Efficient Simulation of Acoustic Measures using Krylov Subspaces and System Simulation

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Abstract

In this work, a method is introduced, which makes it possible to strongly reduce large-scale, structural dynamic Finite Element (FE) models using Krylov Subspace Method (KSM) in order to simulate its full 3D vibrational behavior within the system simulation and to evaluate the acoustic behavior directly on the reduced model. First, the determination of optimal reduction parameters is introduced in order to achieve smallest system size at high accuracy of the system. Second, a method is outlined to directly evaluate the acoustic behavior, namely the Equivalent Radiated Power (ERP), out of the reduced system space. With this procedure, it is possible to efficiently evaluate the acoustic behavior of a system, not only for different specific frequencies and loads, but also for specific load cycles in the time domain, e.g. the behavior of curb crossing or the full lap of a test circuit, which would lead to an untenable amount of resources using common methods. Via the example of a gearbox housing of a full electric vehicle, a comparison to results from a complete 3D FE model is drawn.

Introduction

1D system simulation is widely used for the powertrain system mechanics in the automotive area. It is used in order to efficiently evaluate overall system behavior, such as vibrational behavior, already in the early design phase. The high efficiency is based on the abstraction of the system, leading to very few degrees of freedom. An example of a powertrain model is shown in figure 1, where the simplified representation of the components is illustrated. It is this high degree of abstraction that usually permits the detailed evaluation of local effects in the 3D space, e.g. the acoustic evaluation of the system. During this work, a method will be introduced which combines the advantage of the high degree of abstraction in the system simulation with the high degree of accuracy of the Finite Element Method (FEM) in order to efficiently simulate the acoustic behavior with high accuracy directly in system simulation.

Model Order Reduction

Several Model Order Reduction (MOR) techniques can be used to reduce the calculation effort of a FE model by approximating the original model behavior as far as possible. Thereby the system order is decreased and a speedup in simulation time is reached compared to the

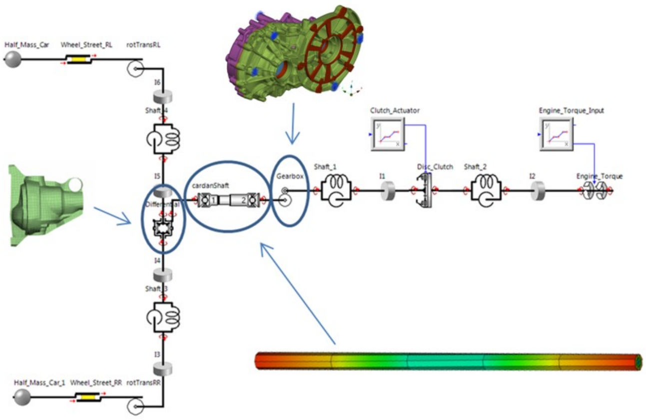

Figure 1: Simplified example of the mechanical part of a drivetrain, modeled in ITI's SimulationX.

simulation of the original model. For all methods using the projection into a subspace, the following holds.

A linear elastic structural dynamic system has to be reduced with n degrees of freedom, described by the equation [3]:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F} \tag{1}$$

with $\mathbf{M}, \mathbf{C}, \mathbf{K} \in \mathbb{R}^{nxn}$, $\mathbf{M}$ being the mass, $\mathbf{C}$ the damping, $\mathbf{K}$ the stiffness matrix, $\mathbf{F} \in \mathbb{R}^{nx1}$ the load vector and $\mathbf{x(t)}$ the displacement vector with its derivatives in time.

To reduce this system, a projection to a subspace $\mathbf{V} \in \mathbb{R}^{mxm}$ with typically $m << n$ is done. For

$$\mathbf{x}(t) \approx \mathbf{V}\mathbf{x_r}(t) \tag{2}$$

equation (1) becomes

$$\mathbf{M_r}\ddot{\mathbf{x_r}}(t) + \mathbf{C_r}\dot{\mathbf{x_r}}(t) + \mathbf{K_r}\mathbf{x_r}(t) = \mathbf{F_r} \tag{3}$$

with $\mathbf{M_r} = \mathbf{V^T M V}$, $\mathbf{C_r} = \mathbf{V^T C V}$, $\mathbf{K_r} = \mathbf{V^T K V}$, $\mathbf{F_r} = \mathbf{V^T F}$ and $\mathbf{x_r}(t)$ being the solution vector in the reduced space. This reduced system can then be solved with much less computational effort for the reduced degrees of freedom. The solution can afterwards be upscaled via the transformation matrix $\mathbf{V}$ and equation (2) into the original space. In the following, the reduction of a linear time-invariant system using the projection into the non physical Krylov Subspace is described in more detail.

Krylov Subspace Method

As there are plenty of methods, only the used KSM is explained in more detail. A deeper insight to other methods can be found in literature e.g. [1] or [2]. Besides the widely used and well-established Modal Truncation and Component Mode Synthesis (CMS), the KSM is a promising subspace method as it is independent from the physical space of the system and only approximates the transfer function $\mathbf{H}(s)$ of the system, which can be, according to Soppa [4], expressed and approximated for the frequency s and the frequency at the expansion point s_0 using:

$$\mathbf{H}(s) = \mathbf{C}(s\mathbf{E} - \mathbf{A})^{-1}\mathbf{B} =$$

$$= \sum_{i=1}^{\infty} -\mathbf{C}\left((\mathbf{A} - s_0\mathbf{E})^{-1}\mathbf{E}\right)^i (\mathbf{A} - s_0\mathbf{E})^{-1}\mathbf{B}(s - s_0)^i. \quad (4)$$

This can be simplified to

$$\mathbf{H}(s) = \sum_{i=1}^{\infty} \mathbf{H}_i (s - s_0)^i \quad (5)$$

with

$$\mathbf{E} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix}, \ \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K} & -\mathbf{D} \end{bmatrix} \ , \ \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{F} \end{bmatrix} \quad (6)$$

and the so-called moments of the transfer function

$$\mathbf{H}_i(s) = -\mathbf{C}((\mathbf{A} - s\mathbf{E})^{-1}\mathbf{E})^i(\mathbf{A} - s\mathbf{E})\mathbf{B}. \quad (7)$$

For the reduction, a Krylov subspace of the dimension q is used, which can be generally defined as:

$$K_q(\mathbf{P}, \mathbf{Q}) = \mathbf{span}\left\{\mathbf{Q}, \mathbf{PQ}, \mathbf{P}^2\mathbf{Q}, ..., \mathbf{P}^{n-1}\mathbf{Q}\right\} \quad (8)$$

with a quadratic matrix $\boldsymbol{A} \in \mathbb{R}^{k \times k}$, an arbitrary starting vector $\boldsymbol{Q} \in \mathbb{C}^{k \times 1}$ and the cardinality of the subspace $n \in \mathbb{N}$. With the use of $\mathbf{P} = \mathbf{A^{-1}E}$ and the start vector $\mathbf{Q} = -\mathbf{A^{-1}B}$, the moments of the Krylov subspace can be used to match the moments of the transfer function of the system. This new Krylov basis normally gets more and more ill-conditioned as n increases [5]. Therefore, the Krylov vectors $\boldsymbol{Q}, \boldsymbol{PQ}, \boldsymbol{P}^2\boldsymbol{Q}, ...$ have to be transformed, here using the Second Order Arnoldi algorithm (SOAR), into a new set consisting of m basis vectors, with $m < n$ [4]. One advantage of the use of Krylov subspaces in combination with the SOAR is, that not only one subspace at one specific expansion point can be used. If there is more than one expansion point used, a Krylov space can be generated for each point and then all subspaces are assembled to one basis using the SOAR in the order of moments, meaning that all first order moments are orthogonalized in the first loop, then all second moments successively. According to Lein [6], this leads to an efficient overall approximation of the system.

Choice of Expansion Points

In order to achieve high accuracy of the reduced model with a low reduced system size m, not only the method for the construction of the basis from the subspaces is important, also the choice of expansion points is crucial for an efficient reduction using the KSM. As stated by Grimme [8], the following three statements can be made for the behavior of expansion points:

- Expansion points that are purely imaginary, result in a very good local approximation of the frequency response function (FRF), but show a slower convergence at frequencies distant from the expansion point.
- Purely real expansion points lead to a more uniform approximation over a large frequency area as purely imaginary expansion points.
- The combination of more than one point results in a better approximation of the system as one single point.

In the following, a new method is introduced, using these properties to get accurate and small reduced models without iterative reduction processes using the modal information of the system.

In order to distribute the expansion points over the frequency range of interest, the behavior of the combination of purely imaginary points is investigated here in more detail. Therefore, a FE model of a differential housing consisting of roughly $16,000$ DOFs having nine Inputs and Outputs (IO) is considered, which is shown in figure 2. The behavior of imaginary expansion points are

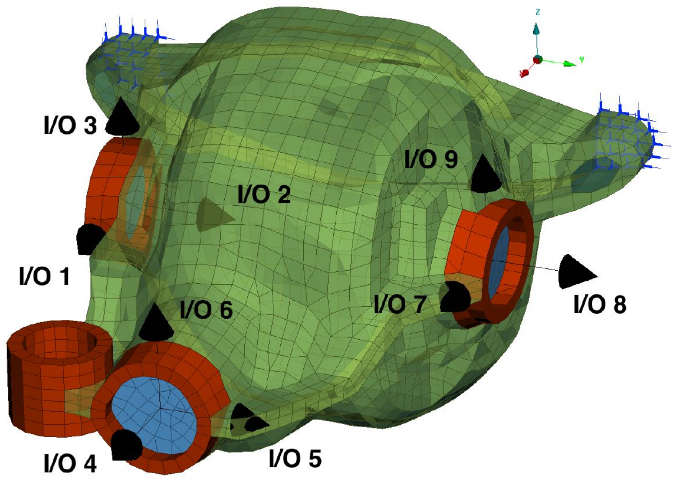

Figure 2: FE model of the differential housing of the Ford Explorer model from the NCAC [9] with roughly 16,000 DOFs.

investigated. Therefore, two expansion points are defined at $\pm 3000i$ Hz and $\pm 6000i$ Hz. A reduction is conducted for each of the points separately increasing the number of expansion length q. A third reduction is conducted, using the combination of both points. The results of the model, shown in figure 3, indicate, that the combination of two points improves the approximation behavior from an error of nearly 100 % to an error below 1 % around $4,500$ Hz for the same system size, even if the isolated points are not approximating the system in that area.

Hence, it can be stated, that areas of single imaginary expansion points with poor approximation quality distant from the expansion frequency, can be combined with neighbor points in order to achieve a better approximation in the combination, than the single points can achieve independently.

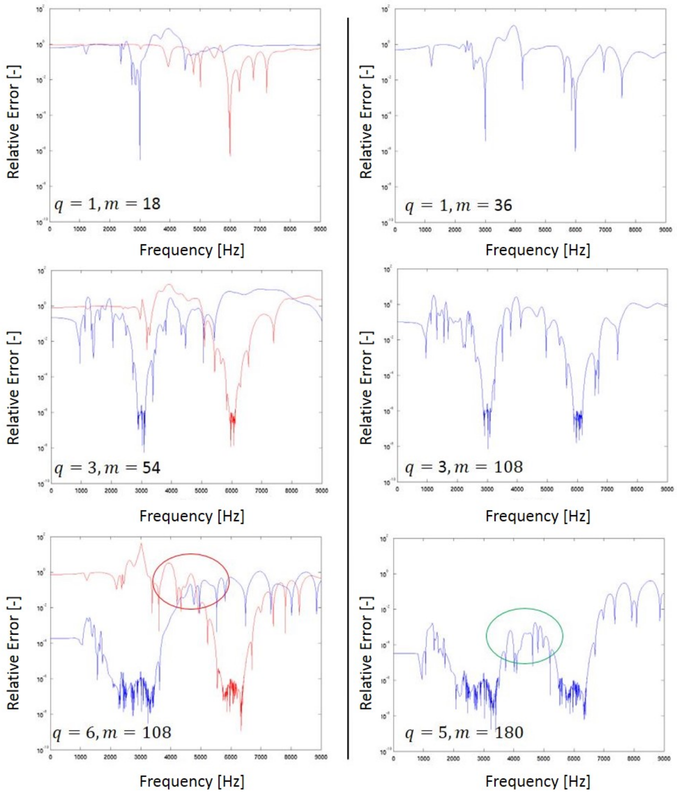

Figure 3: Relative error of the frequency response function for two expansion points separated (left, blue: $3000i$ Hz and red: $6000i$ Hz) and combined (right) with increasing expansion length q and system size m of the differential housing.

The idea of the new method is based on two facts. First, the property that KSM approximates the behavior of the transfer function is considered. As the transfer function shows quite constant behavior in areas without eigenfrequencies and is discontinuous in areas with eigenfrequencies, the information density is high in vicinities of eigenfrequencies of the system. Hence it is advantageous to place the expansion points close to eigenfrequencies of the system, as shown in figure 4. Second, the system size increases by $2qn_{I/O}$ for each expansion point, where $n_{I/O}$ is the number of inputs and outputs of the system. This holds as long as the vectors of the Krylov subspace are orthogonal, which is mostly the case for the described strategies [6]. As both, the expansion length as well as the number of expansion points, increases the quality of the system, it might be advantageous to choose less points and place them according to the modal density of the system instead of increasing their number and size and distribute them constantly in the frequency range. In addition to that, the study performed indicates that the combination of points is of advantage. Considering these two facts, a strategy is proposed here for the a priori choice of expansion points. For reasons of system size, only two points are chosen for the reduction process. Nevertheless more points can be used using the same strategy. Considering the modal distribution of the system, these two points are positioned at the frequency of the $\frac{1}{6}th$ and $\frac{5}{6}th$ mode in the relevant frequency range. This should lead to a good approximation of the edges of the frequency domain and the superposition of the points may lead to a good approximation of the middle area.

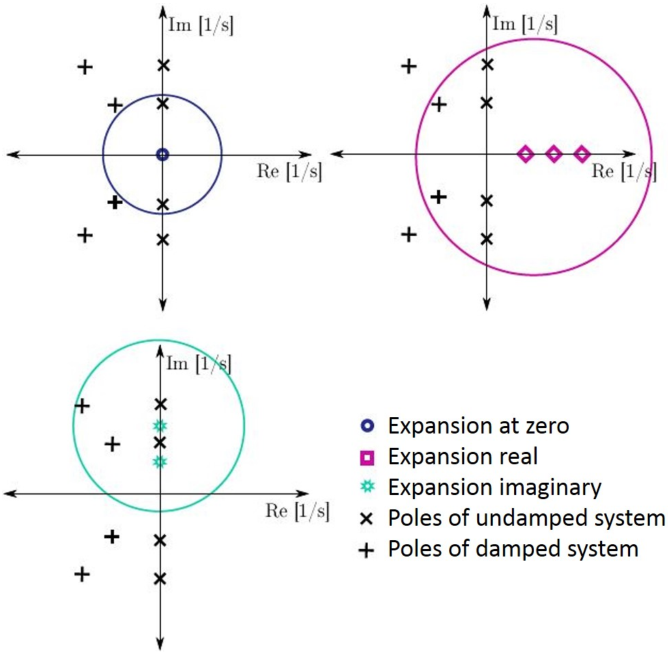

Figure 4: Convergence radius of the different methods according to Grimme [8].

ERP derivation

Using the method introduced above, it is possible to efficiently implement the reduced matrices of the model directly into the system simulation. In order to gain directly an acoustic measure out of the system simulation without the effort of projecting into the full space again, the ERP integral can be derived directly via the reduced matrices:

$$ERP = \frac{1}{2}\rho_F \nu_F \int |\mathbf{v_n}|^{\mathbf{2}} dA \qquad (9)$$

with ρ_F being the density of the surrounding fluid and ν_F its speed of sound, whereas $|\mathbf{v_n}|^{\mathbf{2}}$ is the absolute value of the surface normal velocities corresponding to the surface area A. The integration is formulated directly as a matrix operation of the element distribution, the normalization, and the sum of all areas with the according normalized surface velocities, resulting in an ERP-Matrix $\mathbf{E} \in \mathbb{R}^{mxm}$ of the size of the reduced model. This resulting matrix can be directly multiplied by the reduced solution vectors and thus leading to the ERP value for each time step directly derived from the reduced model within the system simulation environment.

Results

The overall described method was applied to the FE model of a gearbox housing of a full-electric vehicle, shown in figure 5. It was reduced and the resulting matrices were set into the system simulation environment. In the system simulation, a load of a linear harmonic sweep to one input was applied in the time domain, in order to be able to compare it to the frequency response function of the full FE model. Then, the resulting ERP value for each frequency was calculated. For the FE model, it was calculated in the full space whereas the ERP value of the reduced model was calculated directly in the reduced space as described. The results of both models are shown in figure 5. As it can be seen, the FE model

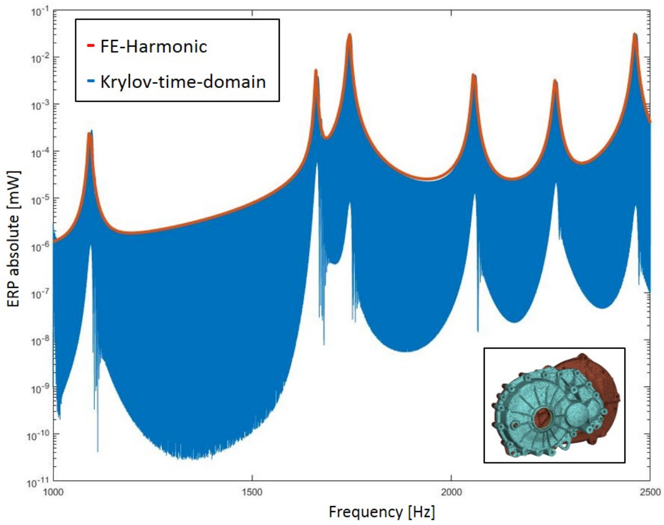

Figure 5: Resulting ERP value of the full model directly in the harmonic space (red) and the reduced model converted from the time domain (blue).

results in one curve over the frequency, consisting of one value for each frequency calculated, whereas the reduced model in system simulation shows a high oscillation in the frequency domain. This effect comes directly from the fact, that the system simulation was performed directly in the time domain. Therefore, the ERP value must oscillate between zero and the full FE-solution, as the surface velocities are not at maximum all the time, they have to go to zero as it is an surface oscillation. For areas with high damping, close to the eigenfrequencies, the oscillation is not as high as in areas where damping is not so relevant, because the higher damping results in a phase shift of the surface velocities leading to a blurred but increased background noise, as not all velocities are zero at one time. For both models, the eigenfrequencies of the system are clearly visible as peaks in the ERP response function. The resulting error is below 5% over the overall frequency range, the eigenfrequencies are all met with high accuracy as well as the absolute value shows high accordance. The resulting error might be further reduced by decreasing the frequency ramp in the time domain as it is assumed to have a perfectly steady-state response at each frequency.

Conclusion

In this work, a method for the simulation of a reduced FE model directly via system simulations in the time domain was introduced. The most promising parameters for the reduction using KSM were shown and an approach to an a priori derivation of the location of the expansion points was given, taking the modal density of the system into account. By use of a matrix multiplication, it is possible to calculate the ERP value directly on the reduced model in system simulation. The comparison to the full FE model showed high agreement between the results from the new method in system simulation and the harmonic FE analysis. Further, it is now possible to simulate the ERP in the time domain which leads to new effects, such as the oscillation of the ERP. Future work might be necessary to investigate these effects in more detail to reveal the full potential of the new method, such as the simulation of acoustic emission in real time or the evaluation of acoustics long-lasting driving cylces and other specific transient effects.

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