Uncertainty Quantification of numerical transmission loss calculations of an aircraft fuselage section

Shreyas M. Guruprasad[1], Christopher Blech[1], Ulrich Römer[2]
Hermann G. Matthies[3], Sabine C. Langer[1]
[1] *Institut für Konstruktionstechnik, TU Braunschweig, Email: c.blech@tu-braunschweig.de*
[2] *Institut für Dynamik und Schwingungen, TU Braunschweig*
[3] *Institut für Wissenschaftliches Rechnen, TU Braunschweig*

Introduction

In early design stages of aircraft, uncertain design parameters are very common. Due to continuous investigation of newer technologies in research, the final design of the aircraft may differ considerably from its initial preliminary design. Nevertheless, a prediction of aircraft cabin noise is important in the preliminary design stages as potential acoustical problems can be detected in these phases to consider damping measures as early as possible. The wave-based approaches are used instead of energy-based methods (which are much faster) as it gives the opportunity to manage wave-based effects as innovative passive damping measures, for instance. The input parameters of deterministic wave-based models, which are used to predict the cabin noise of different aircraft configurations, are typically not known with high precision. This may limit the reliability and significance of the predictions in particular for mechanical models with a high complexity. To overcome this shortcoming, uncertainties in the input and their effect on output parameters of the system should be considered in the simulation. By modelling design parameters as random variables, the deterministic reference model is transformed into a probabilistic model and appropriate numerical techniques for stochastic computations need to be applied [1, 2].

In this contribution, an uncertainty quantification and a global quantitative sensitivity analysis are applied on the example of a numerical transmission loss calculation of an aircraft fuselage section. A non-intrusive surrogate modeling technique is applied in frequency domain to reduce the computational costs significantly. The convergence of the problem is investigated by different polynomial orders and different numbers of samples.

Framework of Uncertainty Quantification

The following process is given in literature [3] and modified to five major steps for the acoustic application. The consecutive steps are shown in Figure 1 and further elaborated in the following sections. The main goal is a global *quantitative sensitivity analysis* to understand the effect of parameter changes on the model's response. An important partial result may be a *statistical analysis* which gives an uncertain frequency response. For the linear frequency domain analysis, this procedure is applied to each frequency step separately.

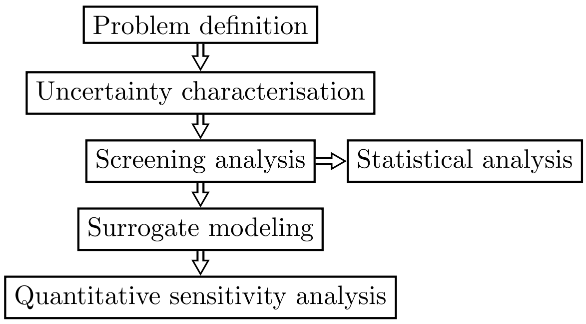

Figure 1: Flowchart of Uncertainty Quantification [3]

Problem definition

The problem of interest in this contribution is a mechanical formulation of the sound transmission through a double-wall section of an aircraft fuselage. Figure 2 shows the simplified cross-section of an aircraft fuselage with the modelled section marked by a black line.

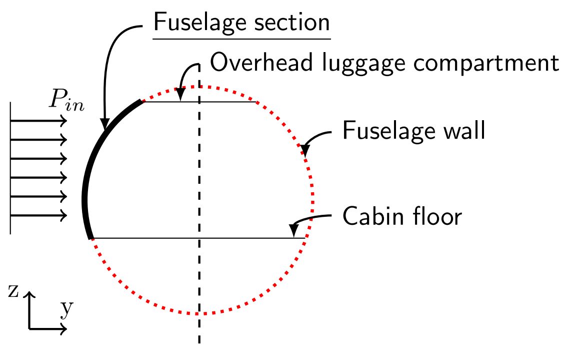

Figure 2: Cross-sectional view of the Aircraft fuselage

The Transmission Loss (TL) is given as

$$\mathrm{TL} = 10\ \log_{10}\left(\frac{P_{in}}{P_{out}}\right)\mathrm{dB} \tag{1}$$

The formula considers the ratio of the incident sound power P_{in} to the radiated sound power P_{out}. As stated in DIN EN ISO 10140-2 for measurements, the incident sound power must be calculated without the influence of the separating component and the radiated sound power has to be calculated for free field conditions. A plane wave excitation is chosen for which input power can easily be calculated by

$$P_{in} = \frac{\tilde{p}^2 A}{\rho c} \tag{2}$$

$\tilde{p}$ is the effective sound pressure amplitude, A the excited surface, ρ the density of the surrounding air and c the speed of sound on the outer surface. The radiated sound power is calculated according to

$$P_{out} = \frac{1}{2} \sum_{u=1}^{n} \mathrm{Re}\{p v_n^*\} \Delta A_u \tag{3}$$

Here, n patches correspond to the finite elements. ΔA_u is the surface of one patch u and Re denotes the real part. v_n^* is the complex conjugated normal velocity of the inner panel given by the finite element solution. The Rayleigh integral is applied to determine the sound pressure p for each panel (for simplicity, though the panel is slightly curved).

The generic double-wall fuselage section, shown in Figure 3, includes the primary outer skin, the inner lining of the cabin, circular frames and the insulation (see [4]). The outer skin and the frames are modelled by a classical shell formulation (Reissner-Mindlin plate & disc), the material is set to be orthotropic linear viscoelastic. The inner side of the sidewall panel consists of a sandwich structure made of a honeycomb core surrounded by GFRP and is modelled by a combination of a continuum and outer shells. The gap between the outer skin and the inner side is filled with glass wool. Its sound insulation effect is considered by an equivalent fluid approach. The model is deterministically solved in the frequency range of $10 - 1000\,\mathrm{Hz}$ with a step size $\delta_f = 10\,\mathrm{Hz}$ using an inhouse Finite Element (FE) implementation (ELPASO). Quadrilateral 9-node elements and 27-node hexahedrons are applied on the basis of a convergence study.

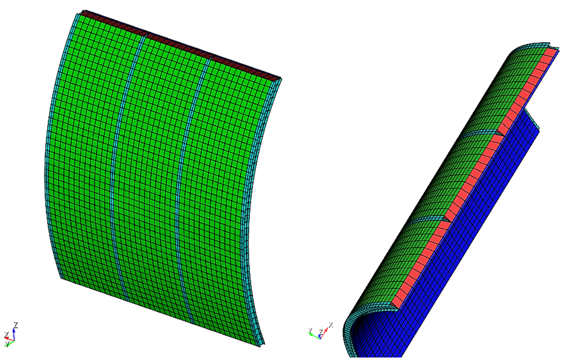

Figure 3: Discretised model of three sections of the fuselage connected by circular frames

Uncertainty characterisation

Uncertainty represents variability in data and is omnipresent due to manufacturing imperfections or the finite precision of measurement equipment. In this work, uncertainty in the specification of parameters of the numerical fuselage section model which leads to uncertain approximation of TL is considered, which is treated in a probabilistic setting. Hence, the parameters must be specified including the name, lower and upper bounds and the form of the probability distribution [3]. The distribution can be determined from measurement data using Bayesian inference. Here, in absence of data, a uniform distribution is assumed according to the principle of indifference or insufficient reason. In table 1, the parameters chosen to be uncertain in the mechanical model of the fuselage section are shown. Each parameter has a $\pm 10\%$ range of uncertainty with a uniform distribution. A variation of the material parameters is expected to have an increasing impact on the TL curve with increasing frequencies.

Table 1: Table of uncertain material parameters

No	Parameter	PDF	Unit
1	$\underline{t_{outSkin}}$	$U(0.0027, 0.0033)$	$[m]$
2	$\underline{E_{x:outSkin}}$	$U(6.714, 8.206 \cdot 10^{10})$	$[N/m^2]$
3	$\underline{E_{y:outSkin}}$	$U(4.824, 5.896 \cdot 10^{10})$	$[N/m^2]$
4	$\underline{\rho_{outSkin}}$	$U(1458, 1782)$	$[kg/m^3]$
5	$\underline{t_{GFRP}}$	$U(0.00045, 0.00055)$	$[m]$
6	$\underline{\rho_{GFRP}}$	$U(1980, 2420)$	$[kg/m^3]$
7	t_{Frame}	$U(0.0027, 0.0033)$	$[m]$
8	ρ_{Frame}	$U(1458, 1782)$	$[kg/m^3]$
9	$\underline{\mathrm{Re}(\rho_{glassWool})}$	$U(5.175, 6.325)$	$[kg/m^3]$
10	$\underline{\mathrm{Im}(\rho_{glassWool})}$	$U(-1.935, -2.365)$	$[kg/m^3]$
11	$\mathrm{Re}(c_{glassWool})$	$U(115.2, 140.8)$	$[m/s]$
12	$\mathrm{Im}(c_{glassWool})$	$U(25.65, 31.35)$	$[m/s]$

Screening analysis

The number of model parameters that need to be considered for dynamic models, such as the fuselage section, can be high. For such rather complex models, a screening step or initial qualitative sensitivity test is performed to filter out the less sensitive parameters from the higher sensitive ones in order to reduce the dimensionality of the problem. The screening step includes a Design of Experiments (DoE), uncertainty propagation and a local sensitivity analysis. DoE is a body of techniques that enable an investigator to conduct better experiments, analyse data efficiently and make the connections between the conclusions from the analysis and the original objectives of the investigations [3]. After performing DoE, uncertainty propagation should be followed by running the simulation model using the parameter sample sets that were generated. The twelve chosen parameters (see table 1) are coarsely varied one-at-a-time. Assuming these parameters to be independent from each other, 10 samples in the uncertainty range using Latin Hypercube Sampling (LHS) are chosen for each parameter (120 samples in total). The result of this *Morris test* is shown in Figure 4. The elementary effect (ee) is calculated using the mean sum level of variations and the sum level when no factor is varied, i.e. $ee(X^{(i)}) = f(X^{(i)} + \Delta X^{(i)}) - f(X)$. 6 parameters are identified, which are underlined in table 1. These parameters seem to be more sensitive compared to the others, as their corresponding points lie farer away from the origin. A **statistical analysis** of the model outputs for the chosen parameter set is performed. The parameters chosen from the Morris test form the 6 random variables of the

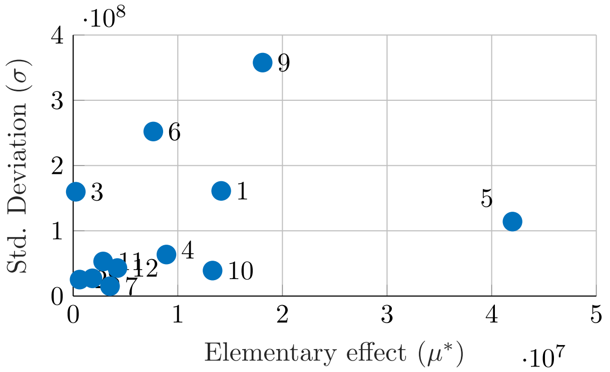

Figure 4: Morris test result for Transmission Loss

random vector (ξ). LHS is used to achieve a better exploration of the sample space (with lower discrepancy). This way a lower number of samples can be used to emulate the distribution functions. A Monte-Carlo (MC) analysis is performed by solving the deterministic system $N = 200$ times taking one realisation of RF $[\xi_1, \xi_2, ...\xi_6]$ at a time. The mean and standard deviation for TL is calculated at each frequency step ω_n. The uncertainty propagated due to the variation of input parameters for TL is shown in Figure 5. The TL increases up to around 70 dB towards the higher frequencies. The frequency response function (FRF) is consistently shifted slightly towards higher or lower values which can be seen from the 99% confidence region. At higher frequencies, the spread of the responses is around ±10 dB due to uncertain input parameters. Instead of a deterministic result of the system's TL, an area can be given as FRF, reflecting the uncertain nature of acoustic problems in early design stages.

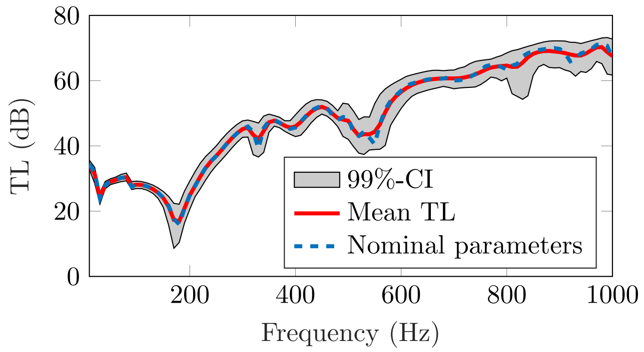

Figure 5: 99% confidence region by MC analysis compared with deterministic FEM solution at nominal values for TL

Surrogate modeling

If a simulation model such as an acoustical model requires significant computational resources to run, surrogate modeling is used to construct a simple statistical emulator of the response surface of the dynamical model. Surrogate modeling is a collection of mathematical and statistical techniques for an empirical model building. The empirical model describes the relationship between inputs and outputs (here: TL) [3]. Once a surrogate model has been constructed, subsequent analysis can rely on this inexpensive surrogate model.

Different surrogate models have been proposed in the literature. Gaussian process models and polynomial surrogate models are among the most popular ones. For the fuselage section, a polynomial approximation based on a least squares fit (regression) is built. Alternatively, the stochastic collocation or Galerkin method can be used. The regression approach collects the MC results from the FEM solver, which is viewed as a black box function f, relating the inputs to the outputs. The generic model structure of the surrogate approximation is $\hat{f}(\xi, \mathbf{a})$, the exact shape of the model being determined by the set of weighting coefficients $\mathbf{a}$. The aim is to estimate these weights and train the model such that it fits to the output. The surrogate model can be represented by,

$$\mathbf{Y}(\omega_n, (\xi_1, \xi_2, \ldots, \xi_6)) = \mathbf{Z}(\xi_1, \xi_2, \ldots, \xi_6)\mathbf{a}(\omega_n) + \epsilon, \quad (4)$$

where ϵ represents the approximation error, $\mathbf{Z} \in \mathbb{R}^{N \times Q}$ is the Vandermonde-like matrix, $\mathbf{a} \in \mathbb{R}^{Q}$ is the coefficient vector, $\mathbf{Y} \in \mathbb{R}^{N}$ is the response vector at each frequency step ω_n and Q is the number of coefficient terms in the polynomial approximation equation. Solving the system by minimising the residual by least square approach for finding $\mathbf{a}$ yields the following equations

$$\mathbf{a}(\omega_n) = \underbrace{(\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T}_{=:\ \mathbf{W}} \mathbf{Y}(\omega_n, (\xi_1, \xi_2, \ldots, \xi_6)) \quad (5)$$

$$\mathbf{a}(\omega_n) = \mathbf{W}\mathbf{Y}(\omega_n, (\xi_1, \xi_2, \ldots, \xi_6)) \quad (6)$$

For an unbiased estimator, cross-validation technique is used to divide the sample data set into k randomly chosen subsets of equal size. One set among them is chosen as the training set, to estimate the weighting coefficients $\mathbf{a}$ of the surrogate model. The remaining subsets are used to estimate the goodness of fit by calculating the RMSE and maximum absolute error (MAE). A generalised RMSE is the average of RMSE over k training subsets and is calculated by,

$$GRMSE = \frac{1}{k}\sum_{i=1}^{k} \epsilon_{RMSE}^{(\sim i)} \quad (7)$$

It can be seen in Figure 6 that the accuracy of the approximation increases by increasing the polynomial order.

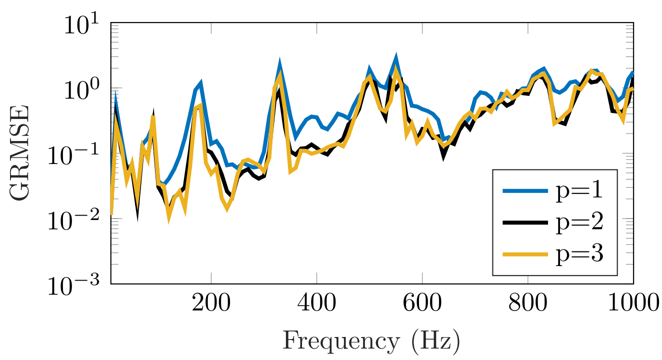

Figure 6: GRMSE for the TL approximation by varying the polynomial order at constant sample size (150)

The error sharply increases in regions of high non-monotonicity (resonances). The function cannot be approximated with sufficient accuracy due to this non-linearity in the frequency domain. Although increasing the order of the polynomial can improve the accuracy, it also increases the computational complexity as the number of coefficients (Q) in $\mathbf{Z}$ increases considerably.

Further, in Figure 7 the GRMSE is shown for different sample sizes for a constant polynomial order of 3. The error decreases significantly by increasing the sample size from the minimum of 84 to higher values. Similar to Figure 6, the error is not converging to zero. Instead, the polynomial approximation does not follow the sharp resonances in the TL curve.

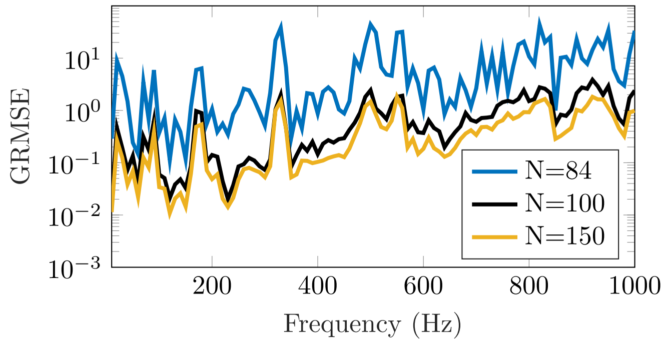

Figure 7: GRMSE for the TL approximation by varying the sample size at constant polynomial order (3)

Quantitative sensitivity analysis

Sensitivity analysis expresses how the uncertainty in the output of a model can be apportioned to different model input parameters. After the surrogate model is established, for the fuselage section, a variance-based approach is applied to determine the sensitivity of the TL to the chosen uncertain input parameters. It is realised by the usage of the Sobol' indices method. The first-order (main) index (S_i),

$$S_i = \frac{V(E[Y|X_i])}{V[Y]} \tag{8}$$

measures the contribution to the output variance by a single model input alone. The total-order index (S_{T_i})

$$S_{T_i} = S_i + \sum_{i \leq j \leq k} S_{ij} + S_{ij\ldots k} \tag{9}$$

measures the contribution caused by a model input, including both its first-order effects and all higher order interactions between parameters. The sensitivities are calculated based on the response of the surrogate model. If the total-order indices are much greater than the first-order indices, non-linear interactions between input parameters can be expected. The result of the total order sensitivity indices for the 6 input parameters of the fuselage model are shown in Figure 8 over the entire frequency range. At low frequency, a high influence of the real part of the density of the insulation layer which is modelled as equivalent fluid can be observed, which is indicating a sound transmission through the insulation at lower frequencies. At higher frequencies, the contribution of the outer skin's density ($\rho_{outSkin}$) and thickness are dominant over other parameters which indicates a high influence of the thin outer surface to the sound transmission.

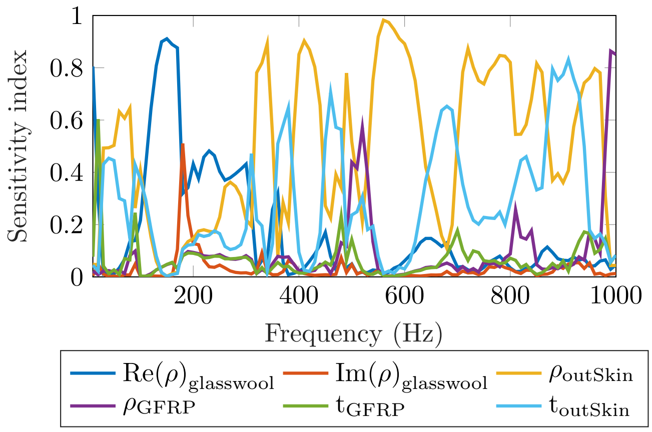

Figure 8: Total order Sobol' sensitivities calculated over the frequency range for TL

Conclusion and Outlook

A well known methodology for uncertainty quantification is applied to a mechanical model of an aircraft's fuselage section in frequency domain. Model parameter uncertainties are considered to finally quantify sensitivities in deterministic transmission loss calculations. The statistical results by the MC analysis with a 10 % parameter uncertainty yields a highly sensitive TL curve, especially in the higher frequency ranges. A surrogate model is applied to reduce the computational costs. It is observed in the frequency domain that the chosen polynomial approximation is inaccurate in the strongly non-monotonic resonances. A speed up may be reached by applying the frequency as an additional (certain) parameter which results in one single surrogate model for the entire frequency range. The problem of high dimensionality due to many engineering parameters can be tackled by a consideration of the physically known relationships of the parameters. Problem-specific parameters as the speed of sound or the bending stiffness could be considered instead of primary parameters as the Young's modulus. Furthermore, sparse grid collocation methods should be used to improve the approximation.

References

[1] Martin Krosche and Wolfgang Heinze. A robustness analysis of a preliminary design of a cestol aircraft. Technical report, Institute of Scientific Computing Technische Universität Braunschweig, Germany, 2014.

[2] O. P. LeMaitre and O. M. Knio. Spectral Methods for Uncertainty Quantification. Springer Verlag, New York, 2010.

[3] Chen Wang, Qingyun Duan, Charles H. Tong, Zhenhua Di, and Wei Gong. A GUI platform for uncertainty quantification of complex dynamical models. Environmental Modeling and Software, 76:1 15.12, 2016.

[4] Christopher Blech and Sabine C. Langer. Aircraft cabin noise reduction by means of acoustic black holes. in Inter.noise, Hongkong, 2017.