A Simple Model for Estimation of Sound Absorption of Perforated Liners with Bias Flow

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Introduction

Due to their absorption properties as well as simple design and mechanical robustness, perforated liners are frequently used as sound absorbers in ducts with mean flow of gas. The mechanism of sound absorption is based on the conversion of sound energy into kinetic energy of vortices, which are formed at the openings of the perforations in the presence of mean flow, and eventually into heat. With regard to the surface of the liner, mean flow can be bias, grazing, or combined grazing-bias flow.

Examples of applications of acoustic liners include bypass ducts of aircraft engines and exhaust mufflers in vehicles with internal combustion engines. We consider liners with low Mach number bias flow, which are used in combustors of aircraft engines and gas turbines [1]. Grazing flow effects can be neglected in them when the ratio of bias and grazing flow velocity is larger than 0.3.

One of the first models for estimation of sound absorption by liners with mean flow was given by Howe [2]. The model assumes an infinitesimally thin perforated screen, low Mach number value of the flow, and small perforation ratio (ratio of total surface of the openings and surface of the screen). Interaction between the perforations is neglected. The model was extended later to include the resonance caused by a rigid backing wall placed behind the screen [3]. The solution demonstrated potentials of bias flow liners for sound absorption, even in the absence of conventional porous absorbing materials. However, the analytical treatment was rather involved, which motivated its simplification in [4]. In addition to this, Jing and Sun [5] provided a correction for finite thickness of the screen (perforated plate).

Betts [6] considered interaction between perforations for high perforation ratios, as well as non-linear effects and high subsonic mean flow velocities. His model also includes contraction of the jet at the perforations (vena contracta effect) by the constant value of discharge coefficient. Durrieu et al. [7] considered a model of orifice based on a quasi-steady theory and investigated the dependence of vena contracta factor and end correction of the orifice length on the Mach number value, perforation ratio, geometry, and Reynolds number value. They showed that vena contracta factor can appreciably increase with Mach number value and that it is larger for a perforated plate than for a single isolated hole with the same open area ratio. It is also argued that the values depend on the details of geometry, in particular sharpness of the edges of the orifice. In addition to this, they reported end correction values at low frequencies which are less than one half of the value in the absence of the mean flow.

The existing models provide insight into the sound-vortex interaction at the perforations and allow prediction of sound absorption of liners and their optimization. However, they seem to be accurate only for very low Mach number and Strouhal number values, when the resonance dominates the absorption characteristic of the liner, or for moderately high Mach number values, when the absorption is broadband[1]. This limits their applicability for design and optimization of liners.

Following Durrieu et al., Phong and Papamoschou [8] presented a model for calculation of sound transmission through a perforated plate. We extend this model to be used for estimation of sound absorption of perforated liners in the presence of low Mach number bias flow. We test it on a cylindrical geometry of a circular duct with a liner at the walls. The calculated values are compared with the simplified Howe's model corrected by Jing and Sun, as well as the experimental data from literature.

Howe's model

The perforated plate of the liner is assumed to be rigid and motionless. The acoustically compact perforations (we consider only circular perforations) are large enough so that the influence of viscosity is negligible at high Reynolds numbers everywhere except very close to the edges of the perforations. Consequently, micro-perforated plates are not covered by the analysis here. All perforations are assumed to have equal size and shape. In Howe's model, they are characterized by the value of Rayleigh conductivity, which is defined as:

$$K_R = -\frac{j\omega\rho_0 Q}{\Delta p}. \qquad \text{[m]} \qquad (1)$$

Here, $Q = \hat{Q}e^{j\omega t}$ represents unsteady volumetric flow with complex amplitude $\hat{Q}$ through a perforation at angular frequency ω and Δp is pressure difference at the two sides of the perforation. Steady density is constant in the following, $\rho_0 = 1.2\text{kg/m}^3$.

In the absence of mean flow, Rayleigh conductivity is closely related to the effective length of the perforation $l_e = l + 2\delta_0$ (l denotes thickness of the plate, see Figure 1, and δ_0 is end correction) by simple relation: $K_R l_e = a^2\pi$, with a denoting radius of the circular perforation. End correction of a circular perforation in a thin plate without mean flow equals $\delta_0 = 0.85a$.

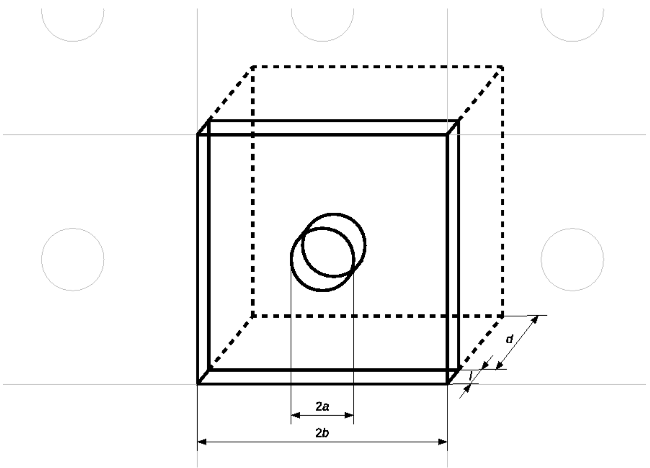

Figure 1: Element of a flat perforated liner.

In the presence of mean flow, such as bias flow through the perforation, part of the incoming sound energy is converted into the kinetic energy of the vortex rings, which are shed from the edges of the perforations. As the vortices are convected away from the edge, the energy is partly absorbed. This can be modelled with a complex value of Rayleigh conductivity. According to the simplified Howe's model [4] for an infinitely thin screen, it equals:

$$K_R = \frac{a^2 \pi \omega}{\omega a \pi/2 + jU/C_c^2}, \tag{2}$$

where U is speed of the bias flow and vena contracta factor C_c quantifies contraction of the cross section area of the jet. This value can be corrected for finite thickness of the perforated plate using the modified model by Jing and Sun [1, 5] to obtain:

$$K_{R,l} = 2a \left(\frac{2a}{K_R} + \frac{2l}{a\pi} \right)^{-1}. \tag{3}$$

For normal incidence, reflection coefficient of a perforated plate backed by a rigid wall at distance d from the plate can be calculated from [3]:

$$R = \frac{\omega/c_0 - j\sigma K_R[1 - j\cot(\omega d/c_0)]/(a^2\pi)}{\omega/c_0 + j\sigma K_R[1 + j\cot(\omega d/c_0)]/(a^2\pi)}, \tag{4}$$

where σ denotes perforation ratio and c_0 is speed of sound in air. Absorption coefficient of the entire assembly equals $\alpha = 1 - |R|^2$. In the following, we restrict ourselves to cold flows with $c_0 = 343$m/s.

Howe's model shows a good match with experimental results for very low Mach number $M = U/c_0$ and low perforation ratios [3], when the absorption coefficient approaches one at the resonance frequency of the liner and the interaction between the perforations is negligible. However, the accuracy seems to be lower at higher Mach number values [1], when the resonance is damped and the absorption is more broadband.

Simple model

As Howe's model, the model considered here is based on the inviscid and linearized theory of mean bias flow and unsteady acoustic perturbation through a single orifice [7, 8, 9]. Therefore, it holds for low perforation ratio (up to several percent) and low subsonic Mach number, when convection of sound is negligible. It is based on the quasi-steady approximation of the jet, which is valid for small Strouhal number value $S_t = \omega a/U = ka/M$. If the bias flow is exhausted from the cavity between the perforation and the wall to the exterior, and we set the coordinate axis in the same direction, linearized conservation of mass reads:

$$V \frac{\partial \rho'_V}{\partial t} + (\rho_0 v'_l + \rho'_i U) C_c a^2 \pi = 0, \tag{5}$$

where V is acoustically compact volume of air behind a single perforation, subscript $_V$ denotes value of a quantity in this volume, subscript $_l$ is used for the values inside the perforation, and $_i$ just outside it. Prime $'$ denotes a small perturbation. Here we used the same approach as in [8], that is, actual perforation is replaced with the one contracted by the factor C_c.

Linearized conservation of unsteady momentum gives:

$$\rho_0 l_e \frac{\partial v'_l}{\partial t} + \rho_0 U v'_l + p'_i - p'_V = 0. \tag{6}$$

Inside the volume V and outside the perforation, we suppose a homentropic flow. The equation of state gives $p'_V = c_0^2 \rho'_V$ and $p'_i = c_0^2 \rho'_i$. Using these equalities and inserting Equation (5) into time derivative of Equation (6) divided with $\rho_0 l_e$, we obtain:

$$\frac{\partial^2 v'_l}{\partial t^2} + \frac{U}{l_e}\frac{\partial v'_l}{\partial t} + \omega_0^2 v'_l + \frac{\omega_0^2 M}{\rho_0 c_0} p'_i + \frac{1}{\rho_0 l_e}\frac{\partial p'_i}{\partial t} = 0. \tag{7}$$

where

$$\omega_0 = \sqrt{\frac{C_c a^2 \pi c_0^2}{l_e V}} \qquad \left[\mathrm{s}^{-1}\right] \tag{8}$$

is the resonance frequency of the resonator with contracted perforation in the absence of mean flow.

After replacing $p'_i = \hat{p}_i e^{j\omega t}$ and $v'_l = \hat{v}_l e^{j\omega t}$, we can estimate impedance of the perforated liner with perforation ratio σ ($v'_i = \sigma C_c v'_l$):

$$Z = -\frac{\hat{p}_i}{\sigma C_c \hat{v}_l} = \frac{\rho_0 c_0}{\sigma C_c} \frac{\omega_0^2 - \omega^2 + jMc_0\omega/l_e}{M\omega_0^2 + jc_0\omega/l_e}. \tag{9}$$

The change of sign is due to the normal pointing into the liner, according to the definition of impedance, which has the direction opposite to the coordinate axis we used.

For compliance with Howe's model, we can calculate apparent Rayleigh conductivity of the backed plate using Equation (4):

$$K_R = -\frac{j\omega C_c a^2 \pi (1 - R)}{c_0 \sigma [R + 1 + j(R-1)\cot(\omega d/c_0)]}, \tag{10}$$

with $R = (Z - \rho_0 c_0)/(Z + \rho_0 c_0)$ and $a^2\pi$ replaced with $C_c a^2 \pi$.

From the Equations (10) and (9), it follows that the quantities of special interest are vena contracta factor C_c and end correction δ in the expression for effective length l_e. Their values are fairly accurately known in the absence of mean flow, but can be substantially different in the case of bias mean flow. Moreover, they can be affected by the interaction between perforations at higher perforation ratios or Mach numbers, as well as geometrical details of the assembly. Computed or measured values can be used in specific problems to take this into account.

Results and discussion

The model presented in the previous section is tested for estimation of sound absorption of a liner in a circular duct. Absorption coefficient of the liner with circular geometry is calculated using the method of Eldredge and Dowling [10] in a similar manner as in [1], with neglected grazing flow. The method uses geometry parameters (radius and length of the perforations, total length and radius of the liner, and radius of the duct), Rayleigh conductivity of the perforations, and perforation ratio as the input parameters. Output values of absorption coefficient α represent fraction of the incoming sound energy which is dissipated by the liner, rather than transmitted or reflected back.

According to [3], appropriate vena contracta factor values are in the range 0.61 to 0.64. However, somewhat larger value $C_c = 0.75$ has been reported in [4] to agree well with the experiments. This is also in a good agreement with the measurement results given in [7] for a perforated plate with perforation ratio 0.25. For initial approximation, we assume that end correction has half of the value in the absence of mean flow, $\delta = \delta_0/2$. That is, $l_e = l + \delta_0$ with zero end correction inside the jet (neglected inertia inside the jet [9]). Still, it should be noted that even lower values of the ratio δ/δ_0 may appear [7], depending on the mean flow velocity, details of the geometry of perforations, and Strouhal number value. Theoretical model and experimental data for circular apertures with bias flow presented in [11] show that for Strouhal number value close to 1, end correction can be up to three times lower than without the mean flow.

Figure 2 compares Howe's model for finite thickness of the plate, which results in Equation (3), simple model considered here, Equation (10), and the experimental results from [1]. Inner radius of the liner is 70mm and its length is 66mm. Thickness of the perforated circular shell is 1mm. Radius of the perforations is 1.25mm and the perforation ratio is 0.01. Vena contracta factor equals $C_c = 0.75$ while end correction is $\delta = \delta_0/2 = 0.85a/2$. The comparison is made for two Mach number values of the bias flow, $M = 0.029$ and $M = 0.049$.

For the lower value of Mach number, resonance of the liner is pronounced, which results in the clear peak of sound absorption. Howe's model seems to underestimate both absorption and the resonance frequency. In contrast to this, simple model provides a good estimation of sound absorption around the resonance frequency. The calculated curve is somewhat narrower, which leads to higher discrepancies above and below the resonance frequency. At higher mean flow velocities, absorption becomes broadband without a clear peak. Although, both models are less suitable for this working regime of the liner, simple model generally gives more accurate approximation of the absorption coefficient.

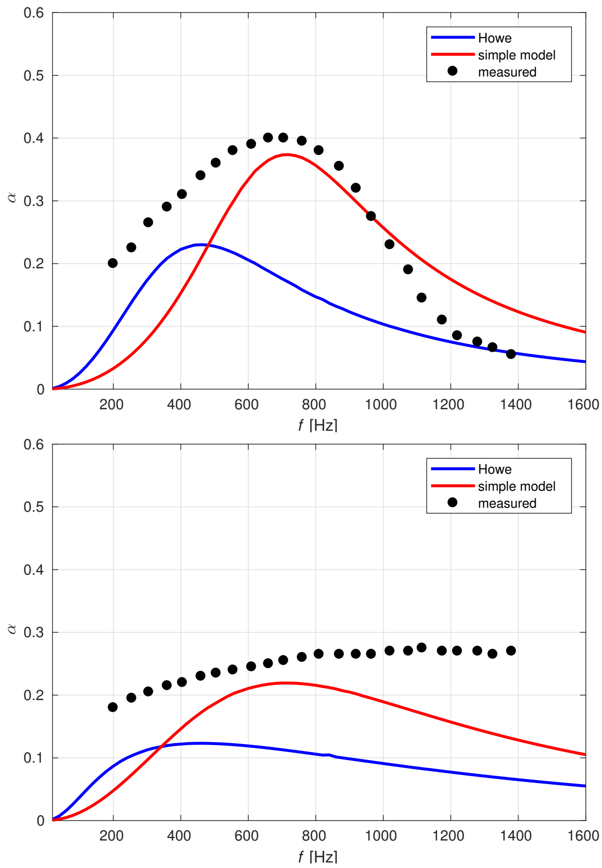

Figure 2: Estimated and measured [1] absorption coefficient of a liner in a circular duct: (above) $M = 0.029$, (below) $M = 0.049$.

The influence of end correction and vena contracta factor is demonstrated in Figure 3. End correction takes three values, δ_0, $\delta_0/2$, and $\delta_0/4$, while vena contracta factor is tested for values 0.7, 0.75, and 0.8. Higher vena contracta factor increases the maximum absorption at only slightly increased resonance frequency. Compared to this, end correction affects mainly the resonance frequency, as shorter effective length of the perforations leads to higher resonance frequency, while the maximum absorption is unaffected. The curves in Figure 3 also indicate that for the analysed geometry and flow conditions, end correction is approximated well with $\delta_0/2$ and vena contracta factor is likely between 0.75 and 0.8.

Quasi steady approximation assumes $S_t < 1$, which, for a given frequency range, sets the minimum value of Mach number around $M_{min} \approx k_{max}a$. Accuracy of the model appears to be the highest around the resonance frequency f_0. Accordingly, Figure 4 shows difference between calculated and measured [1] values of absorption coefficient as a function of Strouhal number and normalized frequency

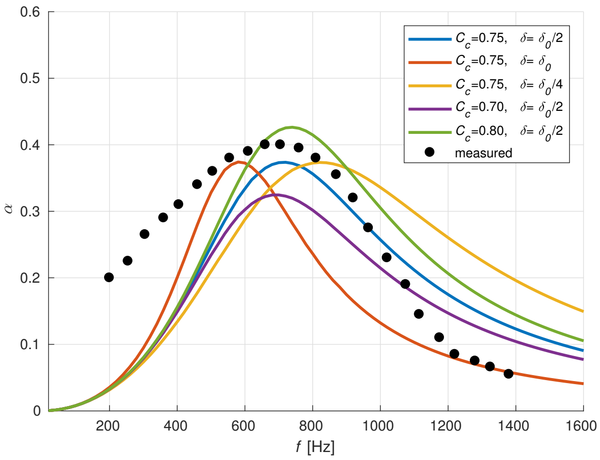

Figure 3: The effect of end correction and vena contracta factor on the estimated values of absorption coefficient.

f/f_0. Besides the two scenarios from Figure 2, additional curve is shown for higher porosity $\sigma = 0.068$ and Mach number value $M = 0.127$. Absorption coefficient curve is qualitatively very similar to the one in Figure 2 (below), except that the simple model slightly overestimates the absorption. The calculated values are reasonably accurate for $S_t < 1$ and two octaves above and below the resonance frequency.

Conclusion

The model considered here can be used for estimation of sound absorption by perforated liners with bias flow at small Strouhal numbers, when the resonance of the liner is not too damped. It is less suitable for estimation of broadband absorption, when other models should be used. Applicability of the model for higher Mach number values and porosities, which imply stronger interaction between the perforations, should be further investigated. Future work may also include grazing and heated flows, as well as the second liner of double-skin liner configurations.

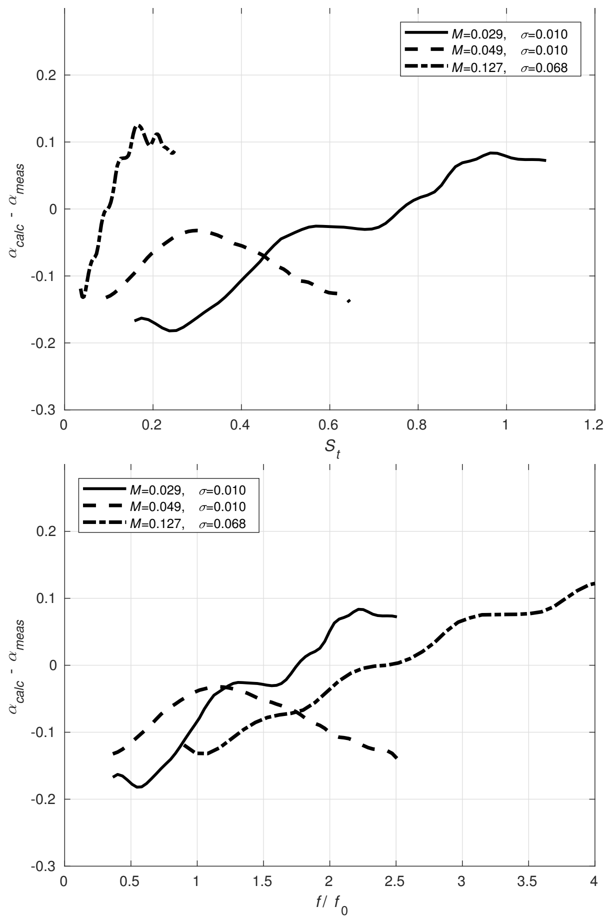

Figure 4: Estimated error as a function of Strouhal number (above) and normalized frequency (below) for $C_c = 0.75$ and $\delta = \delta_0/2$.

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