The 741 family is technically obsolete, the low gain-bandwidth product and slew-rate cause the chip to affect the sound, especially when the gain potentiometer is close to its maximum. In our case, the biggest impediment is that the device is powered by a 9 V battery, which limits the amplitude above the operational amplifier to a rather small maximum value. Whenever the input signal multiplied by the gain exceeds this threshold, the opamp clips hard and further distortion occurs. This condition can be achieved with standard guitar pickups and a gain adjustment in the last third of the control knob.

In this paper we want to content ourselves with the distortions caused by diodes and remain with the assumption of an ideal opamp. The author prefers the sound below this threshold. For the emulation of clipping opamps please refer to [5, 6].

State-Space Model and Nonlinearity

Silicon diodes can best be descripted by the Shockley equation. For germanium diodes, characterized by a smoother current rise, the expression

$$i_D = \begin{cases} I_s v_D^{\xi}, & \text{if } v_D > 0 \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

with saturation current I_s, voltage across diode v_D and power constant ξ has proven to be suitable. Germanium diodes are temperature-sensitive and often have a high dispersion in the characteristic curves. A selection of proper exemplars can be useful, the sound can vary. For this reason, one must not blindly rely on the specifications in the data sheets for modeling.

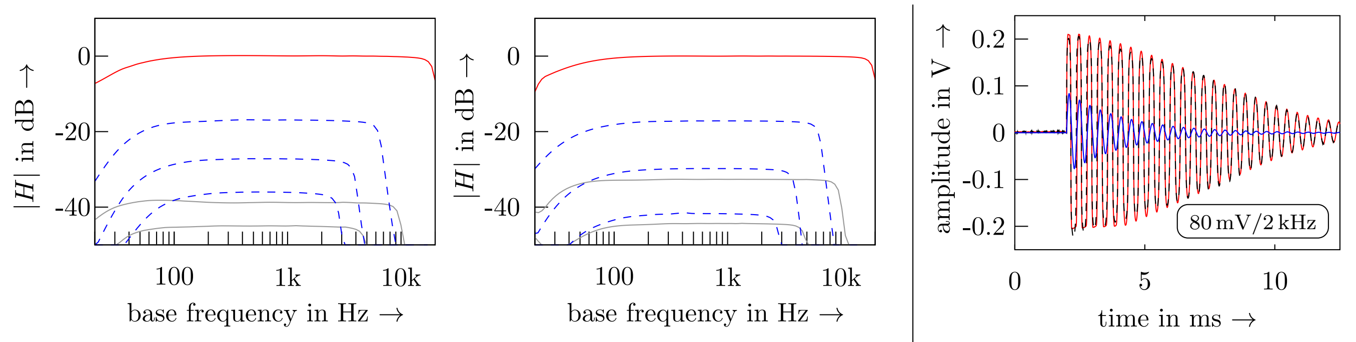
To be able to understand the common variants of the *Distortion +* circuit, a large number of relevant diode types was measured point by point. Thereafter, the parameters were adapted to the proposed equation. Table 1 summarizes suitable parameters for equation (9).

Table 1: Parameters for selected Ge-Diodes.

Type	Manufacturer	I_s	ξ
1N34A	BKC Int. Electr.	0.0180174	2.6429896
1N60A	'cheap chinese'	0.3872400	4.8897675
1N270	Clevite	1.0300958	5.2511100
1N949	BKC Int. Electr.	2.3792045	5.7149062
AA113	ITT	1.9103705	5.8046535
OA7	Philips	1.8167276	5.3653618
OA1150	Tungsram	0.0085940	2.2691389

Results

In addition to a listening test, various comparisons were made between analog reference and digital model to determine whether the simulation was successful. Relatively straight-forward is the comparison of time signals,



(a) Measurement, $v_{in} = 100\,\text{mV}$ (b) Simulation, $v_{in} = 100\,\text{mV}$ (c) sine burst excitation

Figure 2: Frequency response (red), even (gray) und odd harmonics (blue, dashed) of the distortion pedal with diodes of type 1N34A for *gain*-setting 15 o'clock (a, b) and burst signal reaction (c).

where especially *sine bursts* have proven to be suitable. Burst signals allow conclusions to be drawn on transients, temporary operating point shifts (e.g. blocking distortions) and DC parts. In addition, the harmonic content of the output was computed and smoothed using the approved exponential sweep technique [7, 8]. The curves are given in Figure 2. As the curves show, there is a high similarity to the reference. The same is true for the sound reproduction. The result is therefore entirely satisfying.

Nonlinear Model of Cascaded Triode Stages

The second example is a high-gain tube preamplifier that was found in a modern, small combo amplifier.

Circuit Analysis

The schematics in Figure 3 show a two-stage preamplifier with triodes of type 12AX7. Both triodes sections are operated as common-cathode amplifiers and are AC-coupled. The output voltage of the first tube is fed to the second triode via a voltage divider.

A special feature is the capacitive bridging of the anode resistor R_4 by capacitor C_3 causing an additional low-pass filtering with a cutoff frequency of about 10 kHz. The input stage is loaded by the following stage, it must be assumed that the grid voltage gets temporarily positive under normal operating conditions. For this reason, decoupling is not permitted and both stages must be simulated together. The RC-network consisting of C_4, R_9, R_{10}, R_{11} and C_5 emphasizes the output in the for guitar signals relevant band ranging from 10 Hz to 10 kHz.

State-Space Model and Nonlinearity

In the past few years, numerous models for electron tubes have been proposed, explicitly considering the operation in saturation [9, 10, 11]. In this paper we stay with the previously proposed physically motivated triode model [3], where cathode current I_k and grid current I_g are defined by

$$I_k = K\left(\log\left(1+\exp\left(C_a\left(\tfrac{1}{\mu}V_a+V_g\right)\right)\right)\frac{1}{C_a}\right)^{\gamma} \quad (10)$$

$$I_g = K_g\left(\log\left(1+\exp\left(C_gV_g\right)\right)\frac{1}{C_g},\right)^{\xi} + I_{g0}, \quad (11)$$

with anode voltages V_a, grid voltage V_g, perveances K and K_g, exponents γ, ξ, current constant I_{g0} and adaption factors C_a, C_g. Taking the relevant parasitic capacitances C_{ag1} and C_{ag2} [3, 9] into account a state-space model of the order 7 results. With two triode systems involved, the nonlinearity is 4th order.

Results

The output signals of the circuit are shown in Figure 4. It can be seen that with a comparatively small excitation of 100 mV the amplification is still approximately linear, but at higher input amplitudes significant distortion occurs. This observation is confirmed by the evaluation of harmonic distortions. For an input signal with amplitude 1 V, the output signal contains a large amount of both even and odd harmonics. Simulations and measurements show a good match. The simulation is thus to be considered successful.

Conclusion

With the nonlinear state-space representation an approach for the systematic modelling of arbitrary circuits was presented. The general applicability of the approach was evaluated using representative circuits of guitar amplifiers or effect devices. Furthermore, a simple model for germanium diodes was presented. For the simulated circuits good to very good measurements can be obtained and it is confirmed that the resulting models preserve the tonal character of the reference.

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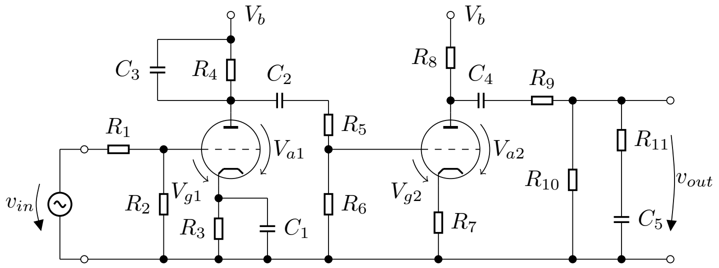
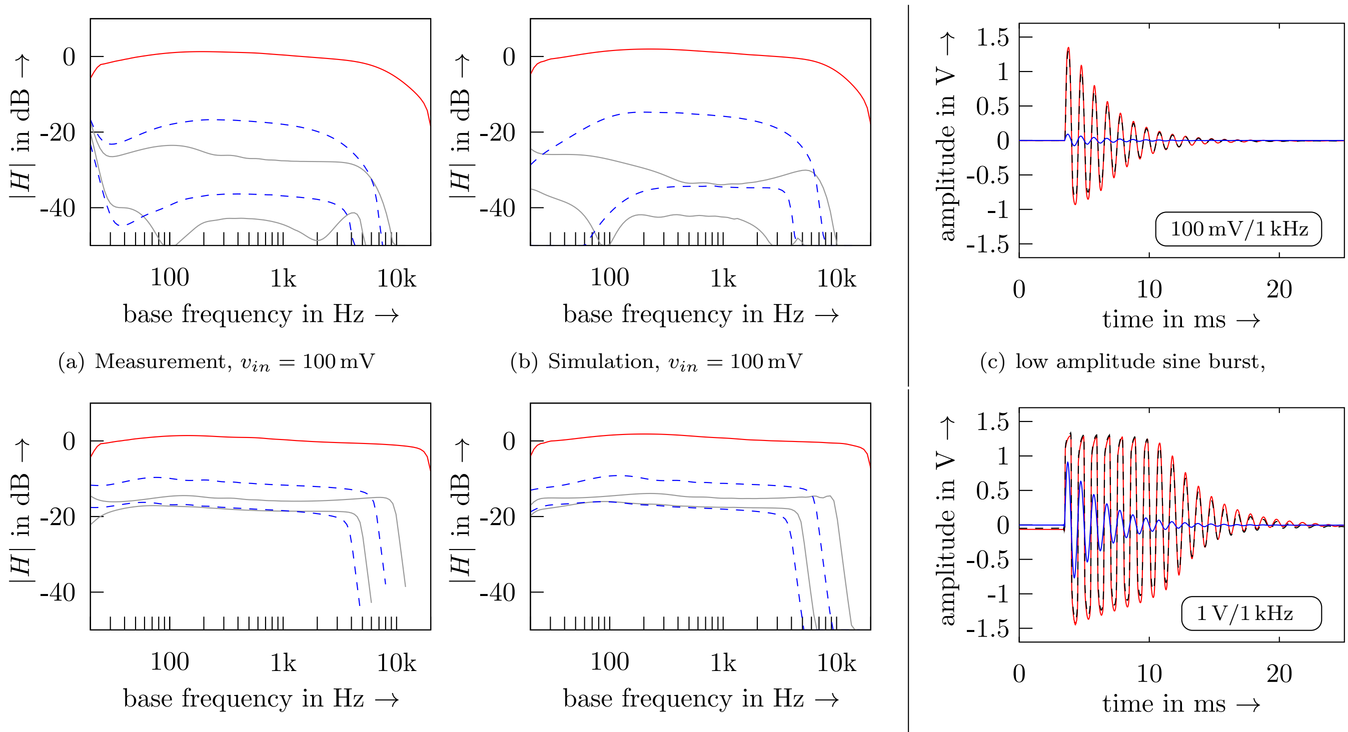


V_b	259 V	R_9	1 MΩ
R_1	10 kΩ	R_{10}	16.5 kΩ
R_2	1 MΩ	R_{11}	33 kΩ
R_3	1.8 kΩ	C_1	2.2 µF
R_4	220 kΩ	C_2	22 nF
R_5	470 kΩ	C_3	220 pF
R_6	470 kΩ	C_4	22 nF
R_7	1 kΩ	C_5	2.2 nF
R_8	100 kΩ		

Figure 3: Schematic of the two-stage preamplifier and component values.



(a) Measurement, $v_{in} = 100\,\text{mV}$
(b) Simulation, $v_{in} = 100\,\text{mV}$
(c) low amplitude sine burst,
(d) Measurement, $v_{in} = 1\,\text{V}$
(e) Simulation, $v_{in} = 1\,\text{V}$
(f) high amplitude sine burst

Figure 4: Frequency response (red), even (gray) und odd harmonics (blue, dashed) of the two-staged preamp (a, b, d, e) and burst signal (c, f).

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