Acoustic topology optimization of porous material distribution by FMBEM–based sensitivity analysis

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Introduction

Porous materials have been widely applied in the noise control, including the sound barrier [1]. However, it is usually not practical to apply a full-coverage design of porous material layer due to various industrial limits. Thus, it is highly desired to obtain the optimal material distribution under a given constraint against vibration and noise, which consists in the problem of topology optimization. In our previous work [2], we optimized the distribution of porous material on the surface of half-Y shaped noise barrier by the boundary element method (BEM). But the sensitivity analysis based on the conventional BEM represents the bottleneck in computational efficiency. Therefore, we develop a fast analysis method to unfreeze the limit caused by the conventional BEM. In this paper, both the acoustic analysis and the sensitivity analysis are accelerated by the fast multipole method (FMM). We validate the proposed topology optimization approach through some numerical examples.

Optimization model

The common optimization problem subjects to a given material volume fraction is formulated as follows:

$$\begin{cases} \min: & \Pi = \Pi(\mathbf{p}, \mathbf{p}_f) \\ \text{s.t.}: & \sum_{e=1}^{N_e} \rho_e v_e - f_v \sum_{e=1}^{N_e} v_e \leqslant 0, \\ : & 0 \leqslant \rho_{\min} \leqslant \rho_e \leqslant 1, \\ : & (\mathbf{H} - \mathbf{GB})\mathbf{p} = \mathbf{p}_i, \\ : & \mathbf{p}_f = -\big(\mathbf{H}_f - \mathbf{G}_f\mathbf{B}\big)\mathbf{p} + \mathbf{p}_f^i, \end{cases} \tag{1}$$

where an objective function Π is expressed as the function of sound pressures $\mathbf{p}$ and $\mathbf{p}_f$, ρ_e and v_e denote the artificial density and volume of element e ($e = 1, 2, \ldots, N_e$) respectively, f_v denotes the volume fraction constraint, and $\rho_{\min}$ is the lower bound to avoid singular value in the calculation. The last two equations in the constraint are the governing equations of the BEM [2]. The method of moving asymptotes (MMA) [3] is employed to solve the optimization problem. Thereby, the sensitivity information is necessary.

Sensitivity analysis

Rewriting the objective function with the governing equations as:

$$\begin{aligned} \Pi =& \Pi(\mathbf{p}, \mathbf{p}_f) + \boldsymbol{\lambda}_1^{\mathrm{T}}\Big[\big(\mathbf{H} - \mathbf{GB}\big)\mathbf{p} - \mathbf{p}_i\Big] + \\ & \boldsymbol{\lambda}_2^{\mathrm{T}}\Big[\mathbf{p}_f + \big(\mathbf{H}_f - \mathbf{G}_f\mathbf{B}\big)\mathbf{p} - \mathbf{p}_f^i\Big], \end{aligned} \tag{2}$$

where $\boldsymbol{\lambda}_1^{\mathrm{T}}$ and $\boldsymbol{\lambda}_2^{\mathrm{T}}$ are the adjoint variable vectors with the degrees of freedom (DOF) of the structure and field points, respectively. Differentiating Eq. (2) with respect to design variable and collecting terms $\partial\mathbf{p}/\partial\rho_e$ and $\partial\mathbf{p}_f/\partial\rho_e$, we have the adjoint equation,

$$\begin{cases} \mathbf{z}_2^{\mathrm{T}} + \boldsymbol{\lambda}_2^{\mathrm{T}} = \mathbf{0} \\ \mathbf{z}_1^{\mathrm{T}} + \boldsymbol{\lambda}_1^{\mathrm{T}}\big(\mathbf{H} - \mathbf{GB}\big) + \boldsymbol{\lambda}_2^{\mathrm{T}}\big(\mathbf{H}_f - \mathbf{G}_f\mathbf{B}\big) = \mathbf{0} \end{cases} \tag{3}$$

where $\mathbf{z}_1^{\mathrm{T}}$, $\mathbf{z}_2^{\mathrm{T}}$ and z_3 are determined by the detailed forms of the objective function as

$$\frac{\partial\Pi}{\partial\rho_e} = \mathbf{z}_1^{\mathrm{T}}\frac{\partial\mathbf{p}}{\partial\rho_e} + \mathbf{z}_2^{\mathrm{T}}\frac{\partial\mathbf{p}_f}{\partial\rho_e} + z_3, \tag{4}$$

where $\mathbf{p}_f$ is regarded as a separate variable independent of $\mathbf{p}$ for common expression in the following analysis, and z_3 does not contain terms $\partial\mathbf{p}/\partial\rho_e$ and $\partial\mathbf{p}_f/\partial\rho_e$. By setting the adjoint vectors as the above values, the calculations of derivatives ($\partial\mathbf{p}/\partial\rho_e$ and $\partial\mathbf{p}_f/\partial\rho_e$) can be avoided. The FMM is also applied to accelerate the vector-matrix product in the adjoint equation like that in the acoustic analysis. Finally, we have the sensitivity as

$$\frac{\partial\Pi}{\partial\rho_e} = \Re\left(z_3 - \boldsymbol{\lambda}_1^{\mathrm{T}}\mathbf{G}\frac{\partial\mathbf{B}}{\partial\rho_e}\mathbf{p} + \boldsymbol{\lambda}_2^{\mathrm{T}}\mathbf{G}_f\frac{\partial\mathbf{B}}{\partial\rho_e}\mathbf{p}\right), \tag{5}$$

where the derivative of the admittance matrix $\mathbf{B}$ can be obtained using the Delany-Bazley-Miki model [4] with rigid backing and the SIMP scheme [5].

Fast multipole method

The main idea of the acceleration is to apply the FMM to accelerate the matrix-vector or vector-matrix product during the iterations in iterative solver without explicitly stored dense matrix, which results in a significant reduction in required memory. The contributions from elements close to the source point are evaluated directly by integration in the usual way, whereas the FMM is used for elements that are far away from the source point. Details for the FMM can be found in References [6, 7].

Single circle design example

The design domain consists of an unit circle and point source at point A as shown in Figure 1. The mass density and sound velocity of the acoustic medium (air) are $\rho_{air} = 1.21$ kg/m^3 and $c = 340$ m/s. The flow resistivity and thickness of porous material are $\sigma = 10^4$ N $\cdot$ /m^4 and $d = 0.05$ m. The excitation frequency is set as $f_p = 500$ Hz.

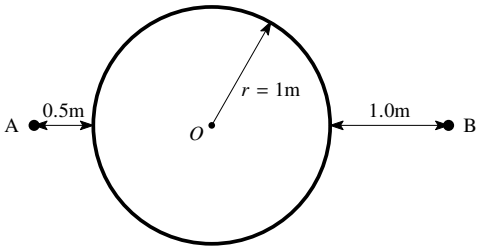

Abbildung 1: The computational domain for the optimization problem.

The cylinder is firstly discretized into 10000 constant elements, and the artificial densities of all porous elements are given a uniform value of 1. Table 1 gives the detailed computational costs of the two approaches for one iteration in the optimization. We can see that the fast multipole BEM consumes much less costs than the conventional one, and this advantage will be further highlighted with a larger degrees of freedom (DOF).

Tabelle 1: Elapsed time and RAM usage for one iteration in the optimization.

Item	Time (s)	RAM (MB)
Conventional BEM	64.4	3104.7
Fast multipole BEM	13.2	75.6

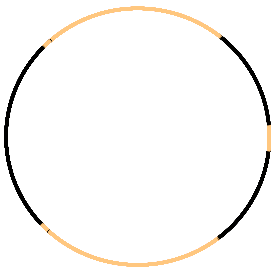

Abbildung 2: The optimal design for the optimization problem.

Figure 2 illustrates the optimal design for the optimization problem. The deep color represents the surface is covered by the porous material, and the light color represents it's not covered by the porous material, i.e., it is sound rigid. Then we consider two different excitation frequencies, $f_p = 400, 600$ Hz. Comparing the three designs in Figures 2, 3 and 4, we can see that the optimal design is strongly influenced by the loading frequency, in other words, the optimal design is frequency-independent.

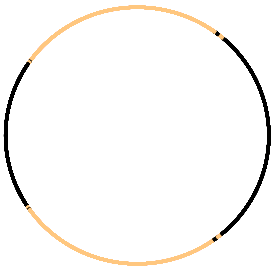

Abbildung 3: The optimal design at 400 Hz.

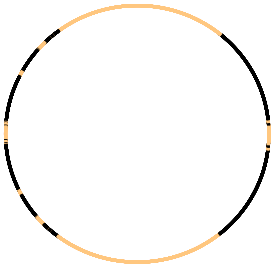

Abbildung 4: The optimal design at 600 Hz.

Conclusions

This paper investigates the topology optimization of porous material distribution on structure surface. The use of fast multipole boundary element method (FMBEM) improves the overall computational efficiency and allow its application in the large-scale problems. Several numerical examples are presented to demonstrate the validity and correctness of the present algorithm. Results show that the optimal distribution is strongly frequency dependent.

Literatur

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